3. Keoki and Nalani are moving in the xy-plane along straight lines at constant speeds. They both start at the same time.

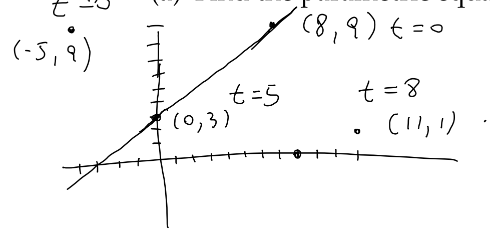
Keoki starts from the point $(-5, 9)$ and heads directly toward the point $(11, 1)$, reaching it in 8 seconds.

Nalani starts from the point $(8, 9)$ and moves toward the y-axis along the line $y = \frac{3}{4}x + 3$.

Nalani takes twice as long to reach the y-axis as it takes Keoki to reach the y-axis.

(a) Find the parametric equations of motion for Keoki.

$t=0$



Keoki $x = -5 + 2t$

$y = 9 - t$

Keoki reeches y exis when $0 = -5 + 2t$ $\quad t = 2.5$

Naleni reechese y exis when $t = 5$

(b) Find the parametric equations of motion for Nalani.

Naleni $x = 8 - \frac{8}{5}t$

$y = 9 - \frac{6}{5}t$

(c) How long has Keoki been moving when the distance between Keoki and Nalani is as small as it ever gets?

$$d = \sqrt{\left(-5+2t-8+\frac{8}{5}t\right)^2 + \left(9-t-9+\frac{6}{5}t\right)^2}$$

$$d^2 = \frac{101}{25}t^2 - \frac{228}{5}t + \frac{3249}{25}$$ hes a min at

$$t = \frac{228}{5} \cdot \frac{25}{202} = \frac{228 \cdot 5}{202} = \frac{1140}{202} = \frac{570}{101} \approx 5.64 \text{ sec}$$

Find parametric equations for Nalani assuming that she starts at $(8, 9)$ and travels along the line $y = \frac{3}{4}x + 3$ towards the y axis at a speed of 5 feet/sec. Assume 1 unit corresponds to 1 ft.

Sol: another point on $y = \frac{3}{4}x + 3$ is for example $(0, 3)$

$d((0\ 3)\ (8\ 9)) = \sqrt{64+36} = 10$

Nalani is at $(0, 3)$ at time $t = \frac{d}{v} = \frac{10}{5} = 2$

$(8\ 9)$ $t=0$

$(0\ 3)$
$t=2$

$v_x = \frac{0-8}{2-0} = -4$

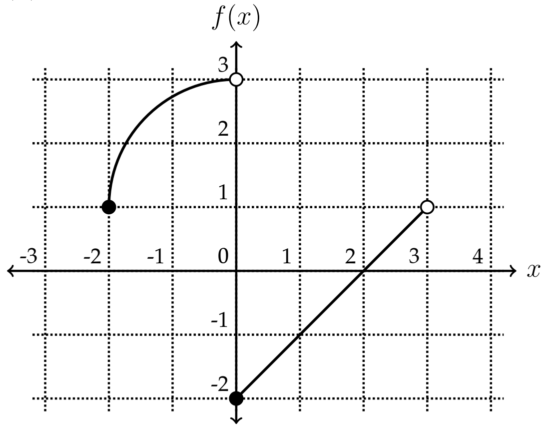
$v_y = \frac{3-9}{2-0} = -3$

so par eq are

$x = 8 - 4t$

$y = 9 - 3t$

4. Here's the graph of $f(x)$.



Use the graph to answer the following questions.

(a) Compute $f(f(f(2)))$ = $f(f(0))$ = $f(-2) = 1$

(b) Is f one-to-one? Why or why not?

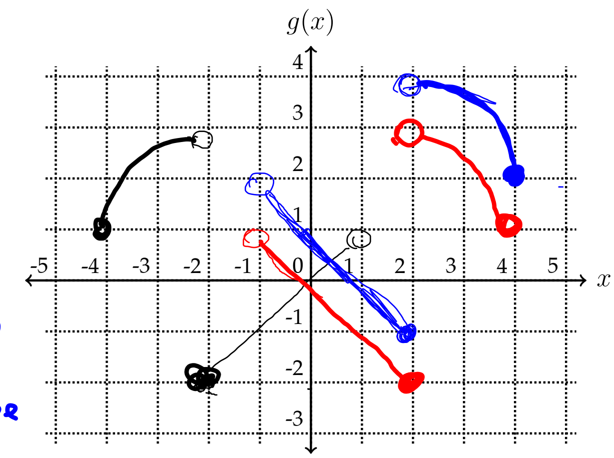
yes it satisfies horizontal line test

(c) Let $g(x) = f(2-x) + 1$.

