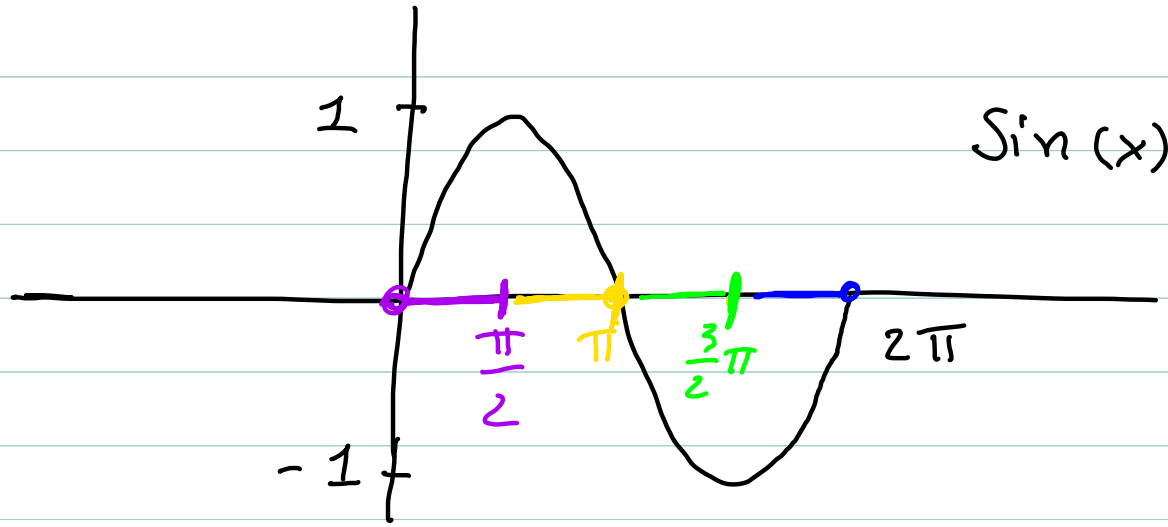
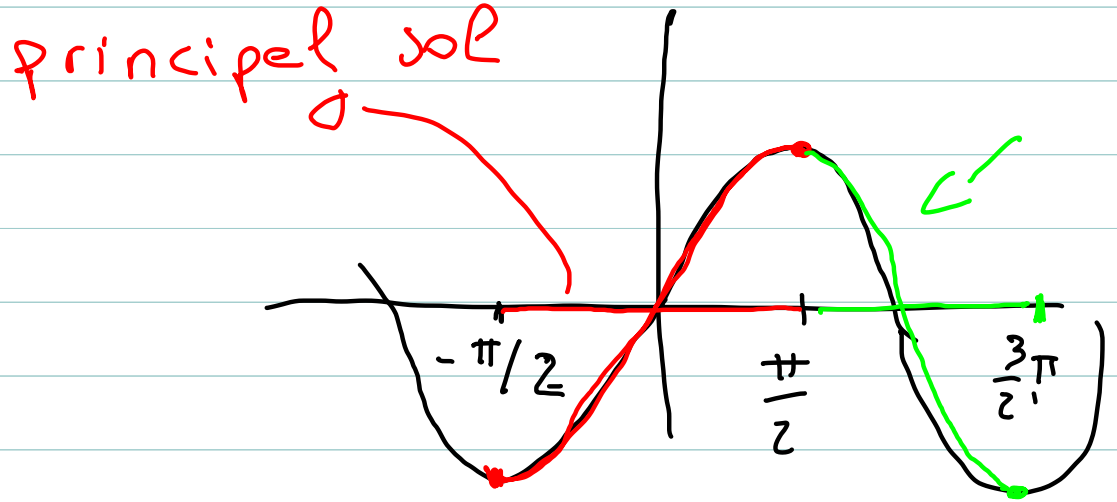
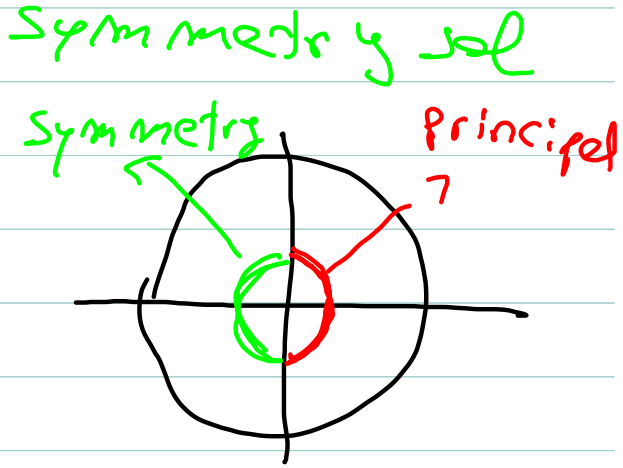
Lesson 24

