Lesson 12

Read Chapter 9

Inverse function

Domain and composition:

If domain of $f(x)$ is $a \le x \le b$
and domain of $g(x)$ is $c \le x \le d$

Then domain $g(f(x))$ is all x that satisfy both:

1) $a \le x \le b$ from inside function
2) $c \le f(x) \le d$ -> solve for x from outside function

Range of Composition

The range of $g(f(x))$ is part of (possibly not equal to) the range of $g(x)$

$$x \longrightarrow \boxed{f} \longrightarrow f(x) \longrightarrow \boxed{g} \longrightarrow \underbrace{g(f(x))}_{\text{output of } g}$$

If for example $g(2)=3$, and 2 is the only input for g that outputs 3 and if $f(x)$ is never equal to 2, 3 is not an output for $g(f(x))$

Suppose $g(x)$ has domain $-5 \leq x \leq 6$ and range $1 \leq y \leq 10$
What are the domain and range of $g(4x-5)$? $\quad f(x) = 4x-5$

$g(f(x))$

Domain $-5 \leq 4x-5 \leq 6$

$0 \leq 4x \leq 11$

$\boxed{0 \leq x \leq \frac{11}{4}}$

Range $1 \leq y \leq 10$

If the inside function is linear then range $g(f(x))$

$=$ range $g(x)$

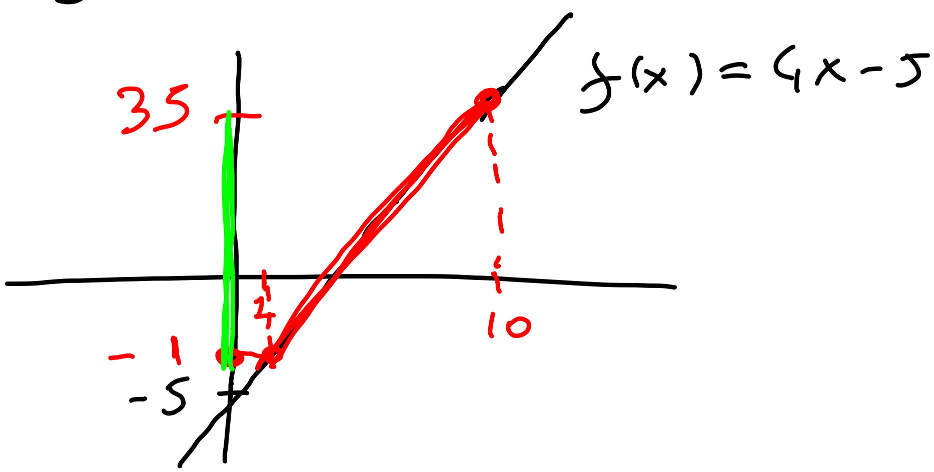
Suppose $g(x)$ has domain $-5 \leq x \leq 6$ and range $1 \leq \overset{g(x)}{y} \leq 10$

What are the domain and range of $4g(x) - 5$? $\quad f(x) = 4x - 5$

$f(\underline{\underline{g(x)}})$

Domain $\quad -5 \leq x \leq 6$

Range



Range is $-1 \leq y \leq 35$

Different method $\qquad 1 \leq g(x) \leq 10$

get $4g(x)-5$ in the middle

$$\underset{\underset{-1}{\shortparallel}}{4 \cdot 1 - 5} \leq 4g(x) - 5 \leq \underset{\underset{35}{\shortparallel}}{4 \cdot 10 - 5}$$

Suppose $h(t) = |t|$ find a formula for $h(h(t) - 2)$ and graph $h(h(t) - 2)$

$$||t|-2| = \begin{cases} |t-2| & \text{if } t \geq 0 \\ |-t-2| & \text{if } t < 0 \end{cases}$$