Sketch a graph of $g(x)$.

1) shift horizontally left 2 units $f(x+2)$
2) reflect across y axis
3) shift vertically up 1 unit

Final graph is the blue one



find domain and range for

$g(x) = f(2-x)+1$

domain f

Domain $\quad -2 \le 2-x < 3$

$-4 \le -x < 1$

$\boxed{4 \ge x > -1}$ or $(-1, 4]$

Range $\quad -2 \le f(x) < 3$

$-2 \le f(2-x) < 3$

$-2+1 \le f(2-x)+1 < 3+1$

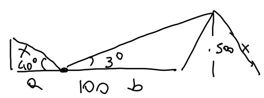
$\boxed{-1 \le y < 4}$ or $[-1, 4)$

6. You are standing somewhere between a mountain and a fountain, which are 100 km apart from each other.

   You know that the mountain is 500 times as tall as the fountain.

   From where you stand, the mountain is at an angle of elevation of $3°$, and the fountain is at an angle of elevation of $40°$.

   How tall is the mountain?



$$\frac{x}{a} = \text{ten } 40 = 0.8391$$

$$\frac{500x}{b} = \tan 3 = 0.0524$$

$$a + b = 100$$

$$x = a \cdot 0.8391$$

$$500\,a\,0.8391 = 0.0524\,(100 - a)$$

$$419.55\,a = 5.24 - 0.0524\,a$$

$$a = \frac{5.24}{419.6024}$$

Mountain is $500\,\frac{5.24}{419.6024} \cdot 0.8391 \approx 5.24$ km

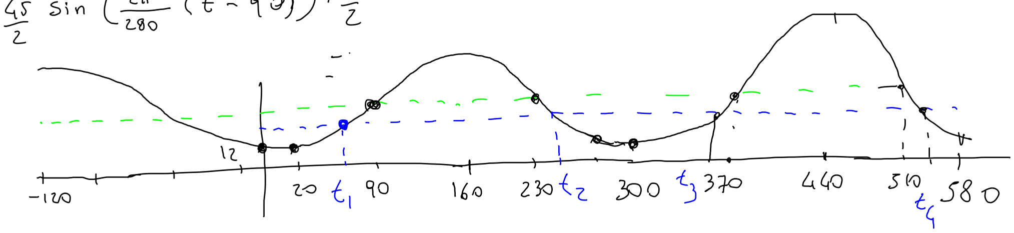
7. The temperature in Skiattle is a sinusoidal function of time.

   120 days ago, the temperature was at its maximum value of 55° F. The temperature has been falling since then, and 20 days from today it will reach its minimum value of 10° F.

   (a) Write a function $f(t)$ for the temperature in Skiattle, in Fahrenheit, t days from today.

$A = \frac{55-10}{2}$ $D = \frac{55+10}{2}$ $B = 2(120+20) = 280$ $C = -120 - 70 = -190$ $+280$ $= 90$

$f(t) = \frac{45}{2} \sin\left(\frac{2\pi}{280}(t-90)\right) + \frac{65}{2}$



   (b) The residents of Skiattle can only ski when the temperature is below 28° F.

   Over the next 500 days (starting from today), for how much time will it be cold enough to ski?

$\frac{45}{2} \sin\left(\frac{2\pi}{280}(t-90)\right) + \frac{65}{2} = 28$

$t_1 = \frac{280}{2\pi} \arcsin\left(\left(28 - \frac{65}{2}\right)\frac{2}{45}\right) + 90 = 81.0268$

$t_2 = 160 + (160 - t_1) = 238.9731$ 280

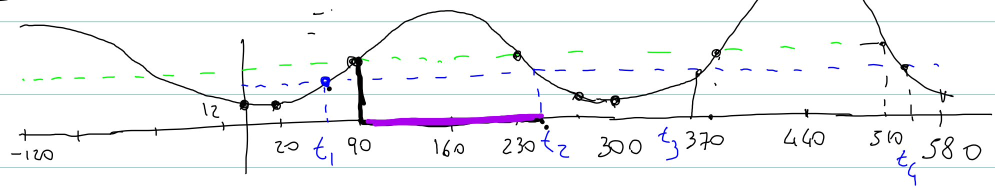
$t_3 = t_1 + 280 = 361.0268$

$t_4 = t_2 + 280 = 518.9731$

Want $t_1 + (t_3 - t_2) \approx 203$ days

If we want the temperature of skiattle to be below 28° for 600 days, how long do we need to wait?

