Lesson 16 part 2

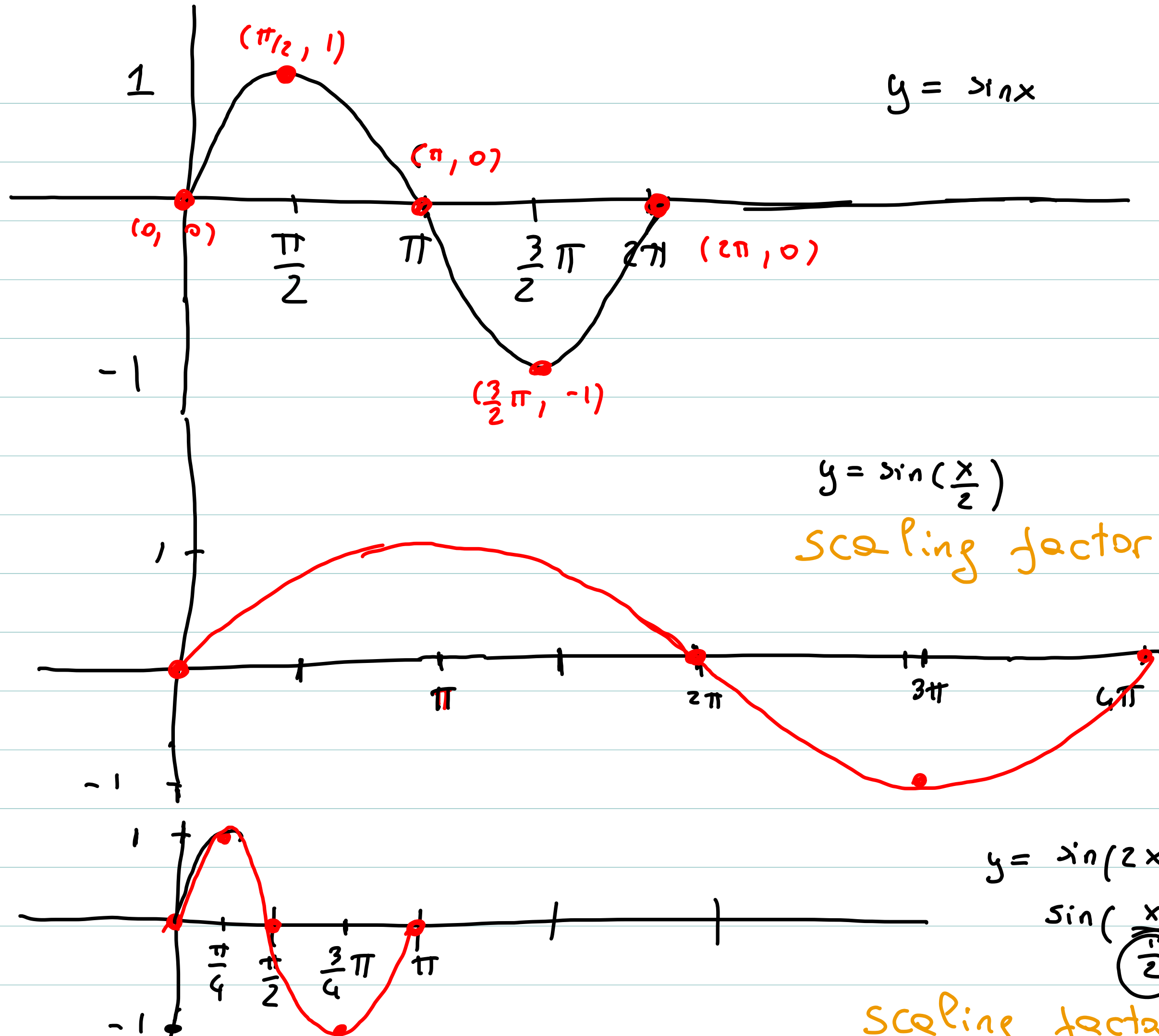
Chapter 13

Horizontal scaling (expansion or compression)

Multiply x by $\frac{1}{b}$ $(b > 0)$ i.e replace x with $\frac{x}{b}$

The graph of $f\left(\frac{x}{b}\right)$ is the graph of $f(x)$ stretched horizontally by a factor of b (if $b > 1$) or compressed horizontally by a factor of b (if $b < 1$)

If (x_1, y_1) on graph of $f(x)$, (bx_1, y_1) on graph of $f\left(\frac{x}{b}\right)$



$y = \sin x$

$y = \sin\left(\frac{x}{2}\right)$

scaling factor: 2

$y = \sin(2x)$

$\sin\left(\frac{x}{\frac{1}{2}}\right)$

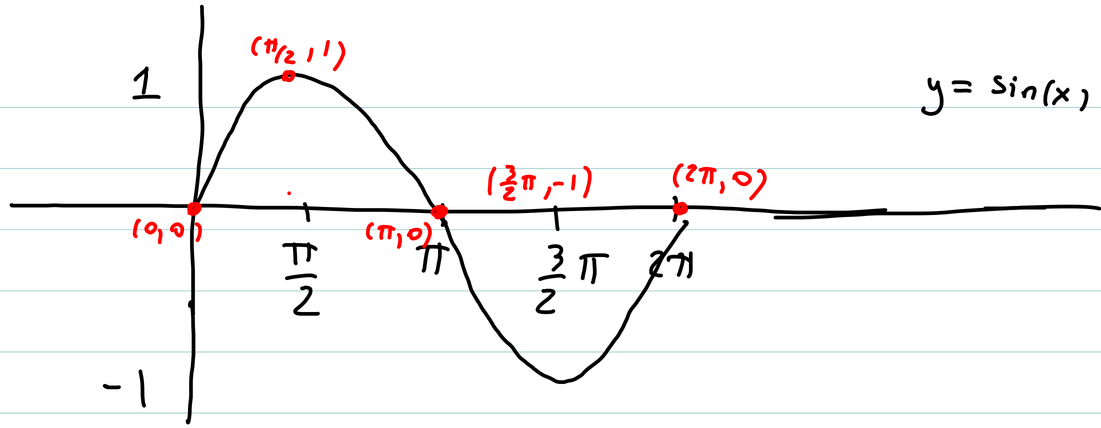
scaling factor $\frac{1}{2}$

Vertical scaling (expansion or compression)