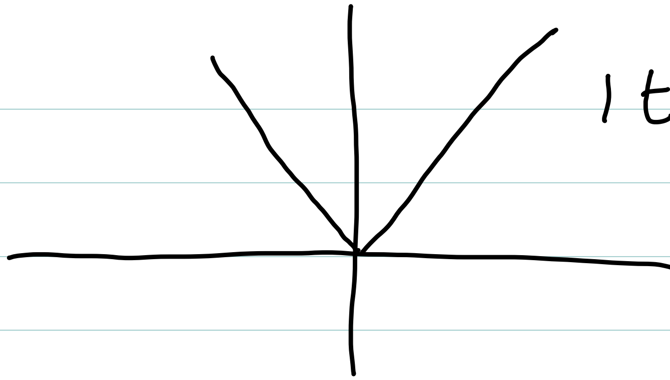
$t-2$ if $t-2 \geq 0$

$-(t-2)$ if $t-2 < 0$

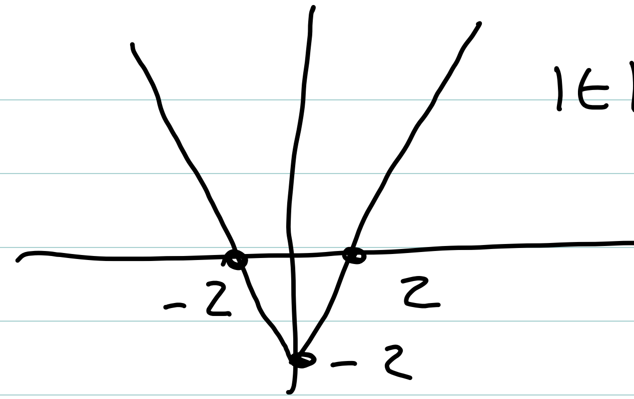
$-t-2$ if $-t-2 \geq 0$

$-(-t-2)$ if $-t-2 < 0$

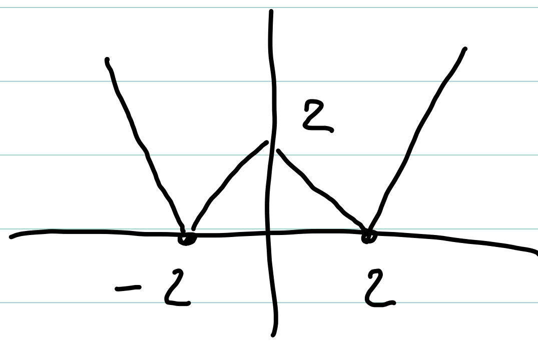
$$\text{so } ||t|-2| = \begin{cases} t-2 & \text{if } t \geq 2 \\ -t+2 & \text{if } 0 \leq t < 2 \\ -t-2 & \text{if } t \leq -2 \\ t+2 & \text{if } -2 < t < 0 \end{cases}$$



$|t|$



$|t| - 2$



$||t| - 2|$

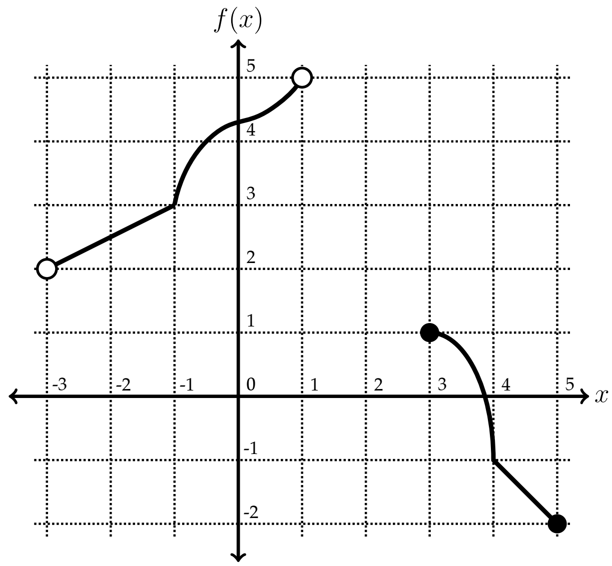
Suppose $f(x)$ is the profit made by selling x barrels of apples and $g(x)$ is the number of barrels of apples produced by x trees. Explain in words the meaning of $f(g(x))$

profit

trees

the profit made by selling apples from x trees

1. Happy Thursday! I bought you this graph.



f

(a) **[4 points]** Compute $f(f(\underbrace{f(4)}_{-1}))$. $= f(\underbrace{f(-1)}_{3}) = f(3) = \boxed{1}$

(b) **[5 points]** Find the domain and range of $f^{-1}(x)$.

(c) **[5 points]** Let $g(x) = f(2x+1)+1$. Find the domain and range of $g(x)$.

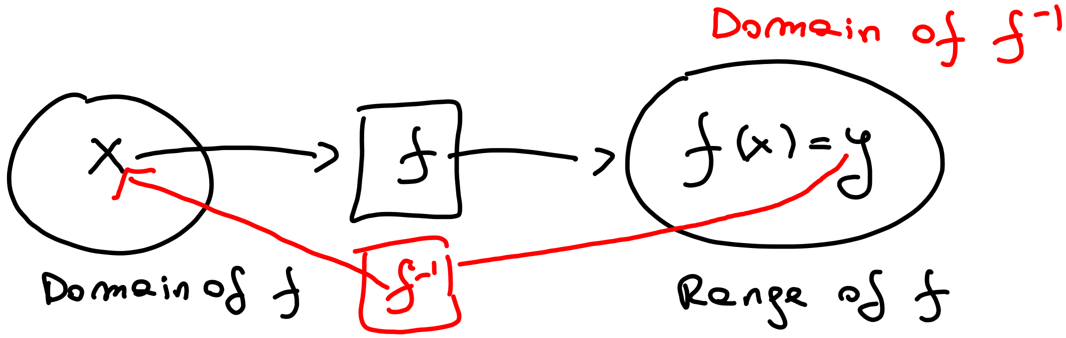
We will talk again about domain/range after chapter 13

