Math 327 Homework 1

1. Prove that the product of a nonzero rational number and an irrational number is irrational. How about the product of two irrational numbers?

2. Find the inf (greatest lower bound), sup (least upper bound), max and min, if they exist, for the following sets. Prove your answers in (d) and (e). To prove m is the inf of a set, you have to show it is a lower bound and that no number bigger than m is a lower bound. To prove M is the sup of a set, you have to show it is an upper bound and that no number smaller than M is an upper bound.

   (a) $S = \{1, 3, 5, 7, 9\}$

   (b) $S = (3, \pi]$

   (c) $S = \{q \in \mathbf{Q} : 3 < q \leq \pi\}$

   (d) $S = \{\frac{1}{a} : a \in \mathbf{Z}, a \neq 0\}$

   (e) $S = \{\frac{n+2}{2n+5} : n \in \mathbf{N}\}$

3. Suppose A and B are non-empty sets of real numbers that are both bounded above.

   (a) Prove that if $A \subset B$, then $\sup A \leq \sup B$.

   (b) Prove that $\sup(A \cup B) = \max\{\sup A, \sup B\}$.

   (c) Prove that if $A \cap B \neq \emptyset$, then $\sup(A \cap B) \leq \min\{\sup A, \sup B\}$. Give an example to show that equality need not hold.