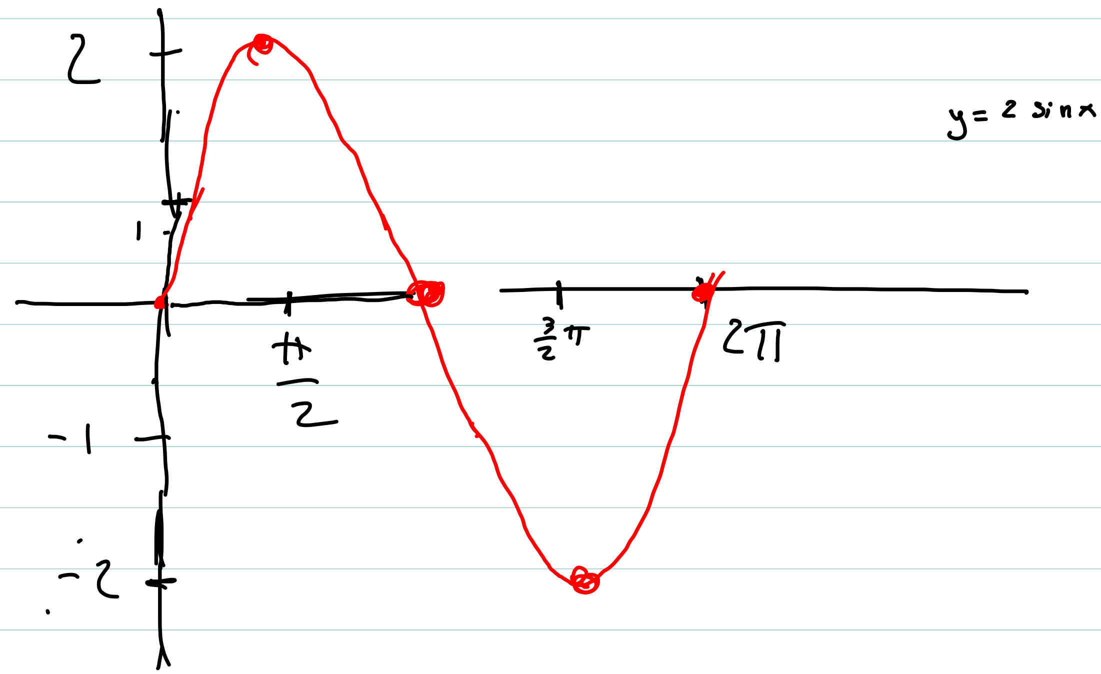
multiply y by $\frac{1}{a}$ or $f(x)$ by a $(a > 0)$

The graph of $a f(x)$ is the graph of $f(x)$ stretched vertically by a factor of a $(a > 1)$ or compressed vertically $(a < 1)$ by a factor of a

If (x_1, y_1) graph of $f(x)$ $(x_1, a y_1)$ on graph of $a f(x)$



y = sin(x)
1
-1
π/2
π
3/2 π
2π
(π/2, 1)
(0, 0)
(π, 0)
(3/2 π, -1)
(2π, 0)



y = 2 sin x
2
1
-1
-2
π/2
3/2 π
2π

How to graph $a\,f(bx+c)+d$

1. Graph $y = f(x)$
   Horizontally :
2. Shift $|c|$ units, left if c is positive, right if c is negative .
3. Scale horizontally of a factor $\frac{1}{|b|}$ (compression if $|b| > 1$, expansion if $|b| < 1$)
4. Reflect across y axis if b is negative . Skip this step if b is positive.
   Vertically:
5. Scale by a factor of $|a|$ (compression if $|a| < 1$, expansion if $|a| > 1$)
6. Reflect across x axis if a is negative . Skip this step if a is positive.
7. Shift $|d|$ units, up if d is positive, down if d is negative .

Note: the order is important.

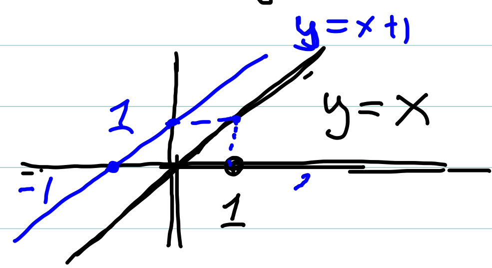
Why is order important?

