Lesson 16

Read Chapter 13

Graphical tools

Exponential function

$A_0 \, a^x$ or $A_0 \, e^{kx}$, $a = e^k$

$k = \ln(a)$

We are often advised to refrigerate foods after opening them. Leftovers that sit outside the refrigerator overnight might "go bad" quickly even though they will last for several days in the refrigerator. Why is this the case? One reason is the presence of bacteria—bacteria are everywhere, and some are good for us and some are bad for us. Bacteria have been shown to display an exponential growth rate (modelled by an equation of the form $A_0 e^{kt}$) in many cases, but the rate of growth (the k) varies significantly depending on temperature.

A study published in 1991 tested growth rates of the bacterium Lactobacillus plantarum, a generally benign bacterium found in fermented foods, at several different temperatures ranging from 6 C to 43 C . After allowing cultures to grow for 24 hours, the authors of the study determined the growth constant (k) for these different temperatures. At 6 C (about what you would find in a refrigerator), they found the growth rate constant of k = 0.0164 . If you put some yogurt with $\underbrace{100}_{A_0}$ bacteria in it in the refrigerator (at 6.C) at 8 pm, how many bacteria will there be at 8 am the following morning ? $t = 12$

$K = 0.0164$, $A_0 = 100$, $t = 12$

$f(t) = 100\, e^{0.0164 \cdot 12} = 122$ bacteria

You forgot some yogurt with 100 bacteria in it on the counter of your house in summer (at 22°) and 12 hours later the following morning there are 10^5 bacteria in the yogurt. What is k ?

$A_0 e^{kt}$

$100\, e^{k\cdot 12} = 10^5$ solve for k

$e^{k\cdot 12} = 10^3$

$\ln e^{k\cdot 12} = \ln(10^3)$

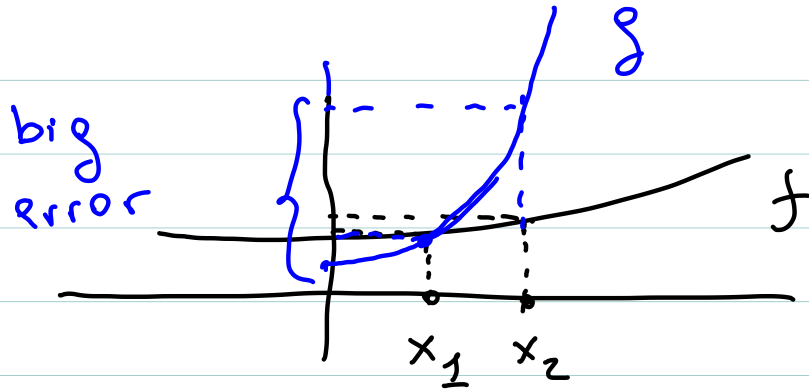
$k \cdot 12 = \ln(10^3)$

$k = \dfrac{\ln(10^3)}{12} \approx 0.5756$

at $6°$ $k = 0.0164$ at $22°$ $k = 0.5756$

$100\, e^{0.0164\cdot 12} = \underline{\underline{122}}$ $100 \cdot e^{0.5756\cdot 12} = \underline{\underline{10^5}}$

big change in values, careful about rounding off


big error
g
f
x_1
x_2

Archaeologists use the exponential, radioactive decay of carbon 14 to estimate the death dates of organic material. The half-life of carbon 14 is 5,730 years. A fossil is found that has 35% carbon 14 compared to the living sample. How old is the fossil?

A_0 amount of carbon 14 present at $t=0$, death time

$f(t) = A_0 a^t$, amount present at time t, years after death

$$a = \sqrt[5730]{1/2}$$

$$\cancel{A_0}\left(\sqrt[5730]{\tfrac{1}{2}}\right)^t = 0.35\,\cancel{A_0} \qquad \text{solve for } t$$

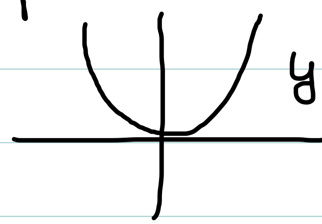
$$t \ln \sqrt[5730]{\tfrac{1}{2}} = \ln 0.35$$

$$t = \frac{\ln(0.35)}{\ln \sqrt[5730]{\frac{1}{2}}} \approx 8679$$

Goal: start from the graph of $y = f(x)$ and graph $y = af(bx+c)+d$

Which graphs should you know?