Given $f : A \to B$

the inverse function $f^{-1} : B \to A$ if it exists, is such that

$f^{-1}(f(x)) = x$, $f(f^{-1}(y)) = y$

or $f(x) = y$ exactly when $f^{-1}(y) = x$



The inverse of e^x is $\ln x$

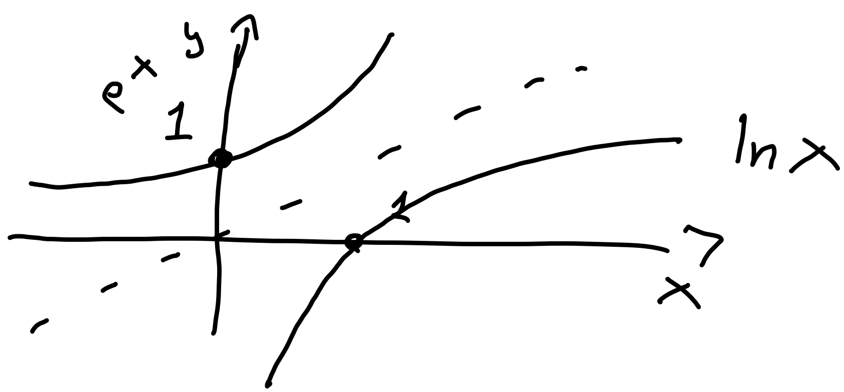
$\ln(e^x) = x$ $\qquad$ $e^{\ln y} = y$

$f^{-1}(x)$ means the inverse of f computed at x

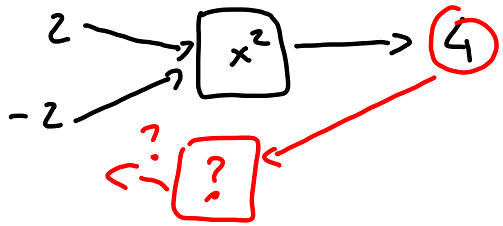
NOT $\dfrac{1}{f(x)}$

Domain f^{-1}= Range f
Range f^{-1} = Domain f

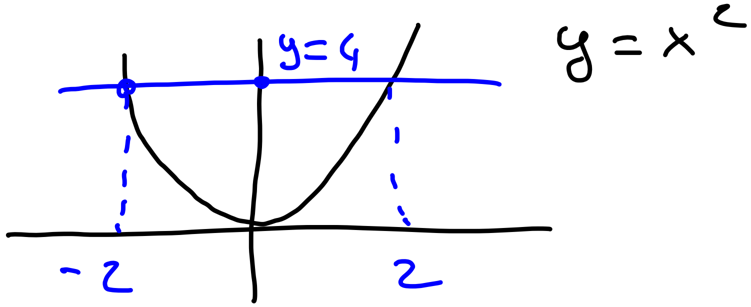
The graph of $f^{-1}(y)$ is the graph of $f(x)$ flipped around the line $y = x$



Does $f(x) = x^2$ have an inverse function ? NO



A function that does not satisfy horizontal line test is not invertible



Given $g(x) = 2x - 1$ ~~on the domain $0 \le x \le 5$~~. Is g invertible ? If it is find the inverse , ~~its domain and its range~~