$y = 2x + 1$

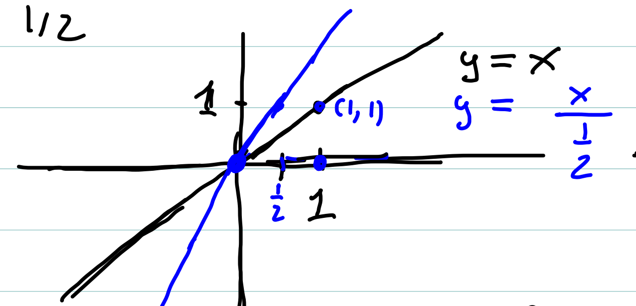
from $y = x$

WRONG

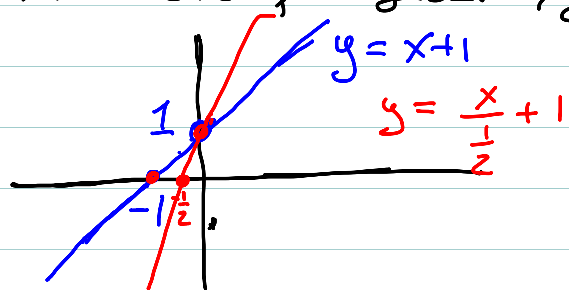
First shift left one



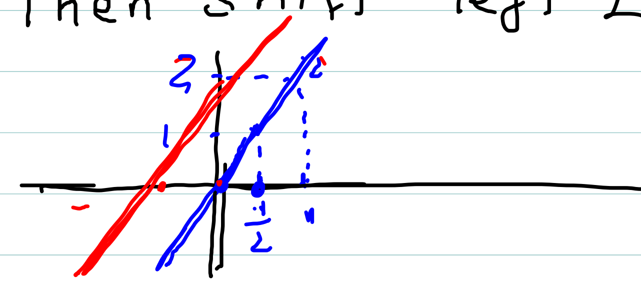
First scale of a factor 1/2



Then scale of a factor 1/2



Then shift left 1



what went wrong?

$y = x$ ... $\rightsquigarrow$ .... $y = 2x+1$

scaling horizontally by a factor of $\frac{1}{2}$

$$y = \frac{x}{\frac{1}{2}} = 2x$$

Translating horizontally 1 unit to the left

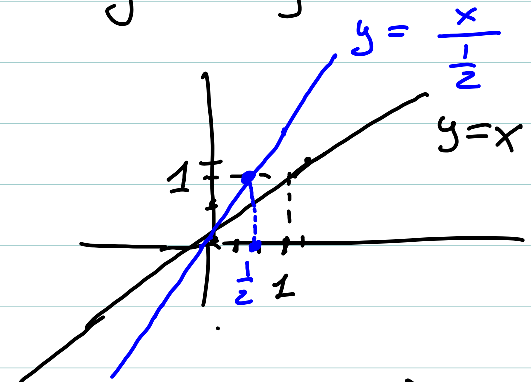
$y = 2(x+1) = 2x+2$ NOT $2x+1$

To scale first: $y = 2x+1$

Other way: $y = 2\left(x+\frac{1}{2}\right)$ from $y=x$

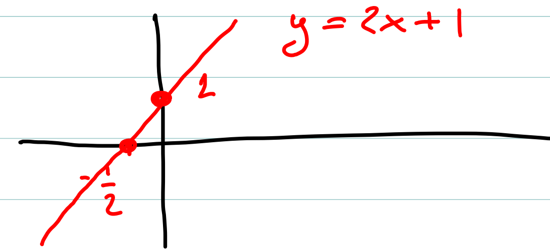
First scale of a factor of $\frac{1}{2}$

