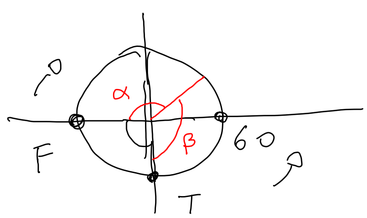
1. Fred and Ted are running around a circular track. The track has a radius of 60 meters.

   Fred starts from the westernmost point of the track, and runs clockwise.

   Ted starts at the same time from the southernmost point of the track, and runs counter-clockwise.

   Fred runs at a constant speed of 9 meters per second, and passes Ted for the first time after 15 seconds.

   (a) What is Ted's speed in meters per second?



$$|\omega_F| = \frac{9}{60} = \frac{3}{20} \text{ rad/sec}$$

$$\frac{\pi}{2} + \overbrace{\frac{9}{60} \cdot 15}^{\alpha} + \overbrace{\omega_T \cdot 15}^{\beta} = 2\pi$$

$$\omega_T = \frac{\frac{3}{2}\pi - \frac{9}{4}}{15} = \frac{6\pi - 9}{60} \text{ rad/sec}$$

$$v_T = 6\pi - 9 \text{ m/sec}$$

   (b) After running for 200 seconds, who is farther North, Fred or Ted? Show all work.

$$y_F = 60 \sin\left(-\frac{3}{20}t + \pi\right), \quad y_T = 60 \sin\left(\frac{6\pi - 9}{60}t - \frac{\pi}{2}\right)$$

$$y_F(200) = -59.28$$

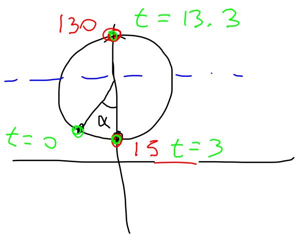
$$y_T(200) = -9.26$$

Ted is farther North

Spring 2007

5. Jessie is riding a ferris wheel. From when the ride starts, it takes her 3 seconds to reach the lowest point on the ride, and 13.3 seconds (from the start) to reach the highest point on the ride. The highest point on the ride is 130 feet off the ground, and the lowest point is 15 feet off the ground.

   How high off the ground is she after riding the ferris wheel for 2 minutes?



$$r = \frac{130 - 15}{2} = \frac{115}{2} \qquad C\left(0, \frac{145}{2}\right)$$

$$y = \frac{145}{2} + \frac{115}{2}\sin(\omega t + \varphi)$$

$$T = 2 \cdot 10.3 = 20.6$$

$$\omega = \frac{2\pi}{20.6}$$

$$\alpha = \frac{2\pi}{20.6} \cdot 3 \text{ rad}$$

$$\varphi = -\frac{\pi}{2} - \alpha$$

$$y(120) = \frac{145}{2} + \frac{115}{2}\sin\left(\frac{2\pi}{20.6} \cdot 120 - \left(\frac{\pi}{2} + \overbrace{\frac{6\pi}{20.6}}^{\alpha}\right)\right) \approx 97.11$$

2. At the beginning of 2001, Clovis invested \$ 10,000 in an account. In 2013 his investment was worth \$ 14,764.

   Isobel made an investment at the same time as Clovis. Her investment doubles every 7 years. In 2014, Clovis had 8 times as much in his account as Isobel had in hers.

   Assume the values of both investments are exponential functions of time.

   Take $t = 0$ in 2001.

   (a) Give an exponential function relating the value of Clovis's investment y to the year t.

$y(t) = 10,000\, a^t$

$14\,764 = 10\,000\, a^{12}$ $\quad a = \sqrt[12]{1.4764}$

$y(t) = 10,000\left(\sqrt[12]{1.4764}\right)^t$

   (b) What is the annual growth rate of Clovis' investment?

If $y(t) = 10,000\,(1+r)^t$ what is r?

$10,000\,\sqrt[12]{1.4764}^{\,t} = 10,000\,(1+r)^t$

$r = \sqrt[12]{1.4764} - 1$

   (c) What was the value of Isobel's investment at the beginning of 2001?

$f(t) = A_0\, \sqrt[7]{2}^{\,t}$

$A_0\, \sqrt[7]{2}^{\,13} = \frac{1}{8} \cdot 10,000\, \sqrt[12]{1.4764}^{\,13}$

$A_0 = \frac{10\,000}{8}\left(\frac{\sqrt[12]{1.4764}}{\sqrt[7]{2}}\right)^{13} \approx 526.21$

   (d) How many years after 2001 will Clovis have five times as much money as Isobel?

$10\,000\, \sqrt[12]{1.4764}^{\,t} = 5 \cdot 526.21\, \sqrt[7]{2}^{\,t}$

$\left(\frac{\sqrt[12]{1.4764}}{\sqrt[7]{2}}\right)^t = \frac{5 \times 526.21}{10000}$

$t = \frac{\ln\left(\frac{5 \cdot 526.21}{10\,000}\right)}{\ln\left(\frac{\sqrt[12]{1.4764}}{\sqrt[7]{2}}\right)} \approx 20$ years