

MATH 120 (Pezzoli)
SPRING 2016
MIDTERM #2

NAME _____

TA: _____

SECTION: _____

Instructions:

- Your exam contains 3 problems.
 - Your exam should contain 5 pages; please make sure you have a complete exam.
 - Box in your final answer when appropriate.
 - Unless stated otherwise, you **MUST** show work for credit. No credit for answers only. If in doubt, ask for clarification.
 - Your work needs to be neat and legible.
 - You are allowed one 8.5×11 sheet of notes (both sides). The only calculator allowed is Texas Instruments ti 30x iis.
 - Round off your answers to 2 decimal places, unless you are asked for exact answers.
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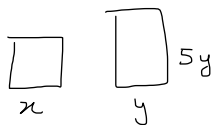
Problem #1 (15 pts) _____

Problem #2 (15 pts) _____

Problem #3 (15 pts) _____

TOTAL (45 pts) _____

1. You have 2000 meters of fencing with which to build two enclosures. One enclosure will be a square, and the other will be a rectangle that is 5 times as long as it is wide. What should the dimensions of the rectangular enclosure be in order to minimize the combined area of the two enclosures?



$$A = x^2 + 5y^2, \quad 2000 = 4x + 12y, \quad x = 500 - 3y$$

$$A = (500 - 3y)^2 + 5y^2 = 14y^2 - 3000y + C \quad \cup$$

$$\text{min at } y = -\frac{b}{2a} = \frac{3000}{28}$$

Dimensions

$$y = \frac{3000}{28} \approx 107.14 \text{ m}$$

$$5y = \frac{15000}{28} \approx 535.71 \text{ m}$$

2. Weeds are taking over Mary's garden. 30 days ago she noticed 4 dandelions and 3 thistle weeds. Today she counted 20 dandelions. Assuming that the number of dandelions and thistle in Mary's garden grows exponentially, and that the number of thistle doubles every two weeks (14 days), when will there be twice as many ^{dandelion} thistle than dandelions in the garden? Give the answer in days from today (for example 230 days from today).

$$D(t) = A_0 a^t \text{ goes through } (4, -30) \text{ } (0, 20) ; A_0 = 20$$

$$4 = 20 a^{-30}, a^{30} = 5, a = \sqrt[30]{5} \approx 1.055113$$

$$D(t) = 20 \left(\sqrt[30]{5} \right)^t$$

$$T(t) = B_0 b^t, \text{ we know } 14 = \frac{\ln 2}{\ln b} \quad \ln b = \frac{\ln 2}{14}, b = e^{\frac{\ln 2}{14}} = \sqrt[14]{2} \approx 1.050757, \text{ we know } 3 = B_0 \left(\sqrt[14]{2} \right)^{-30}$$

$$B_0 = 3 \left(\sqrt[14]{2} \right)^{30} \approx 13.2491$$

$$\text{We want } D(t) = 2T(t), 20 \left(\sqrt[30]{5} \right)^t = 6 \left(\sqrt[14]{2} \right)^{30} \sqrt[14]{2}^t$$

$$\left(\frac{\sqrt[30]{5}}{\sqrt[14]{2}} \right)^t = \frac{6 \sqrt[14]{2}^{30}}{20}$$

$$\ln \left(\frac{6 \sqrt[14]{2}^{30}}{20} \right)$$

$$t = \frac{\ln \left(\frac{6 \sqrt[14]{2}^{30}}{20} \right)}{\ln \left(\frac{\sqrt[30]{5}}{\sqrt[14]{2}} \right)} \approx 68 \text{ days}$$

3. Water is filling a trough. The width of the water, in feet, at time t , in hours is given by

$$f(t) = 2\sqrt{49 - \left(-7 + \frac{t}{3}\right)^2}$$

the domain of $f(t)$ is $0 \leq t \leq 21$ and the range of $f(t)$ is $0 \leq w \leq 14$.

(a) Give the domain and range of $f^{-1}(w)$.

DOMAIN: $0 \leq w \leq 14$

RANGE: $0 \leq t \leq 21$

(b) Find a formula for $f^{-1}(w)$, the inverse function of $f(t)$.

$$w = 2\sqrt{49 - \left(-7 + \frac{t}{3}\right)^2}, \quad \frac{w^2}{4} = 49 - \left(-7 + \frac{t}{3}\right)^2$$

$$\left(-7 + \frac{t}{3}\right)^2 = 49 - \frac{w^2}{4}, \quad -7 + \frac{t}{3} = \pm \sqrt{49 - \frac{w^2}{4}}$$

$$t = 21 - 3\sqrt{49 - \frac{w^2}{4}}, \quad \text{choose } - \text{ since } t \leq 21$$

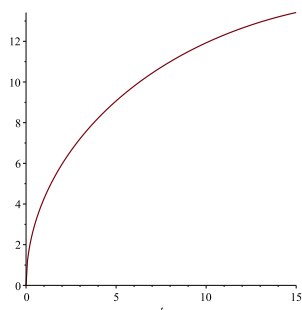
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(c) calculate $f^{-1}(3)$ ~~exactly~~. $f^{-1}(3) \approx 0.49$

(d) Explain in words what $f^{-1}(3)$ represents

After 0.49 hours the width of the water in the trough will be 3 feet

(continued from the previous page) The graph of $f(t)$ is given below.



Which of the following graphs is the graph of $-2f(3t - 4) + 5$? Circle the correct graph.

$$-2f\left(3\left(t - \frac{4}{3}\right)\right) + 5$$