$y = 2x$

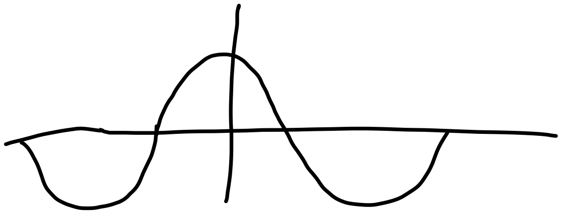


then shift $\frac{1}{2}$ (NOT 1) horizontally

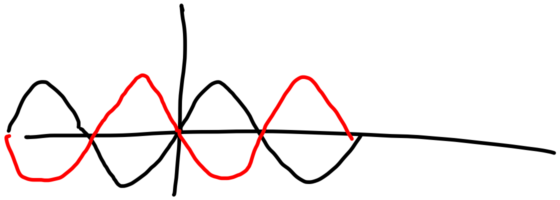
to the left



f is even if $f(x) = f(-x)$.
EX: $\cos x$ is even



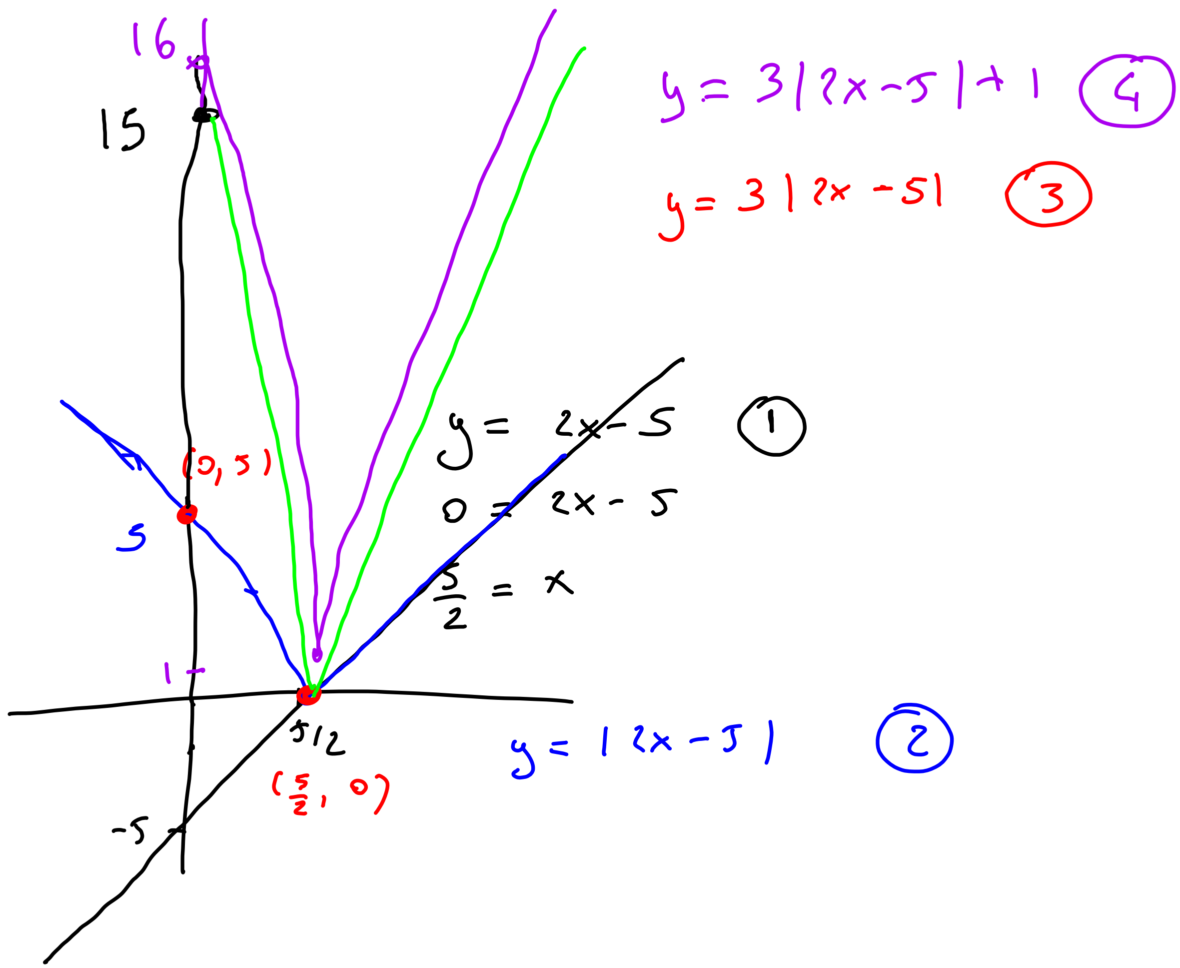
f is odd if $f(x) = -f(-x)$.
EX: $\sin x$ is odd



How to graph $a|bx+c|+d$ Shortcut

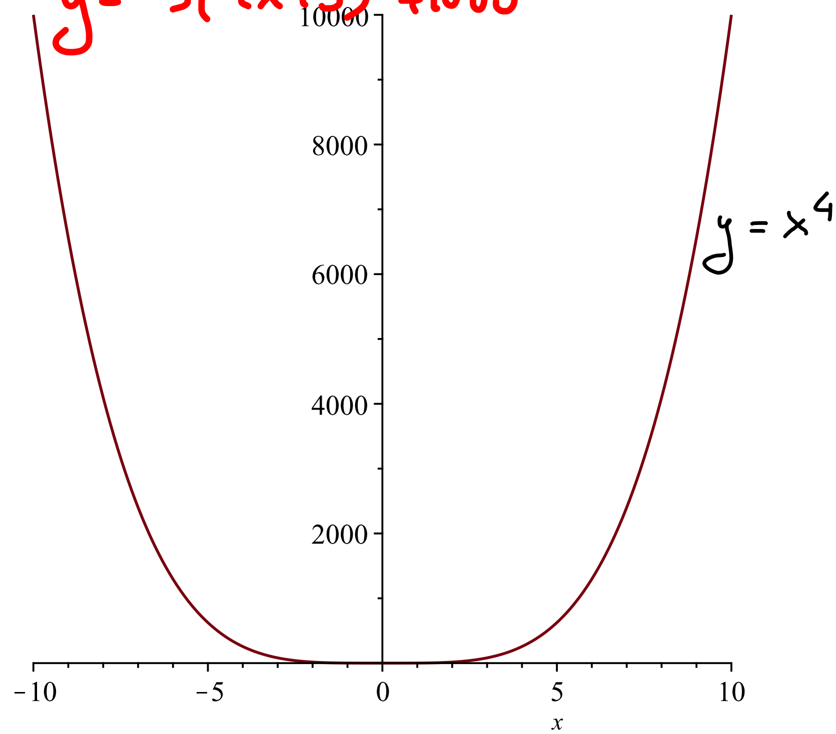
1. Graph $y = bx + c$ and flip it into V shape. This gives you the graph of $|bx+c|$
2. Graph $a\,|bx+c|$. Scale by $|a|$, reflect if $a < 0$
3. Graph $a\,|bx+c| + d$. Shift vertically

Sketch the graph of $g(x) = 3|2x - 5| + 1$



Goal from graph of $y = x^4$ below graph

$y = -3(-2x+5)^4 + 1000$



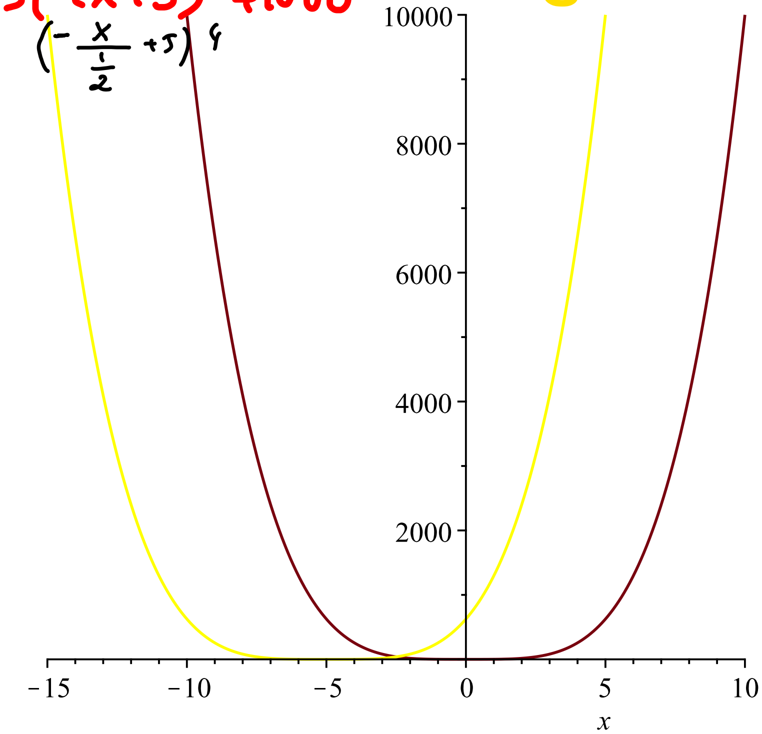
> $q2 := plot\left((x+5)^4, x = -15..5, color = yellow\right);$

