Math 135, Winter 2015, Homework 7

For practice - do not hand in

1. From TP, page 311, problems 12-23.

2. Prove Theorem 1 from lecture and deduce Example 2 as a corollary.

3. Show that \( f(t) = e^{t^2} \) is not of exponential order.

4. Compute the Laplace transforms of the following functions:
   
   (a) \( f_1(t) = te^{4t} \cos(-2t), \ f_2(t) = \cos^2(t), \ f_3(t) = \sqrt{t}e^t. \)
   
   (b) \( f(t) = \begin{cases} 4, & t < 2, \\ t + 2, & 2 \leq t \leq 5, \\ e^{-t}, & t > 5. \end{cases} \)

5. Compute the inverse Laplace transforms of the following functions:
   
   (a) \( F_1(s) = \frac{1}{s^2 + 2s + 10}, \ F_2(s) = \frac{3s}{s^2 + 4s + 13}, \ F_3(s) = \frac{2s + 7}{s^2 + 6s + 9}. \)
   
   (b) \( F_1(s) = \frac{s^2 - 6}{s^3 + 4s^2 + 3s}, \ F_2(s) = \frac{16}{s(s^2 + 4)}, \ F_3(s) = \frac{6s - 3}{s(s + 1)^2}. \)
   
   (c) \( F(s) = \frac{(1 - e^{-2s})(1 - 3e^{-2s})}{s^2}. \)

To hand in

1. (a) Use
   \[
   \int \text{Re} \left( e^{(a+bi)t} \right) dt = \text{Re} \left( \int e^{(a+bi)t} dt \right)
   \]
   where \( \text{Re}(z) \) is the real part of a complex number \( z \), to compute \( \int e^{at} \cos bt dt. \)

   (b) Find \( \mathcal{L}\{\cos(bt)\}. \)
   
   (c) Use Theorem 3 from lecture to find \( \mathcal{L}\{\sin(bt)\}. \)
   
   (d) Use Theorem 5 from lecture to find \( \mathcal{L}\{t \sin(bt)\}. \)
   
   (e) Use Theorem 1 from lecture to find \( \mathcal{L}\{e^{at} \sin(bt)\}. \)
   
   (f) Compute \( \mathcal{L}\{te^{at} \sin(bt)\}. \)

2. Solve the initial value problem \( y'' + y = f(t), \ y(0) = 0 \) and \( y'(0) = 0 \) where
   \[
   f(t) = \begin{cases} 4, & 0 \leq t \leq 2, \\ t + 2, & t > 2. \end{cases}
   \]

3. Compute \( \mathcal{L}\{h\} \) where
   \[
   h(t) = \begin{cases} t, & 0 \leq t < 1, \\ h(t-1), & t \geq 1, \end{cases}
   \]

   is the sawtooth wave function.