## Part I - Vectors and Their Components

Given a vector $\mathbf{v}=\langle a, b\rangle$, you can use the Pythagorean Theorem to find its length and trigonometry to find the angle it makes with the $x$ or $y$ axes. Conversely, given the length of the vector and the angle it makes with the $x$ or $y$ axes, you can write down its components.

1. Find the length $|\mathbf{v}|$ and the angle $\theta$ for the vector below.

2. Write the vector below in therms of its components as $\mathbf{v}=<a, b>$.
3. Below is a regular hexagon of each side equal to 2 units.

(a) First, use geometry to figure out the angle $\theta$ shown in the picture.
(b) Use the angle and the length 2 of each vector to write down the vectors $\mathbf{u}, \mathbf{v}$ and $\mathbf{w}$.
(c) Now, use the symmetry to write down the vectors $\mathbf{a}, \mathbf{b}$ and $\mathbf{c}$.
(d) Finally, use the vectors and the fact that the hexagon has one of its vertices at the origin to compute the coordinates of all its six vertices.

## Part II - Properties of Vector Addition and Scalar Multiplication

Vector addition and scalar multiplication have properties just like ordinary addition of numbers, for example, $x+y=y+x$ and $c(x+y)=c x+c y$ which we use all the time without much thinking about them. Verify the below properties for the given vectors and think about why they should hold all the time, regardless of the specific vector and scalars given. A scalar is an ordinary number in the vector world.

## Checking the Addition and Scalar Multiplication Properties Arithmetically

For the following vectors

$$
\mathbf{a}=\langle 1,0,-5\rangle, \mathbf{b}=\langle 2,3,6\rangle, \mathbf{c}=\langle 7,6,2\rangle
$$

and the scalars

$$
d=2, e=5
$$

check the following properties by computing both sides. Think about why the property would hold for all vectors. The zero vector is $\mathbf{0}=\langle 0,0,0\rangle$. The first one is done as an example.

1. For any two vectors $\mathbf{a}$ and $b$, we have $\mathbf{a}+\mathbf{b}=\mathbf{b}+\mathbf{a}$.

$$
\begin{aligned}
& \langle 1,0,-5\rangle+\langle 2,3,6\rangle=\langle 3,3,1\rangle \\
& \langle 2,3,6\rangle+\langle 1,0,-5\rangle=\langle 3,3,1\rangle
\end{aligned}
$$

2. For any three vectors $\mathbf{a}, \mathbf{b}$, and $\mathbf{c}$ we have $(\mathbf{a}+\mathbf{b})+\mathbf{c}=\mathbf{a}+(\mathbf{b}+\mathbf{c})$.
3. For any two vectors a and $b$ and scalar $d$, we have $d(\mathbf{a}+\mathbf{b})=d \mathbf{a}+d \mathbf{b}$.
4. For any vector a and scalars $d$ and $e$, we have $(d+e) \mathbf{a}=d \mathbf{a}+e \mathbf{a}$.
5. For any vector a and scalars $d$ and $e$, we have $(d e) \mathbf{a}=d(e \mathbf{a})$.

## Checking the Addition and Scalar Multiplication Properties Geometrically

Now, you will check the same Properties 1-5 by drawing both sides of the equalities to confirm that they are the same vector. Here, the examples vector are two dimensional to make it easy to sketch:

$$
\mathbf{a}=\langle 1,2\rangle, \mathbf{b}=\langle 3,-1\rangle, \mathbf{c}=\langle 4,0\rangle, d=2, e=3
$$

The first one is done as an example.


Here is another page for more space to work. The vectors and scalars were

$$
\mathbf{a}=\langle 1,2\rangle, \mathbf{b}=\langle 3,-1\rangle, \mathbf{c}=\langle 4,0\rangle, d=2, e=3
$$