Goal

$y = -3(-2x+5)^4 + 1000$

$\left(-\frac{x}{\frac{1}{2}}+5\right)^4$

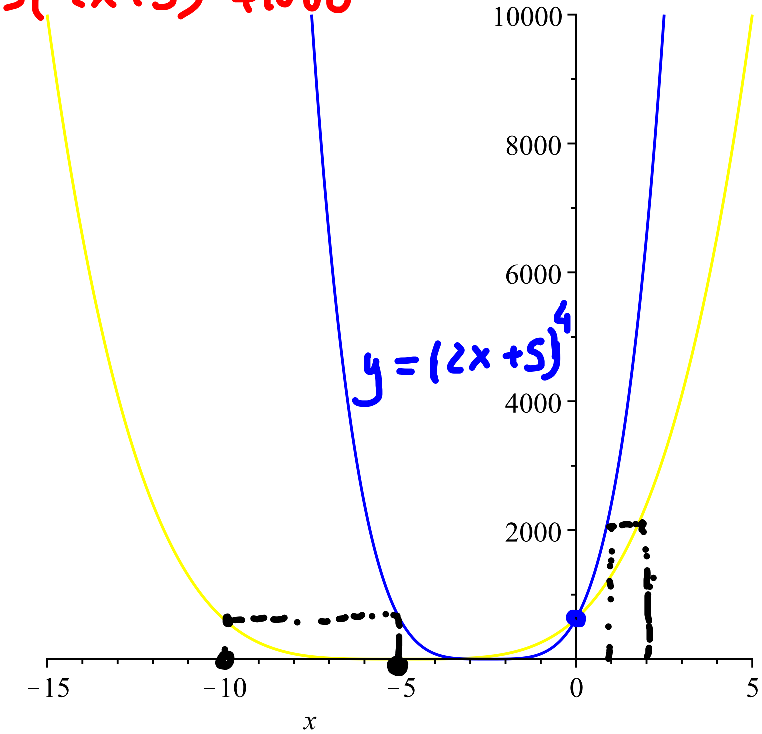
$y=(x+5)^4$



> $q3 := plot\left((2\cdot x+5)^4, x = -\frac{15}{2} .. \frac{5}{2}, color = blue\right);$

Goal

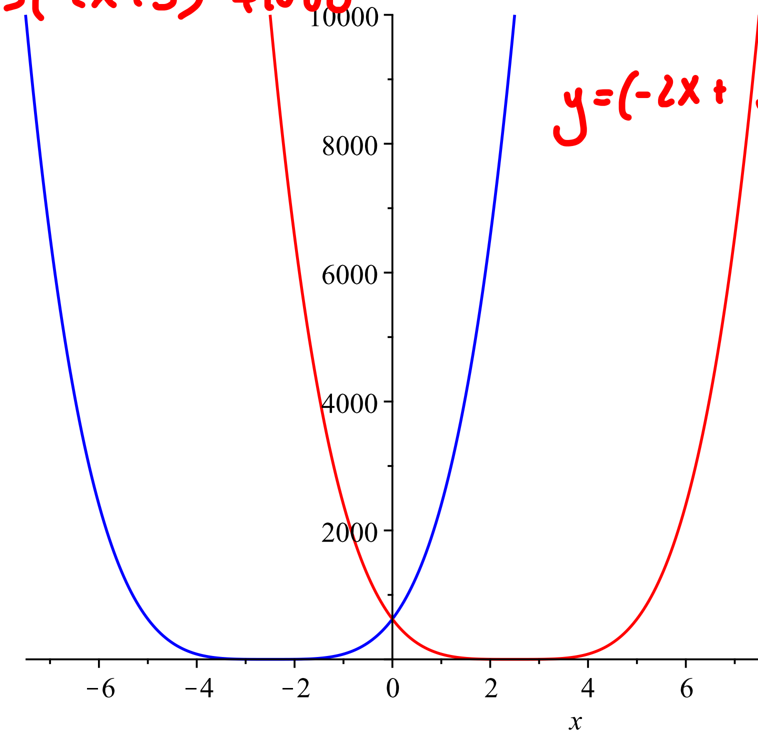
$y = -3(-2x+5)^4 + 1000$



> $q4 := plot\left((-2\cdot x+5)^4, x = -\frac{5}{2} .. \frac{15}{2}, color = red\right);$