Sinusoidal functions problems

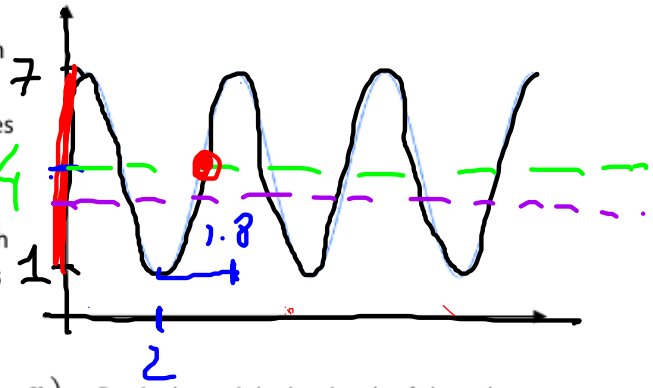

1
-1
Sin (x)
π/2
π
3/2π
2π


principel sol
-π/2
π/2
3/2π


Symmetry sol
Symmetry
principel
7

Spring 2012

Problem 7 (16 pts) The **depth** of a swimming salmon below the water surface can be modeled by a sinusoidal function of time. The salmon's depth varies between a minimum of 1 foot and a maximum of 7 feet below the surface of the water. It takes the salmon 1.8 minutes to move from its minimum depth to its successive maximum depth, and it first reaches the minimum depth at $t = 2$ minutes.



a) Find the sinusoidal function $d(t) = A\sin\left(\frac{2\pi}{B}(t - C)\right) + D$ which models the depth of the salmon after t minutes.

$A = \frac{y_{max} - y_{min}}{2} = \frac{7-1}{2} = 3$, $D = \frac{y_{max} + y_{min}}{2} = \frac{7+1}{2} = 4$

Half period $= \frac{B}{2} = 1.8$ so $B = 3.6$

$C = x_{max} - \frac{B}{4} = 2 + 1.8 - 0.9 = 2.9$

b) Compute all the times during the first 5 minutes when the depth of the fish is exactly 3 feet.

$0 \le t \le 5$

$3\sin\left(\frac{2\pi}{3.6}(t - 2.9)\right) + 4 = 3$

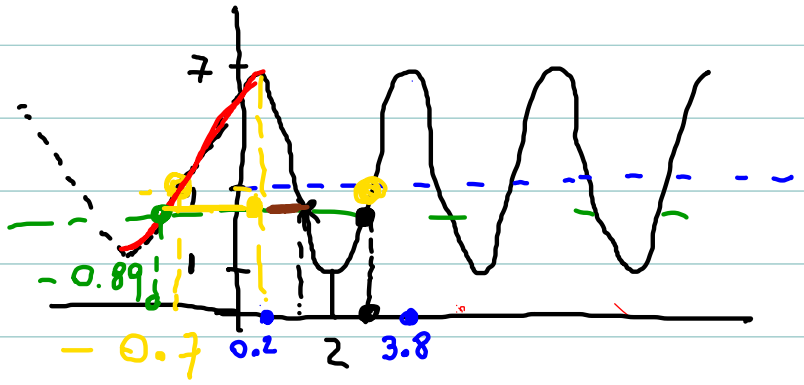
$3\sin\left(\frac{2\pi}{3.6}(t - 2.9)\right) = -1$

$\sin\left(\frac{2\pi}{3.6}(t - 2.9)\right) = -\frac{1}{3}$

$\frac{2\pi}{3.6}(t - 2.9) = \sin^{-1}\left(-\frac{1}{3}\right)$

$t - 2.9 = \frac{3.6}{2\pi}\sin^{-1}\left(-\frac{1}{3}\right)$

$t = \frac{3.6}{2\pi}\sin^{-1}\left(-\frac{1}{3}\right) + 2.9 \simeq 2.7$



To find symmetry solution

$s = 0.2 + (0.2 - (-0.89))$

$x_{max} + (x_{max} - p) = 1.29$

$1.29 + 3.6 = 4.89$

other formula: $s = 2c + \frac{B}{2} - p$

So times are 1.29, 2.7, 4.89

Win 2013

3. The temperature at your house in the desert is a sinusoidal function of time with a 24 hour period. The maximum daily temperature is 45 degrees Celsius and occurs at 5:00 PM. The minimum daily temperature is 11 degrees Celsius.

17

(a) Let t be hours after midnight last night. Find the function $f(t)$ that gives the temperature at time t.

$B = 24$ $\quad A = \frac{45-11}{2} = 17$

$D = \frac{45+11}{2} = 28$ $\quad C = 17 - \frac{24}{4} = 11$

$f(t) = 17 \sin\left(\frac{2\pi}{24}(t-11)\right) + 28$

$0 \le t < 24$

(b) For how much of each day is the temperature above 35 degrees Celsius?