$f(t)=\frac{45}{2}\sin\left(\frac{2\pi}{280}(t-90)\right)+\frac{65}{2}$



If we want the temperature of skiattle to be below 28° for 600 days, how long do we need to wait?

In one period (from $t=90$ $t=370$)

the temperature is below 28 for $t_3-t_2=122.05$ days

$\frac{600}{122.05}\approx 4.91$

total time	90	$4\cdot 280$	?
time below 28	$t_1=81.03$	$122.05\cdot 4$	?

at this point I still need $600-(81.03+122.05\cdot 4)$

$=102.83$ days, so look at the beginning of another period: the period starts with temperature above 28: I need to wait t_2-90 $=148.97$ days for the temperature to reach 28 then another 102.83 days

total time	90	$4\cdot 280$	$148.97+102.83\approx\boxed{1462}$	
time below 28	$90-t_1=81.03$	$122.05\cdot 4$	102.83	$=600$

Rosalie is organizing a circus performance to raise money for a charity. She is trying to decide how much to charge for tickets. From past experience she knows that the number of tickets sold is a linear function of the price. If she charges 5 dollars per ticket , she can sell 1000 tickets, if she charges 7 dollars she can only sell 900 tickets. How much should she charge per tickets to make the most money ?

$(5, 1000)$ $(7, 900)$

$f(x) = 1000 - \frac{100}{2}(x-5)$ number of tickets sold at price x

$g(x) = x(1000 - 50(x-5))$ money made if price of ticket is x

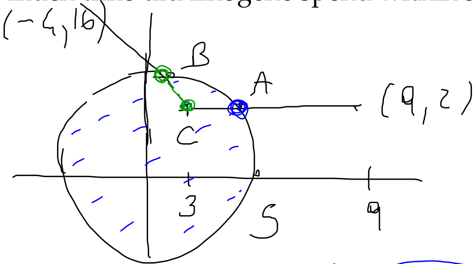
$g(x) = 1000x - 50x^2 + 250x = -50x^2 + 1250x$

max at $x = \frac{1250}{2 \cdot 50} = \boxed{12.50}$

7. Imogene is sailing near a buoy with a radar antenna on it. She sails at a constant 4 miles per hour. The buoy's radar can detect her boat when she is within 5 miles of it. Imogene starts sailing from a point 9 miles EAST and 2 miles NORTH of the buoy. She sails due WEST for 1.5 hours, then turns and sails due directly to a point 16 miles NORTH and 4 miles WEST of the buoy.

How much time did Imogene spend within 5 miles of the buoy?

D: (−4, 16)



$$\underbrace{x^2 + y^2 = 25}_{\text{rader area}}$$

Find A : (y = 2) : $\overbrace{x^2 + (2)^2 = 25}$ $x = \sqrt{21}$ $A = (\sqrt{21}, 2)$

Find C : $y = 2$ $x = 9 - \overset{v}{4} \cdot \overset{t}{1.5}$ $C = (3, 2)$

Find B : Line CD $y = 2 - 2(x - 3)$; $y = -2x + 8$

$$x^2 + (-2x + 8)^2 = 25$$

$$5x^2 - 32x + 39 = 0$$

$$x = \frac{32 \pm 15.62}{10} \quad \begin{cases} 1.638 \\ 4.762 \end{cases}$$

Look at picture choose $\underbrace{\text{smaller}}$ sol

$B = (1.638, 4.724 \ (= -2 \cdot x + 8))$

$d(A\,C) = \sqrt{21} - 3$ $d(C\,B) = 3.046$

$$\text{time} = \frac{\sqrt{21} - 3 + 3.046}{4} \approx 1.157 \text{ hr}$$