Goal

$y = -3(-2x+5)^4 + 1000$

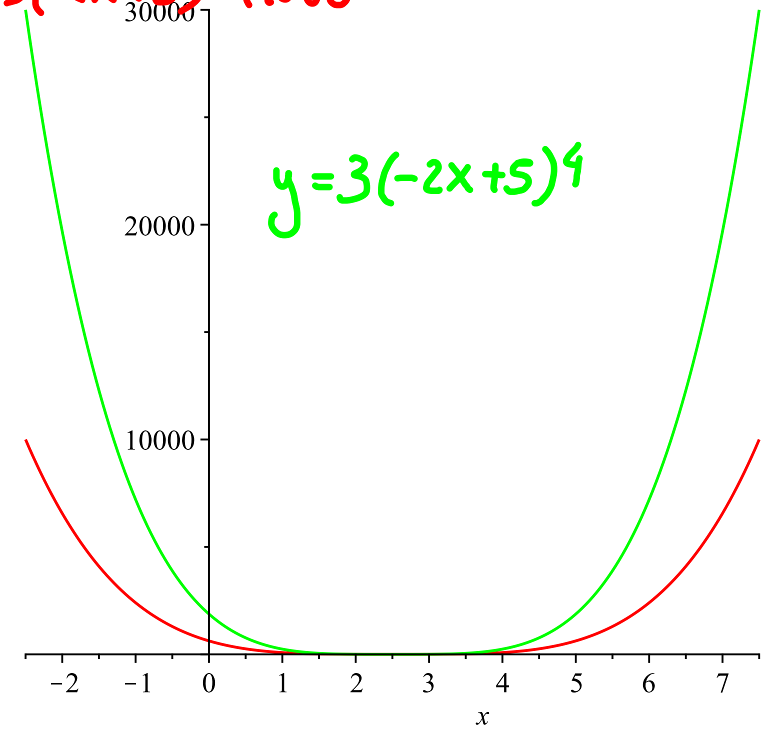


$y = (-2x+5)^4$

> $q5 := plot\left(3\cdot(-2\cdot x+5)^4, x=-\frac{5}{2}..\frac{15}{2}, color=green\right);$

Goal

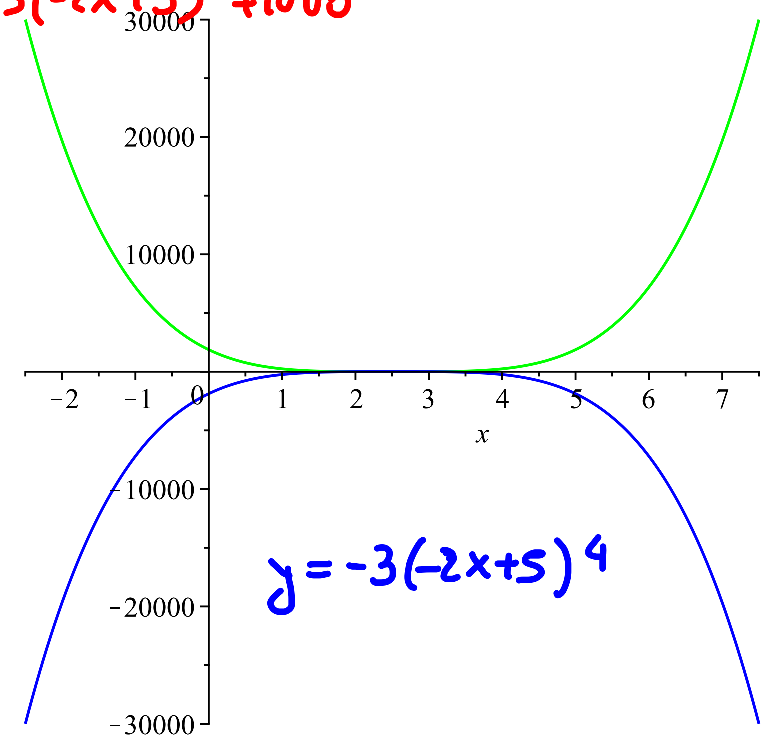
$y = -3(-2x+5)^4 + 1000$



> $q6 := plot\left(-3 \cdot (-2 \cdot x + 5)^4, x = -\frac{5}{2} .. \frac{15}{2}, color = blue\right);$

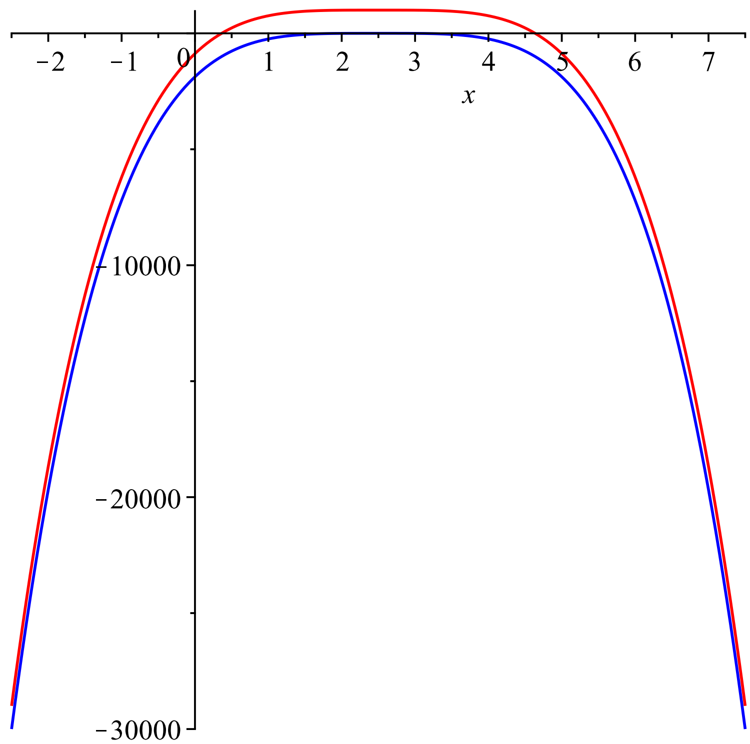
Goal

$y = -3(-2x+5)^4 + 1000$



> $q7 := plot\left(-3 \cdot (-2 \cdot x + 5)^4 + 1000, x = -\frac{5}{2} .. \frac{15}{2}, color = red\right);$

