Lesson 1~~5~~4

Read Chapter 12

Logarithms

1000 at 2% compounded annualy

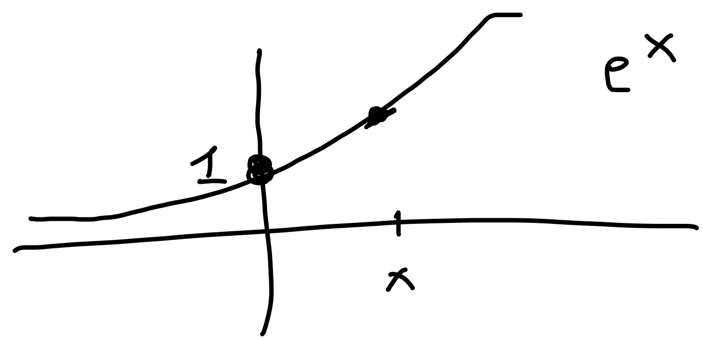
after 1 year you have $1 \cdot \underline{1000} + 0.02 \cdot \underline{1000} = 1000(1+0.02)^1$

after t years you have $1000(1+0.02)^t$

1000 at 2% compounded continuosly

after t years $1000 \cdot e^{0.02 \cdot t}$

$e \approx 2.71 \ldots\ldots$ irrational



e^x

ln x is the inverse of e^x . This means

$$\begin{tikzcd} x \arrow[r, bend left, "e^x"] & y \arrow[l, bend left, "\ln y"] \end{tikzcd} \quad y = e^x$$

