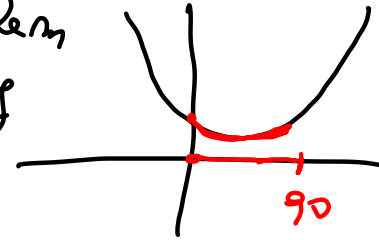


What is a function?
Just a formula?

$$f(x) = 2x + 3$$

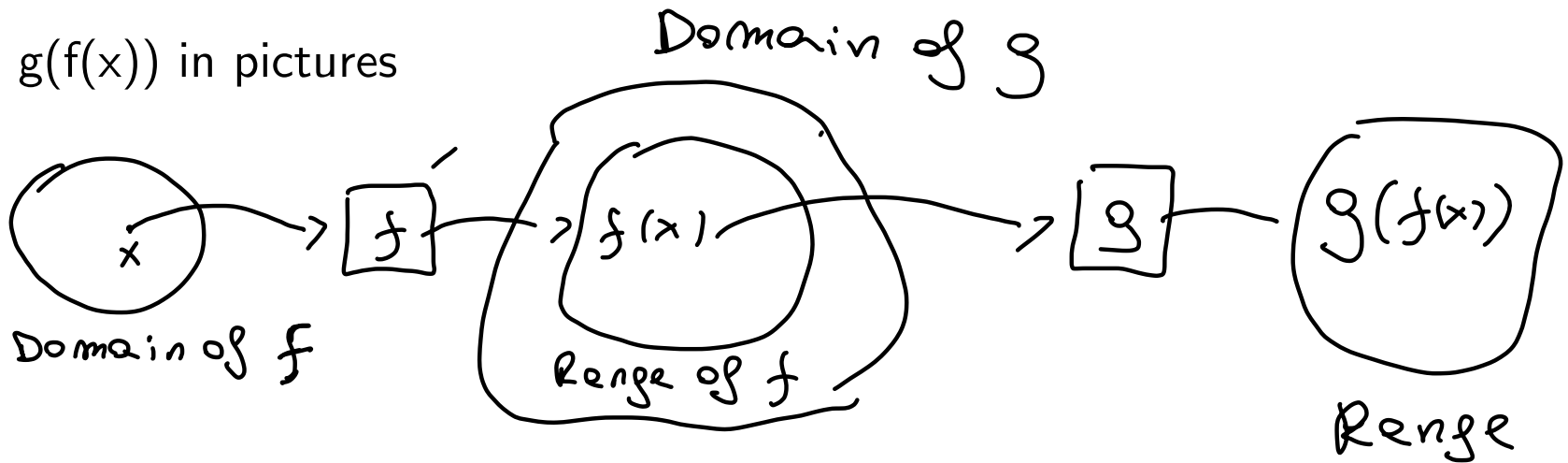
Recall problem
from Friday

$$0 \leq x \leq 90$$



Domain is
important

$g(f(x))$ in pictures



Example $f(x) = x^2 + 1$, $g(x) = 2x + 3$

$$g(f(x)) = g(\underbrace{x^2 + 1}_{f(x)}) = 2 \cdot (x^2 + 1) + 3 = 2x^2 + 5$$

DIFFERENT

$$f(g(x)) = f(2x + 3) = (2x + 3)^2 + 1 = 4x^2 + 2 \cdot 2x \cdot 3 + 9 + 1 = 4x^2 + 12x + 10$$

$$\text{Example } f(x) = \begin{cases} x + 1 & \text{if } x \leq 0 \\ 2x^2 + x + 1 & \text{if } x > 0 \end{cases} \quad g(x) = 2x + 3$$

$$2f(x) + 3 \\ \parallel \\ \underline{\underline{g(f(x))}} = \begin{cases} 2 \frac{(x+1)}{f(x)} + 3 & \text{if } x \leq 0 \\ 2 \frac{(2x^2 + x + 1)}{f(x)} + 3 & \text{if } x > 0 \end{cases}$$

$$f(g(x)) = f(\underline{\underline{2x+3}}) = \begin{cases} (2x+3) + 1 & \text{if } 2x+3 \leq 0 \\ 2(2x+3)^2 + (2x+3) + 1 & \text{if } 2x+3 > 0 \end{cases}$$

$$= \begin{cases} 2x + 4 & \text{if } x \leq -\frac{3}{2} \\ \underline{\underline{8x^2 + 26x + 22}} & \text{if } x > -\frac{3}{2} \end{cases}$$

Write the following functions as composition of two functions:

$$e^{x^3} = g(f(x))$$

$$x \longrightarrow x^3 = y \longrightarrow e^y$$

$$f(x) = x^3$$

$$g(x) = e^x$$

$$\sqrt{x^3 + 1} = g(f(x))$$

$$x \longrightarrow x^3 + 1 = y \longrightarrow \sqrt{y}$$

$$f(x) = x^3 + 1$$

$$g(x) = \sqrt{x}$$

Domain of $g(f(x))$

1) $x \rightarrow f(x)$ x must be in the domain of f

2) $f(x) \rightarrow g(\underline{\underline{f(x)}})$ $f(x)$ must be in the domain of g

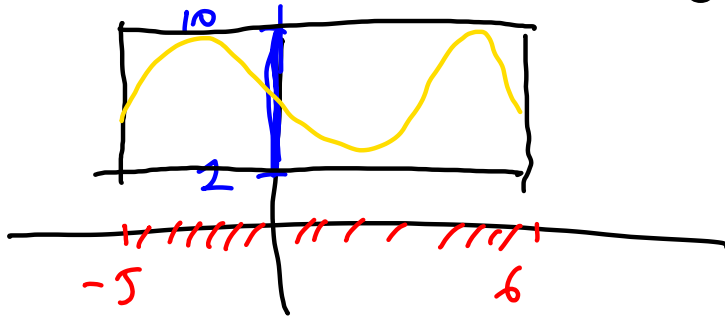
Range of $g(f(x))$

$x \rightarrow \boxed{f} \rightarrow f(x) \rightarrow \boxed{g} \rightarrow \underline{\underline{g(f(x))}}$

$g(f(x))$ is an output for g

Range of $g(f(x))$ is a part of (not always equal to) the range of g .

Suppose $g(x)$ has domain $-5 \leq x \leq 6$ and range $1 \leq y \leq 10$
 What are the domain and range of $g(4x - 5)$?



$$f(x) = 4x - 5$$

Domain : 1) Given x

compute $4x - 5$

fine for all x

2) Compute $g(\underline{\underline{\underline{4x-5}}})$

Need

$$-5 \leq 4x - 5 \leq 6$$

solve for x

$$\frac{0}{4} \leq \frac{4x}{4} \leq \frac{11}{4}$$

$$0 \leq x \leq \frac{11}{4}$$

Suppose $g(x)$ has domain $-5 \leq x \leq 6$ and range $1 \leq y \leq 10$
What are the domain and range of $4g(x) - 5$?

$$f(x) = 4x - 5$$

$$"f(g(x))"$$

Domain: x need to compute $g(x)$

need $-5 \leq x \leq 6$

Then feed $g(x)$ to f

compute $f(g(x)) = 4 \cdot \underline{\underline{g(x)}} - 5$ Ang

value $g(x)$ will be fine

Domain $\boxed{-5 \leq x \leq 6}$