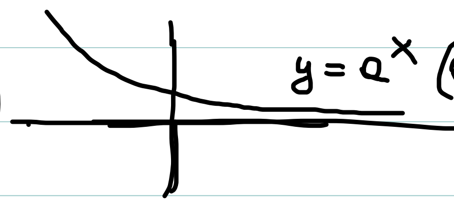
1) Line

2) 

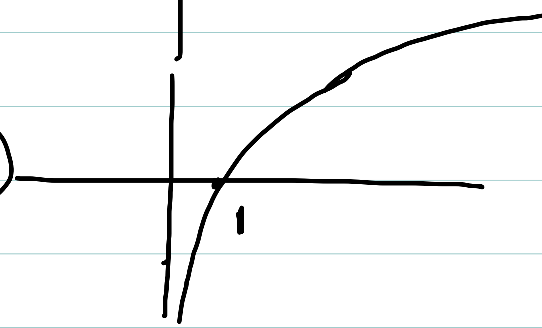
$y = x^2$

3) 

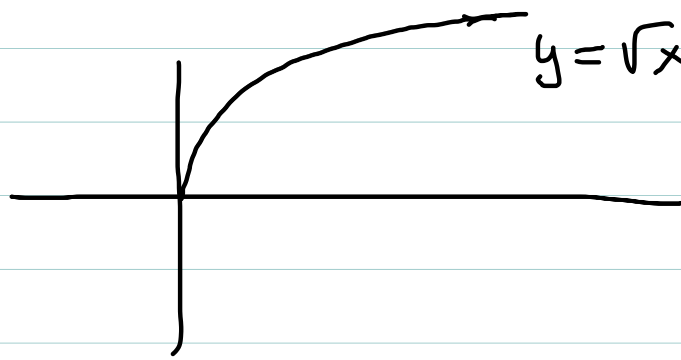
$y = e^x$ $\left(y = a^x \atop a>1\right)$

4) 

$y = a^x$ $(a<1)$

5) 

$y = \ln x$

6) 

$y = \sqrt{x}$

Horizontal translation

$f(x) = x^2$

ADD OR SUBTRACT C FROM X $(C \geq 0)$

I.E. REPLACE X with $x+C$ or $X-C$

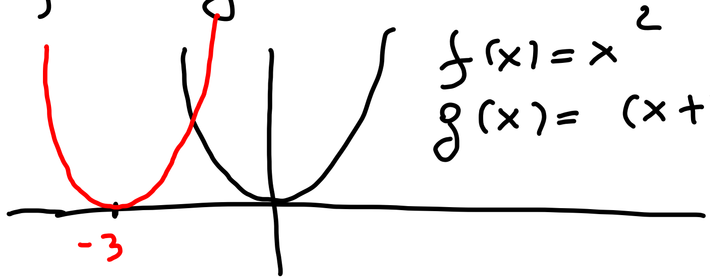
$x \to x+C$ (x_1, y_1) on graph of $f(x)$ $(x_1 - C, y_1)$ on graph of $f(x+c)$

$x \to x-C$ (x_1, y_1) on graph of $f(x)$ $(x_1 + C, y_1)$ on " " $f(x-c)$

graph of $f(x+c)$ $(c \geq 0)$ is the graph of f shifted horizontally of c units to left

graph $f(x-c)$ $(c \geq 0)$ is the graph of f shifted horizontally c units to the right

Ex:



$f(x) = x^2$

$g(x) = (x+3)^2 = (x-(-3))^2 + 0$

Vertical translation

$f(x) = x^2$

$y = x^2$

ADD or SUBTRACT d to y (or $f(x)$)