$$e^x = y$$

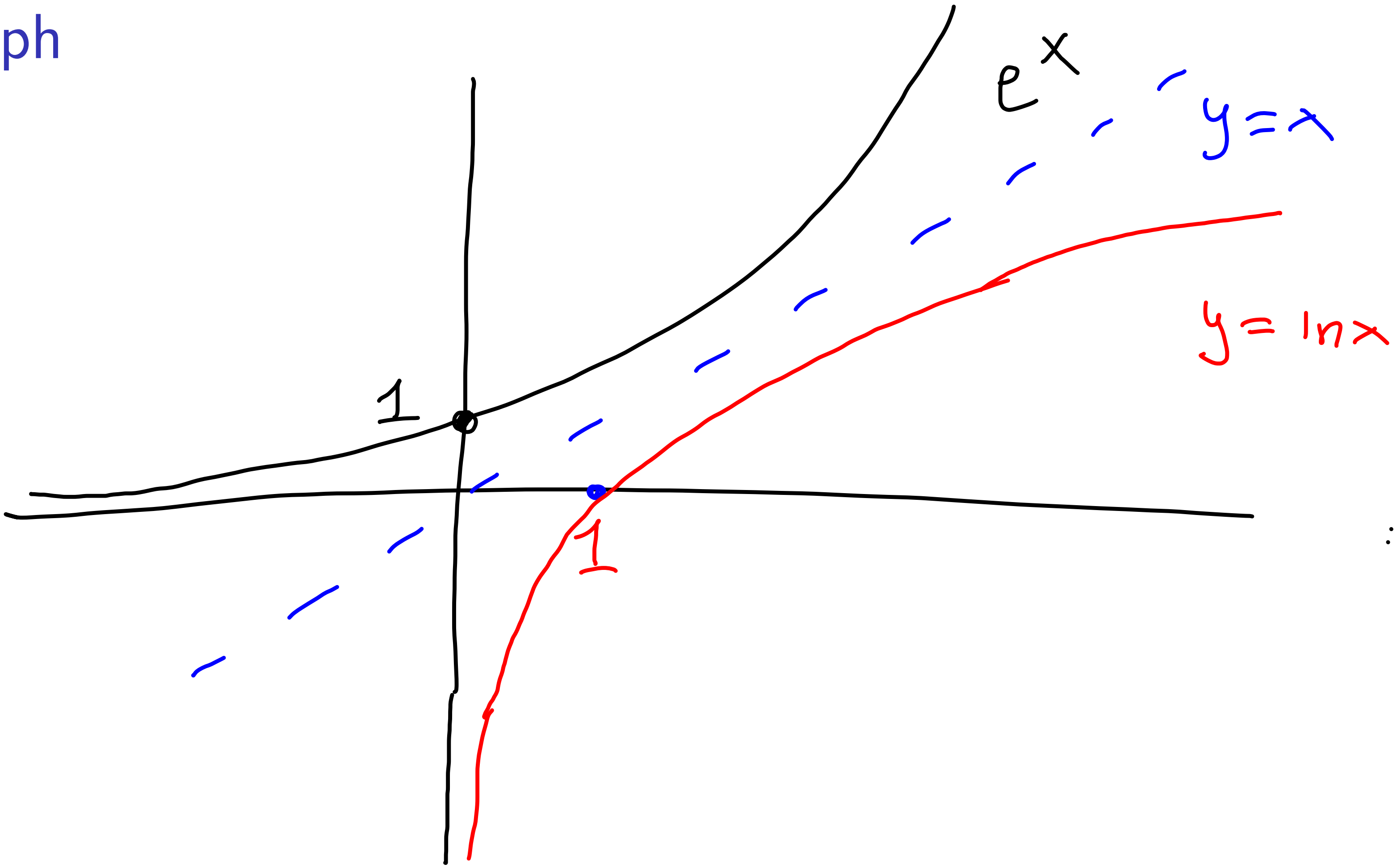
$$x = \ln y$$

$$\ln e^x = x$$

$$e^{\ln y} = y$$

Slope at x is equal to e^x

Graph



$\ln 1 = 0$

Domain of $\ln x$ $\quad x > 0$

Range of $\ln x$ $\quad (-\infty, +\infty)$

Other log functions

$\log_a x$ is the inverse of a^x

$\log_3 x$ is the inverse of 3^x

In WebAssign $\log x$ for $\log_{10} x$

properties of log

- $\ln(x^y) = y \ln x$ $\qquad \log_b x^y = y \log_b x$
- $log_b x = \frac{\ln x}{\ln b}$
- $a^x = e^{(\ln a)x}$ $\qquad a = e^{\ln(a)} \qquad a^x = (e^{\ln a})^x$
- $\ln(xy) = \ln(x) + \ln(y)$
- $\ln \frac{x}{y} = \ln x - \ln y$
- $\ln 1 = 0$
- $\ln \frac{1}{x} = - \ln x$

$\underbrace{\ln 1}_{=0} - \ln x$

Solve the following equations

1. $5e^{x-4} = 2$

$$\ln(e^{x-4}) = \ln\left(\frac{2}{5}\right)$$

$$x-4 = \ln\left(\frac{2}{5}\right)$$

$$x = 4 + \ln(2/5)$$

2. $5 \cdot 3^{x-4} = 2$

$$3^{x-4} = \frac{2}{5}$$

$$\log_3 3^{x-4} = \log_3 \frac{2}{5}$$

$$x-4 = \log_3 \frac{2}{5}$$

$$x = 4 + \log_3 \frac{2}{5}$$

$$\ln(3^{x-4}) = \ln\left(\frac{2}{5}\right)$$

$$(x-4)\ln 3 = \ln\left(\frac{2}{5}\right)$$

$$x\ln 3 - 4\ln 3 = \ln\left(\frac{2}{5}\right)$$

$$x\cancel{\ln 3} = \frac{4\ln 3 + \ln\left(\frac{2}{5}\right)}{\ln 3}$$

$$x = \frac{4\cancel{\ln 3}}{\cancel{\ln 3}} + \frac{\ln(2/5)}{\ln 3}$$

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Solve the following equations

1. $5\ln(5x+2)=3$

$$\ln(5x+2)=3/5$$

$$e^{\ln(5x+2)}=e^{3/5}$$

$$5x+2=e^{3/5}$$

$$\cancel{5}x=\frac{e^{3/5}-2}{5}$$

2. $log_2(5x+2)=3$

$$2^{\log_2(5x+2)}=2^3$$

$$5x+2=8$$

$$\cancel{5}x=\frac{6}{5}$$

or

$$\frac{\ln(5x+2)}{\ln(2)}=3$$

$$\ln(5x+2)=3\cdot\ln(2)$$

$$e^{\ln(5x+2)}=e^{3\cdot\ln(2)}$$

$$5x+2=e^{3\cdot\ln(2)}$$

$$\cancel{5}x=\frac{e^{\ln(2)\cdot 3}-2}{5}$$

$\ln(\ln x)$

$x = \frac{1}{2}$, $\ln(\ln \frac{1}{2})$

① $\ln(\frac{1}{2}) = -0.69$

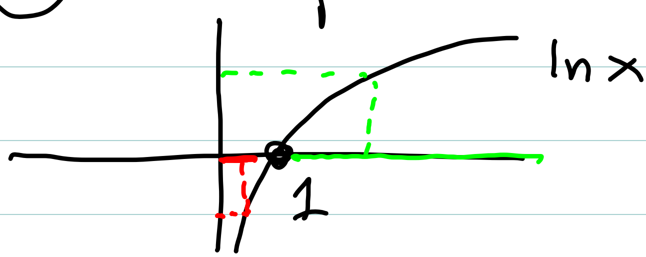
② $\ln(-0.69)$??