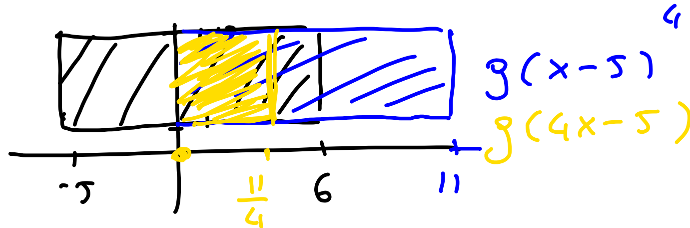
$y = -3(-2x+5)^4 + 1000$



>
>

Suppose $g(x)$ has domain $-5 \leq x \leq 6$ and range $1 \leq y \leq 10$
What are the domain and range of $g(4x-5)$?

$g\left(\frac{x}{\frac{1}{4}} - 5\right)$



domain $\quad g(4x-5) \quad 0 \leq x \leq \frac{11}{4}$

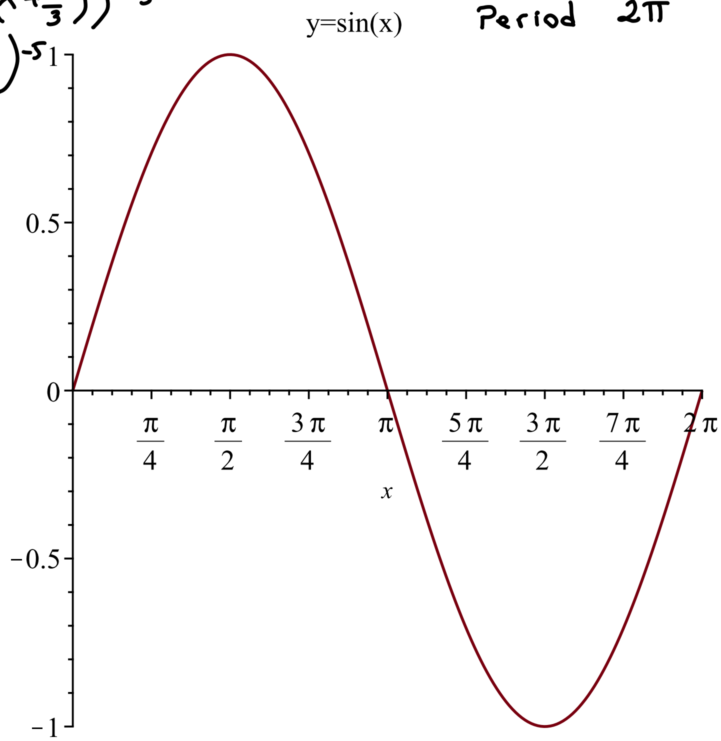
range $\quad g(4x-5) \quad$ sames as $g(x) \quad 1 \leq y \leq 10$

Goal : graph $2\sin(3x+1)-5$

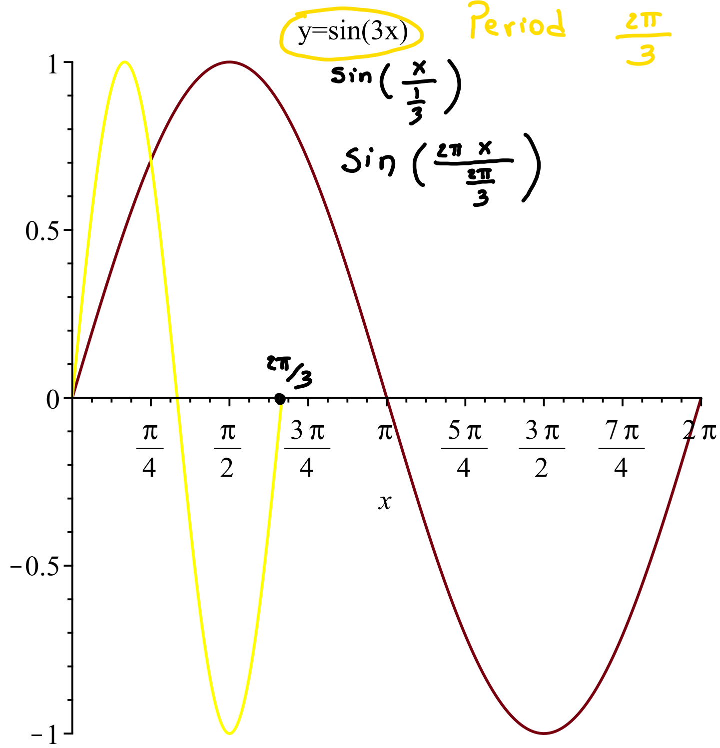
$2\sin\left(3\left(x+\frac{1}{3}\right)\right)-5$