$(d \geq 0)$

$f(x) \to f(x)+d$ $\quad (x_1, y_1)$ on graph of $f(x)$ $\quad (x_1, y_1+d)$ on graph of $f(x)+d$

$f(x) \to f(x)-d$ $\quad (x_1, y_1)$ on graph of $f(x)$ $\quad (x_1, y_1-d)$ on " " $f(x)-d$

Graph of $f(x)+d$ $(d \geq 0)$ is graph of $f(x)$ shifted vertically up of d units

Graph of $f(x)-d$ $(d \geq 0)$ is graph of $f(x)$ shifted vertically down of d units

$f(x) = x^2$

$g(x) = x^2 + 3$

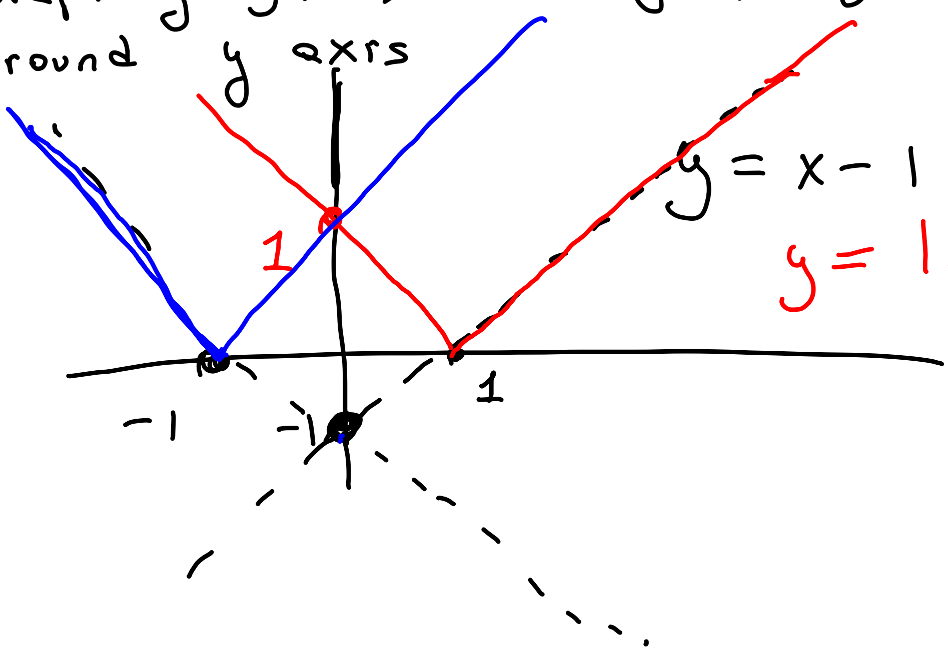
Horizontal Reflections

$$f(x) = |x - 1|$$

Replace x with $-x$

(x_1, y_1) on graph of $f(x)$, $(-x_1, y_1)$ on graph of $f(-x)$

Graph of $f(-x)$ is graph of $f(x)$ reflected around y axis



$y = x - 1$

$y = |x - 1|$

$y = |-x - 1|$

$y = -x - 1$

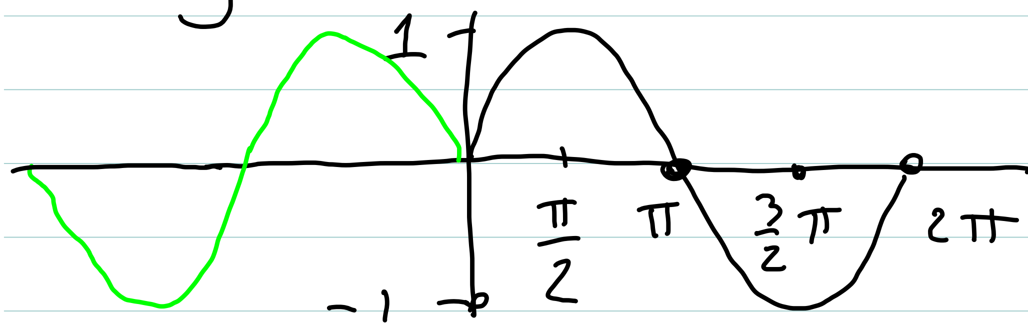
$$y = |x-1|$$

$$y = |-x-1|$$ horizontal reflection

$y = |x-1+2| = |x+1|$ horizontal translation two units to the left.