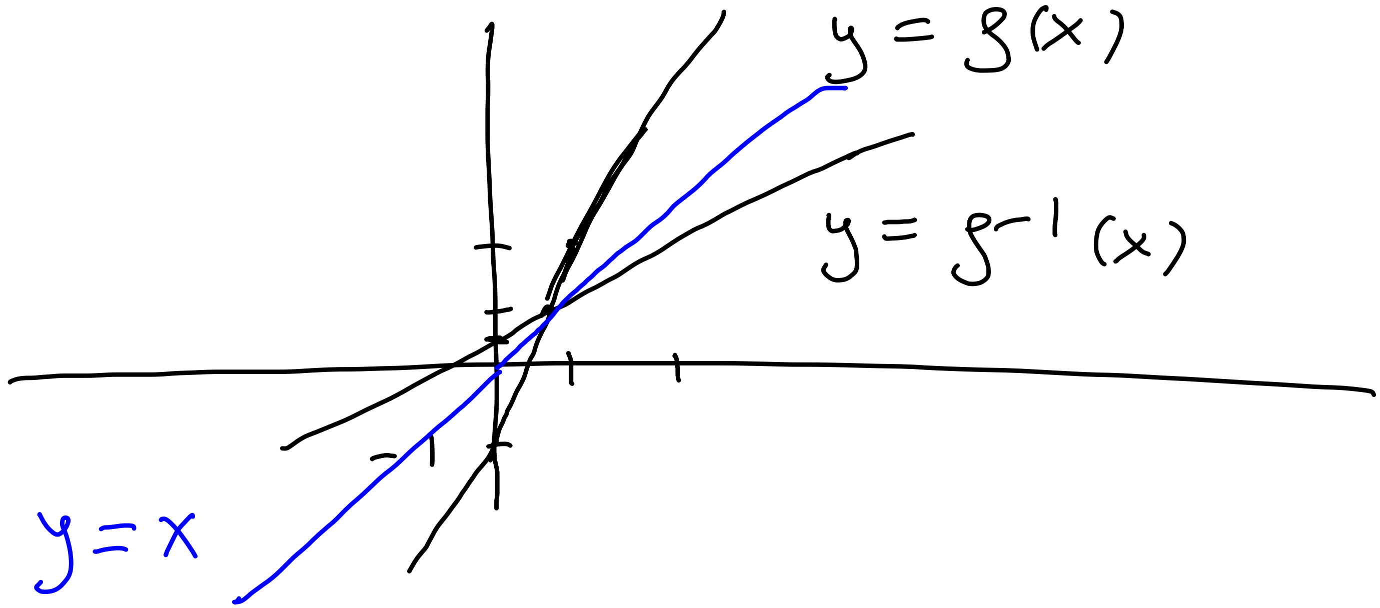
yes

To find $g^{-1}(y)$: $y = 2x - 1$ $(y = g(x))$ solve for x

$$\frac{y+1}{2} = \frac{\cancel{2}x}{\cancel{2}}$$

$$g^{-1}(y) = \frac{y+1}{2} = \frac{1}{2}y + \frac{1}{2}$$

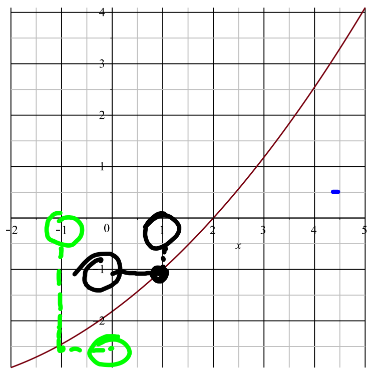
(In WebAssign pay attention if they ask for $g^{-1}(x) = \frac{1}{2}x + \frac{1}{2}$)



2. Below is the graph of the function $y = f(x)$ on the domain $-2 \leq x \leq 5$

$f^{-1}(-1) = 1$

$f(-1) \approx 2.5$



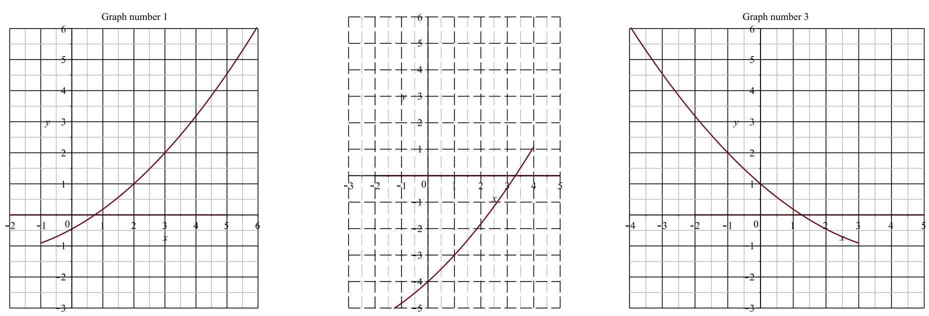
$f(x)$

c) Find $f^{-1}(-1)$

$f(x) = y$

$x = f^{-1}(y)$

(a) Which of the graphs below is the graph of $y = 2 + f(x-1)$? Circle the correct graph.



(b) If the domain of f is $-2 \leq x \leq 5$ what is the domain of the function $\frac{f(3x)+5}{x-1}$?

