Lesson 10

Min/max problems

Want to maximize / minimize a quantity

① Choose your variable(s) and find a formula for q $\boxed{q = q(x)}$

② Most likely $q(x)$ is a quadratic function its graph is a parabola $\cup$ $\cap$ and you need to find vertex.

③ Pay attention wether the problem is asking for an x value (h) or a value of the quantity q (k)

Tricks / problems

1) The quantity is a distance
2) The quantity depends on more than one variable
3) Min / max <u>not</u> at vertex

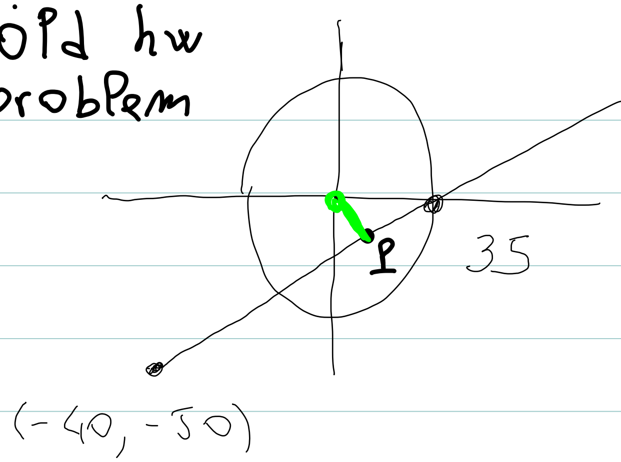
If $\sqrt{f(t)}$ and $f(t)$ have a min/max, they both reach the min/max at the same t value t_1

$$\text{Min: } \sqrt{f(t_1)} \leq \sqrt{f(t)}$$

$$f(t_1) \leq f(t)$$

Trick: remove square root to find t

old hw problem



$$x = -40 + \frac{30}{\sqrt{13}} t$$

$$y = -50 + \frac{20}{\sqrt{13}} t$$

Second method : want minimize distance between Ball and center

$$d(\text{Ball}, \underset{(0,0)}{\text{center}}) = \sqrt{\left(-40 + \frac{30}{\sqrt{13}} t\right)^2 + \left(-50 + \frac{20}{\sqrt{13}} t\right)^2}$$

we want to minimize d

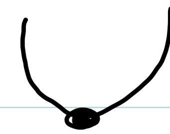
Trick minimize d^2. SIMPLIFY

remove square root

$$1600 - \frac{2 \cdot 40 \cdot 30}{\sqrt{13}} t + \frac{900}{13} t^2 +$$

$$2500 - \frac{2 \cdot 50 \cdot 20}{\sqrt{13}} t + \frac{400}{13} t^2$$

Add together similar terms



$$d^2 = 100t^2 - \frac{4400}{\sqrt{13}}t + 4100 \qquad (h, k)$$

$$\text{min at } h = \frac{\frac{4400}{\sqrt{13}}}{200} = \boxed{\frac{22}{\sqrt{13}}}$$

This is time when bell is closest to center

To find P, plug $t = \frac{22}{\sqrt{13}}$ into parametric coordinates

$$x = -40 + \frac{30}{\sqrt{13}} \cancel{t} \frac{22}{\sqrt{13}}$$

$$y = -50 + \frac{20}{\sqrt{13}} \cancel{t} \frac{22}{\sqrt{13}}$$

simplify

To find minimum distance compute $d\left(\frac{22}{\sqrt{13}}\right) =$ (here you need $\sqrt{\ }$)

$$= \sqrt{100 \cdot \left(\frac{22}{\sqrt{13}}\right)^2 - \frac{4400}{\sqrt{13}} \cdot \frac{22}{\sqrt{13}} + 4100}$$

$$= \sqrt{\frac{48400}{13} - \frac{96800}{13} + \frac{4100 \cdot 13}{13}}$$

$$= \sqrt{\frac{4900}{13}} = \boxed{\frac{70}{\sqrt{13}}}$$

Rosalie is organizing a circus performance to raise money for a charity. She is trying to decide how much to charge for tickets. From past experience she knows that the number of tickets sold is a linear function of the price. If she charges $\overset{x}{5}$ dollars per ticket, she can sell 1000 tickets, if she charges 7 dollars she can only sell 900 tickets. How much should she charge per tickets to make the most money ?