$$|-x-1| = |-(x+1)| = |x+1|$$

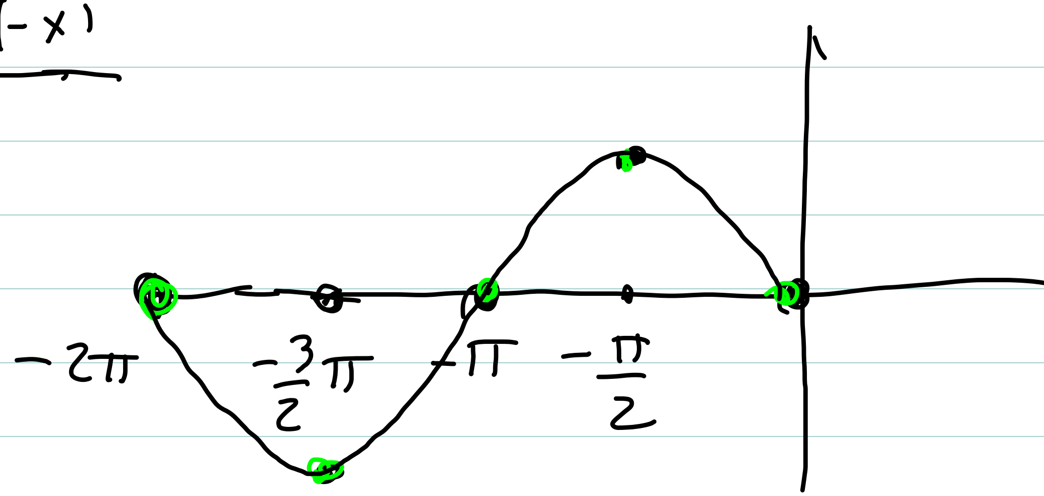
$f(x) = \sin x$ on $0 \le x \le 2\pi$



$0 \le -x \le 2\pi$

$g(x) = f(-x) = \sin(-x)$ domain $0 \ge x \ge -2\pi$

x	$\sin(-x)$
-2π	0
$-\frac{3}{2}\pi$	-1
$-\pi$	0
$-\frac{\pi}{2}$	1
0	0



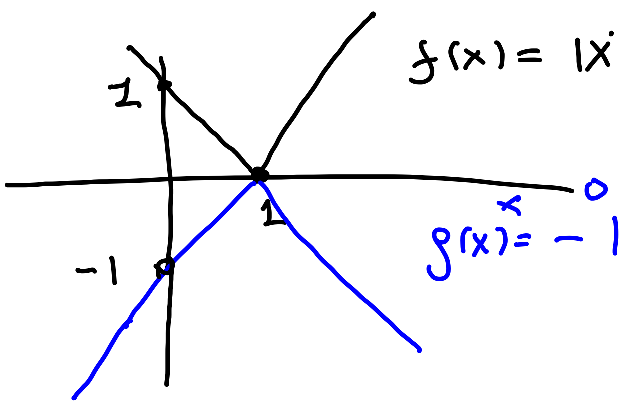
Vertical Reflections

$f(x) = |x - 1|$, $y = |x-1|$

Replace y with $-y$ (or $f(x)$ with $-f(x)$)

(x_1, y_1) on graph of $f(x)$, $(x_1, -y_1)$ on graph of $-f(x)$

The graph of $-f(x)$ is the graph of $f(x)$ reflected around the x axis



$f(x) = |x - 1|$

$$g(x) = -|x-1| = \begin{cases} -(x-1) & \text{if } x \geq 1 \\ x-1 & \text{if } x < 1 \end{cases}$$