Solve $17 \sin\left(\frac{2\pi}{24}(t-11)\right) + 28 > 35$

First solve

$17 \sin\left(\frac{2\pi}{24}(t-11)\right) + 28 = 35$

see next page

(c) Starting from midnight last night, how long will it be until the temperature has been above 35 degrees Celsius for 22 hours?

solve

$$17 \sin\left(\frac{2\pi}{24}(t-11)\right) + 28 = 35$$

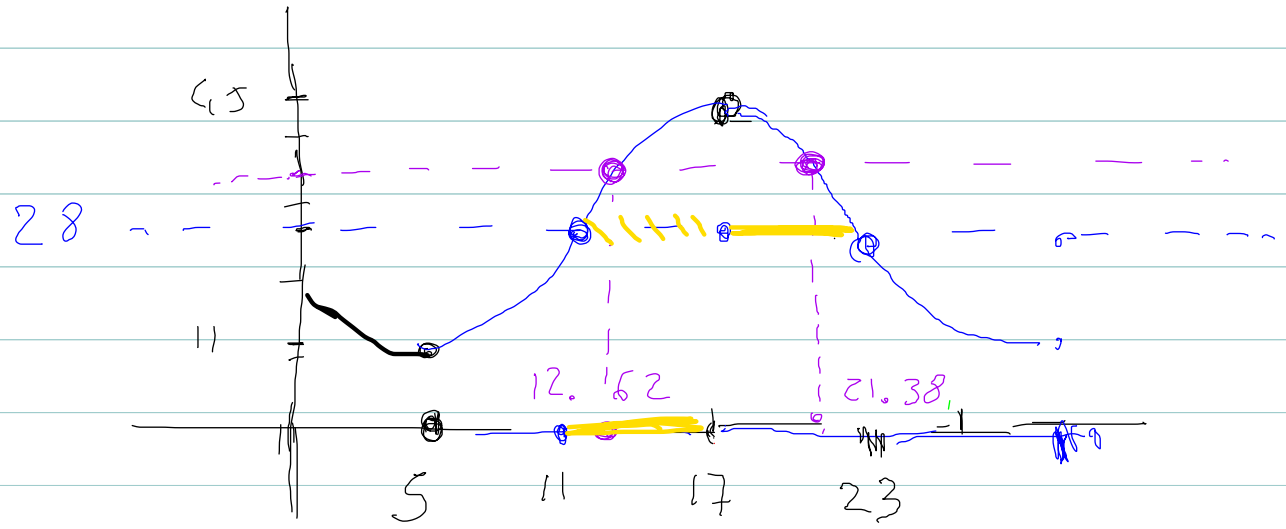
$$17 \sin\left(\frac{2\pi}{24}(t-11)\right) = \frac{7}{17}$$

$$\frac{2\pi}{24}(t-11) = \sin^{-1}\left(\frac{7}{17}\right)$$

$$t = \frac{24}{2\pi} \sin^{-1}\left(\frac{7}{17}\right) + 11$$

$$t = 12.62$$

only principal solution in range $0 \le t \le 24$

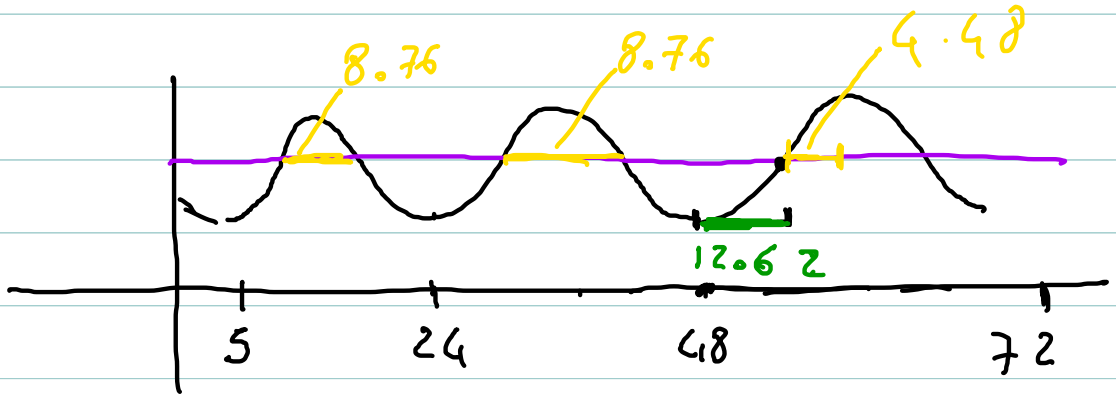


symmetry = $17 + (17 - 12.62) = 21.38$

time it stays above 35

is $21.38 - 12.62 = 8.76$ hours

Part c)