$2\sin\left(\frac{x+\frac{1}{3}}{\frac{1}{3}}\right)-5$

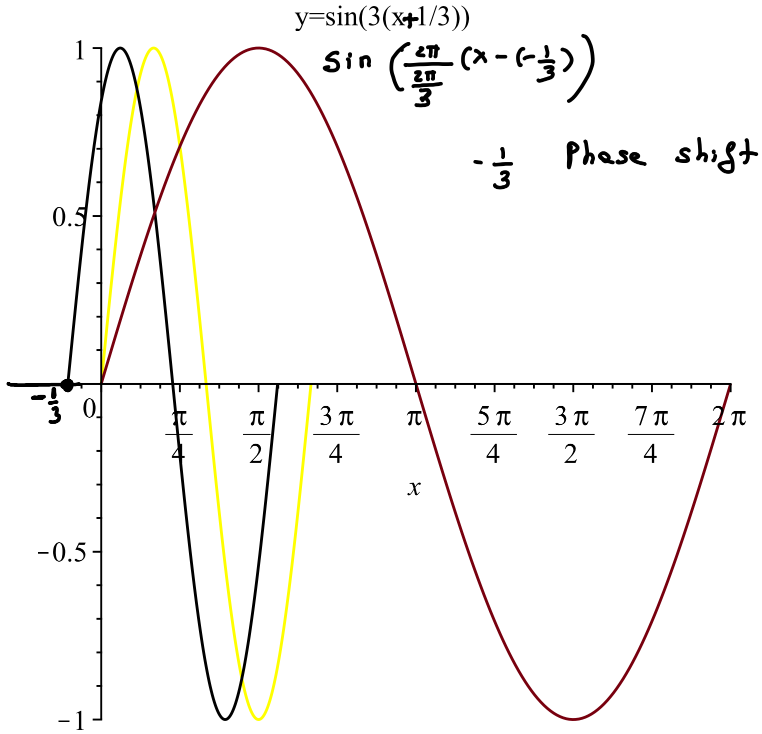
Period 2π



Goal $2\sin\left(3\left(x+\frac{1}{3}\right)\right)-5$



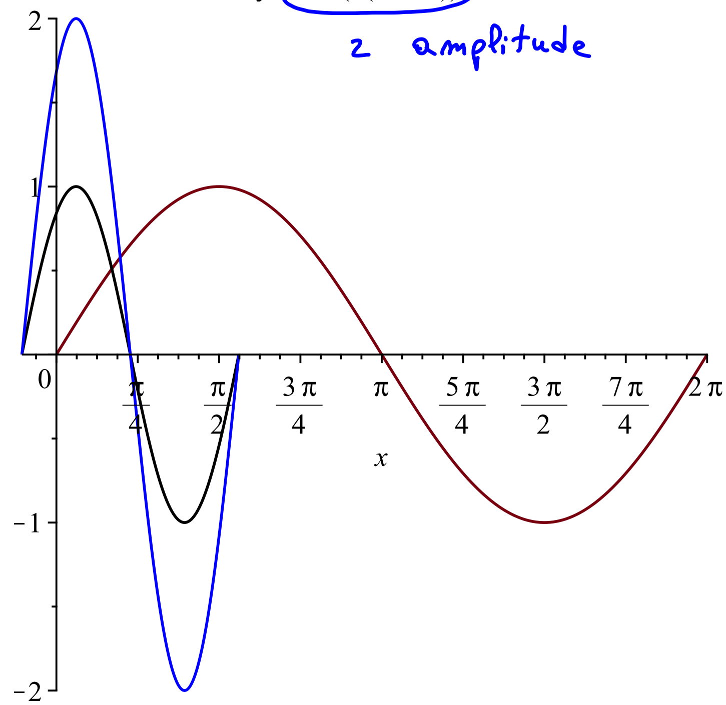
Goal $2\sin(3(x+\frac{1}{3}))-5$



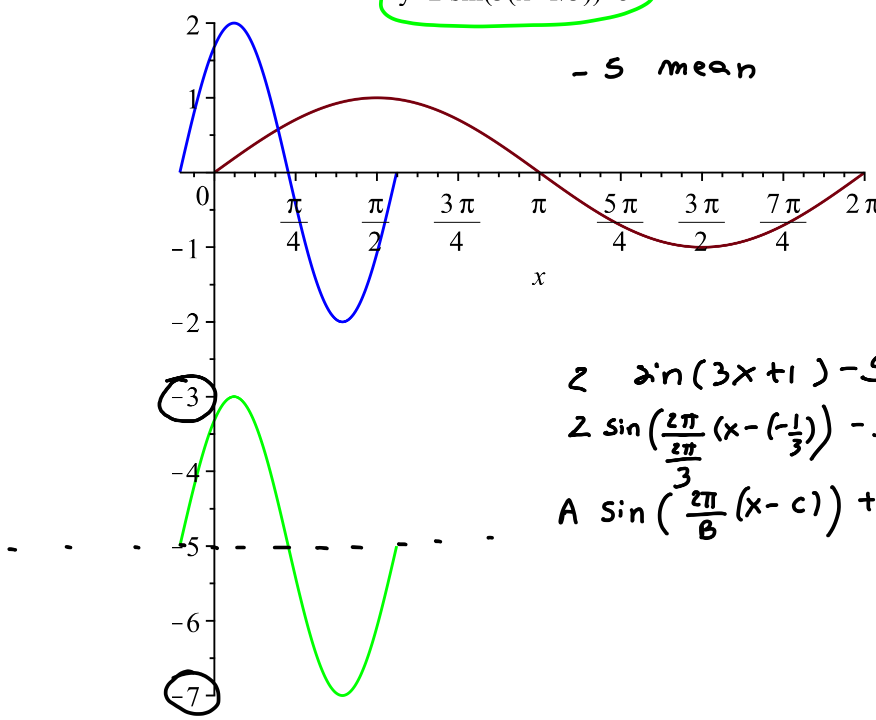
Goal $2\sin\left(3\left(x+\frac{1}{3}\right)\right)-5$

y=2 sin(3(x+1/3))

2 amplitude



y=2 sin(3(x+1/3)) -5



-5 mean

$2 \sin(3x+1) - 5$

$2 \sin\left(\frac{2\pi}{\frac{2\pi}{3}}\left(x-\left(-\frac{1}{3}\right)\right)\right) - 5$

$A \sin\left(\frac{2\pi}{B}(x-C)\right) + D$

NOTE $\quad -5 = \dfrac{-3+(-7)}{2} \quad$ D

$2 = \dfrac{-3-(-7)}{2} \quad$ A