price of one ticket

Maximize the $\underbrace{\text{money Rosalie makes}}_{g}$

$g = g(x)$ Choose x : price of one ticket

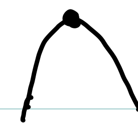
$g = x \cdot$ number of tickets sold

$f(x)$ linear function $(5, 1000)$ $(7, 900)$

$f(x) = 1000 + \dfrac{1000-900}{5-7}(x-5)$; $f(x) = 1000 - 50(x-5)$

$g(x) = x \cdot \left(1000 - 50(x-5)\right) = 1000x - 50x^2 + 250x = -50x^2 + 1250x$

$q(x) = -50x^2 + 1250x$

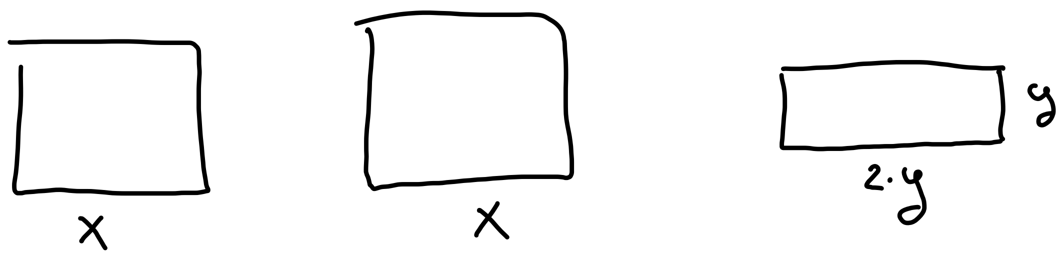


$h = -\dfrac{1250}{2(-50)} = \boxed{12.5}$

To find the max money Rosalie can make calculate $q(12.5) = -50\cdot(12.5)^2 + 1250\cdot 12.5$ (k)

and simplify

You have 720 m of fencing with which to build 3 enclosures. Two are identical squares and one is a rectangle that is twice as long as it is wide. What should be the dimensions of the squares to minimize the combined area of all three enclosures ? What should be the dimensions of the squares to maximize the combined area of all three enclosures ? (dimensions = 0 are OK)



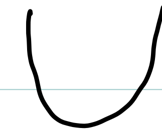
quantity Area: $q = A(x)$; $A = x^2 + x^2 + 2y\cdot y$

want to "get rid of y"; $4x + 4x + 2y + y + 2y + y = 720$

$\cancel{6}y = \frac{720 - 8x}{6} = \frac{360-4x}{3}$; $A(x) = 2x^2 + 2\left(\frac{360-4x}{3}\right)^2$

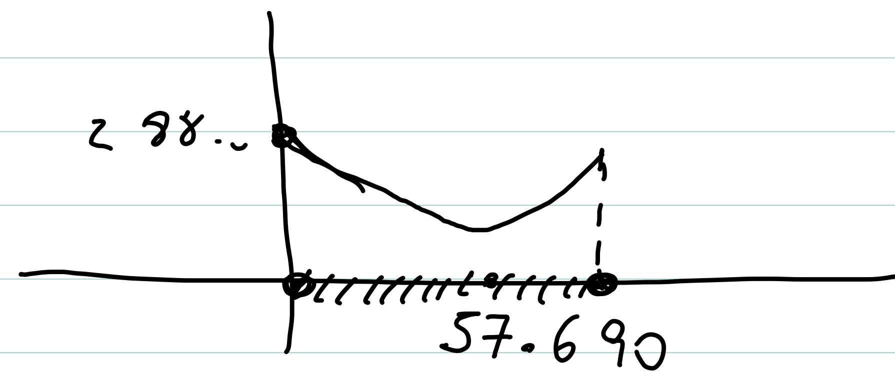
$A(x) = 2x^2 + \frac{2}{9}\left(360^2 - 360\cdot 2\cdot 4x + 16x^2\right)$

$$A(x) = \frac{50}{9}x^2 - 640x + 288000$$



minimum at vertex $h = \dfrac{640}{2 \cdot \frac{50}{9}} \approx \boxed{57.6}$

Maximum at ?



$0 \le x \le 90$

$8x = 720$

$x = \dfrac{720}{8} = 90$

$A(0) = 288000$

$A(90) = 275000$

Area is max for $\boxed{x = 0}$