$\ln(\ln x)$ what is the domain?

① Compute $\ln x$ $\underline{x > 0}$

② Compute $\ln(\underline{\underline{\ln x}})$ $\ln x > 0$ solve for x

$\boxed{x > 1}$



Solve the following equations

1. $log_2 5 = \log_3(7-x)$

$$3^{\log_2 5} = 3^{\log_3(7-x)} = (7-x)$$

$$x = 7 - 3^{\log_2 5}$$

or

$$\frac{\ln 5}{\ln 2} = \frac{\ln(7-x)}{\ln 3}$$

$$\frac{\ln 3 \cdot \ln 5}{\ln 2} = \ln(7-x)$$

$$e^{\frac{\ln 3 \cdot \ln 5}{\ln 2}} = e^{\ln(7-x)}$$

$$e^{\frac{\ln 3 \cdot \ln 5}{\ln 2}} = 7-x$$

$$x = 7 - e^{\frac{\ln 3 \cdot \ln 5}{\ln 2}}$$

2. $5y = 10^x$ solve for x

$$\log(5y) = \log 10^x$$

$$\log(5y) = x$$

Exponential functions in standard form

$$f(x) = A_0\, a^x$$ ← standard form

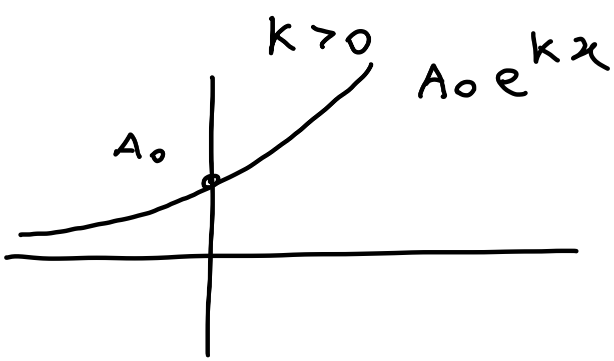
$$a = e^{\ln a}$$

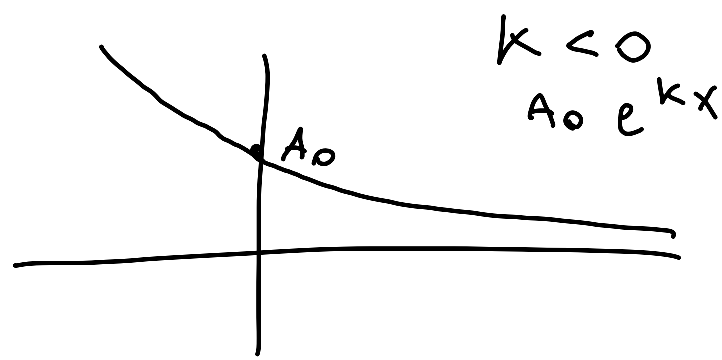
$$a^x = (e^{\ln a})^x$$

or

$$f(x) = A_0 e^{(\ln a)\,x} = A_0 e^{kx}$$ ← e form

$$k = \ln a$$





Rewrite in e form

- $y = 5 \cdot 7^t$

$7 = e^{\ln 7}$

$5 \cdot e^{\ln 7 \cdot t}$

- $y = 3 \cdot 2^{3t-1}$

$y = 3 \cdot 2^{3t} \cdot 2^{-1} = \frac{3}{2} \cdot 8^t$

$\frac{3}{2} e^{\ln 8 \cdot t}$

Convert $f(x) = \frac{2}{3^{x-4}}$ to the form $A_0 e^{kx}$

$$2 \cdot \frac{1}{3^{x-4}} = 2 \cdot \left(\frac{1}{3}\right)^{x-4} = 2\left(\frac{1}{3}\right)^{x} \cdot \left(\frac{1}{3}\right)^{-4}$$

$$2 \cdot 3^4 \left(\frac{1}{3}\right)^{x} = 162 \cdot \left(\frac{1}{3}\right)^{x}$$

"e form" $\quad 162 \cdot e^{\ln\left(\frac{1}{3}\right) \cdot x}$