$22 - 8.76 \times 2 = 4.48$ ← time needed in day 3

So the temperature stayed above 35

22 hours for

24 + 24 + 12.62 + 4.48

↑ day 1 ↑ day 2 ↑ day 3

8.76 h 8.76 h time we have to wait for temperature to reach 35

$= 65.1$ hrs

Fall 2012

NOT DONE IN CLASS

5. The diameter of a certain cloud in the sky above Seattle is a sinusoidal function of time.
   At 7 AM this morning, the diameter of the cloud was at its minimum, 20 meters.
   The cloud then expanded, and reached its maximum diameter of 26 meters at 11:30 AM this morning.
   From 3 AM this morning to 3 PM this afternoon, for how much time was the cloud's diameter less than 21.5 meters?

$t = 0$ corresponds to 12 am

$A = \frac{26-20}{2} = 3$, $D = \frac{26+20}{2} = 23$

$\frac{B}{2} = 4.5$ so $B = 9$, $C = 11.5 - \frac{9}{4} = 9.25$

$y(t) = 3\sin\left(\frac{2\pi}{9}(t-9.25)\right) + 23$

solve

$3\sin\left(\frac{2\pi}{9}(t-9.25)\right) + 23 = 21.5$ for

$3 \le t \le 15$ → 3 pm

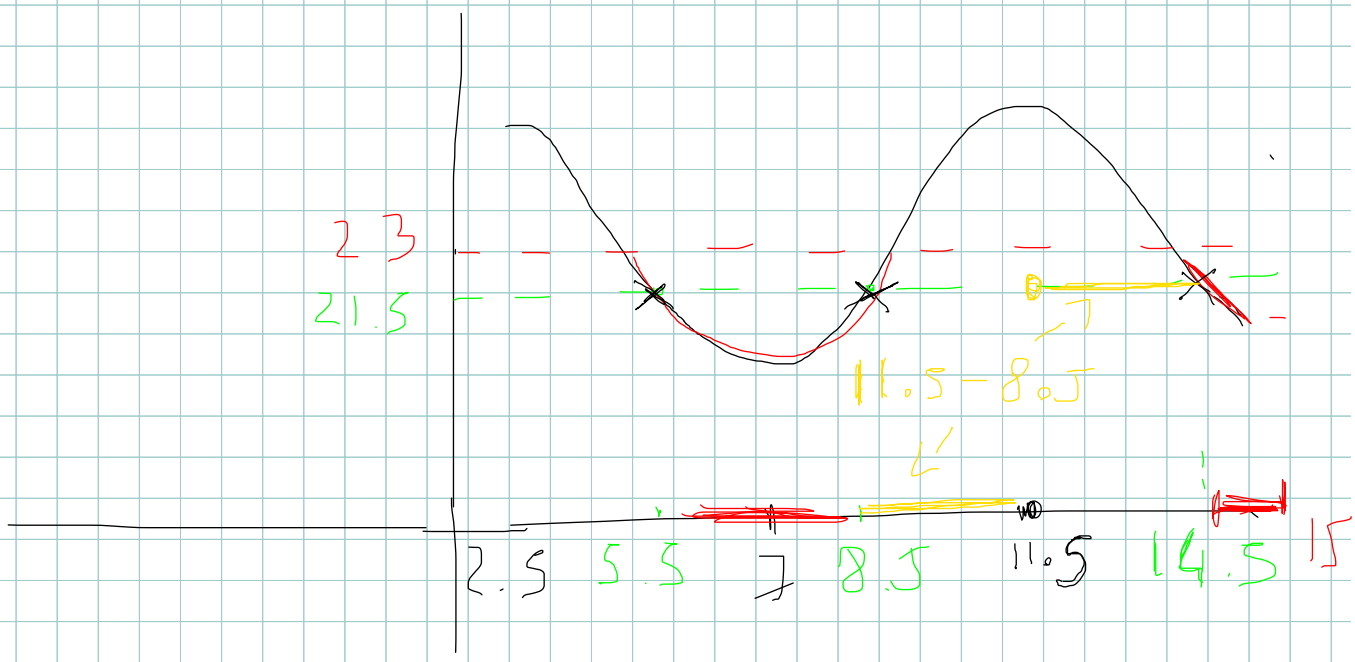
1) $\frac{2\pi}{9}(t-9.25) = \sin^{-1}\left(-\frac{1.5}{3}\right)$

$t = 8.5$ principal solution

If we add or subtract 9 we get solutions outside the range $3 \le t \le 15$

2) Symmetry solution

see graph next page



Symmetry = $11.5 + (11.5 - 8.5) = 14.5$

other symm. sol $14.5 - 9 = 5.5$

Less than 21.5 m (looking at graph) for

$(8.5 - 5.5) + (15 - 14.5) = 3.5$ h