1. Determine if the following statements are true or false. If true, prove the statement and/or cite a relevant theorem. If false, give a counterexample, where $a$ and $b$ are numbers and the function $f$ is specific and explain why your counterexample works.

(a) If $f : [a, b] \to \mathbb{R}$ is integrable and $\int_a^b f = 0$, then $f(x) = 0$ for all $x$ in $[a, b]$.
(b) If $f : [a, b] \to \mathbb{R}$ is continuous and $\int_a^b f = 0$, then $f(x) = 0$ for all $x$ in $[a, b]$.
(c) If $f : [a, b] \to \mathbb{R}$ is integrable, then $f$ is continuous on $[a, b]$.
(d) If $f : [a, b] \to \mathbb{R}$ is integrable and $f(x) \geq 0$ for all $x$ in $[a, b]$, then $\int_a^b f \geq 0$.
(e) If $f : [a, b] \to \mathbb{R}$ is continuous on $[a, b]$, then $f$ is integrable over $[a, b]$.
(f) If $f : [a, b] \to \mathbb{R}$ is continuous on $(a, b)$, then $f$ is integrable over $[a, b]$.

2. Let $f : [0, 1] \to \mathbb{R}$ be given by

$$f(x) = \begin{cases} 
  x & \text{if } x \text{ is rational,} \\
  -x & \text{if } x \text{ is irrational.}
\end{cases}$$

Prove that $f : [0, 1] \to \mathbb{R}$ is not integrable.

3. Suppose that the continuous function $f : [0, 1] \to \mathbb{R}$ has the property that $f(x) \geq 0$ for all $x \in [0, 1]$. Prove that $\int_0^1 f > 0$ if and only if there exists an $x_0$ with $f(x_0) > 0$.

Hint: See Question 3 in Homework 3.

4. Compute the following derivatives. You may remember doing these applications of the Fundamental Theorem of Calculus in Math 125.

(a) $\frac{d}{dx} \int_0^5 \frac{1}{1 + t^4} dt$.
(b) $\frac{d}{dx} \int_1^x \frac{e^t}{1 + t} dt$.
(c) $\frac{d}{dx} \int_{-1}^{x^3} e^{t^2} dt$.
(d) $\frac{d}{dx} \int_{e^x}^{\sin x} \sqrt{1 + 7t^6} dt$. 