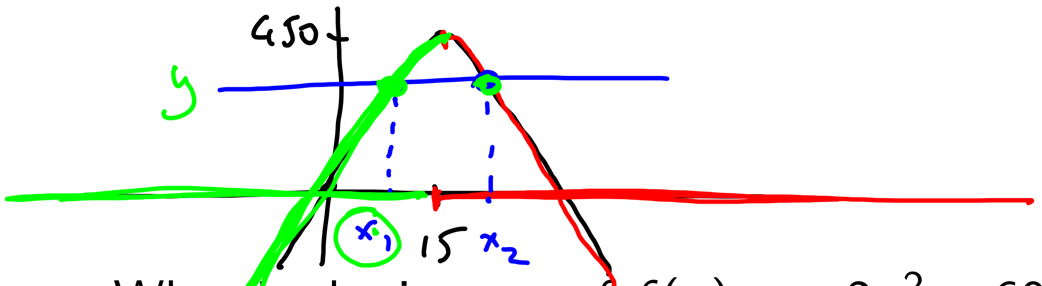
(c) Compute $f^{-1}(-1)$

(d) If $h(x) = e^{f(x)}$ Which of the values below is closest to $h^{-1}(2)$? Circle the the right answer.

0.6 ,-1, 2.5, -2, 3.5

Explain why $f(x) = -2x^2 + 60x$ is not invertible.

$h = 15$
$k = 450$



What is the inverse of $f(x) = -2x^2 + 60x$ on $[15, +\infty)$

$y = -2x^2 + 60x$; $\underbrace{2}_{a}x^2 \underbrace{-60}_{b}x + \underbrace{y}_{c} = 0$

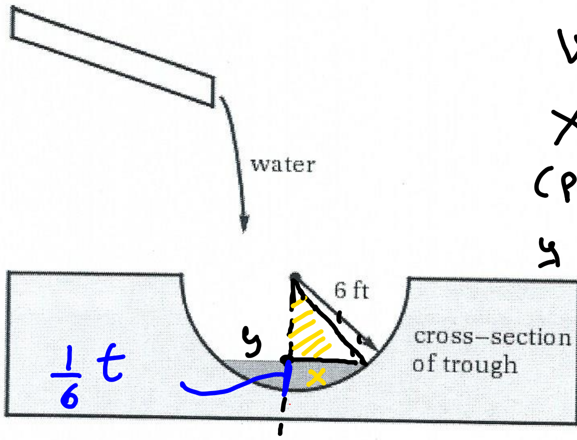
$$x = \frac{60 \pm \sqrt{60^2 - 4 \cdot 2 \cdot y}}{2 \cdot 2}$$

$$f^{-1}(y) = \frac{60 + \sqrt{60^2 - 8y}}{4}$$

What is the inverse of $f(x) = -2x^2 + 60x$ on $(-\infty, 15]$

$$f^{-1}(y) = \frac{60 - \sqrt{60^2 - 8y}}{4} = \underbrace{15 - \frac{\sqrt{60^2 - 8y}}{15}}_{x < 15}$$

A trough has a semicircular cross section with a radius of 6 feet. Water starts flowing into the trough in such a way that the depth of the water is increasing at a rate of 2 inches per hour.



$W = 2x$

$x = \sqrt{6^2 - y^2}$

(pythagorean th)

$y = 6 - 2\ \text{in/hr} \cdot \frac{1}{12}\ \frac{\text{feet}}{\text{in}} \cdot t = 6 - \frac{1}{6}t$

(a) Give a function
$w = f(t)$
relating the width w, in feet of the surface of the water to the time t, in hours. Make sure to specify the domain and compute the range too.

(b) After how many hours will the surface of the water have width of 7 feet? (Round your answer to two decimal places.)

(c) Give a function
$t = f^{-1}(w)$
relating the time to the width of the surface of the water. Make sure to specify the domain and compute the range too.

a) $w = 2\sqrt{36 - (6 - \frac{1}{6}t)^2}$ domain $0 \le t \le 36$ (the time it takes to fill the trough)

range $0 \le w \le 12$ (the width at top)

c) solve for t : $\frac{w^2}{4} = \frac{4}{4}(36 - (6 - \frac{1}{6}t)^2)$

$36 - \frac{w^2}{4} = (6 - \frac{1}{6}t)^2$

$\pm\sqrt{36 - \frac{w^2}{4}} = 6 - \frac{1}{6}t$

$\frac{1}{6}t = 6 \pm \sqrt{36 - \frac{w^2}{4}}$

$t = 36 \pm 6\sqrt{36 - \frac{w^2}{4}}$

since $t \le 36$ $t = f^{-1}(w) = 36 - 6\sqrt{36 - \frac{w^2}{4}}$