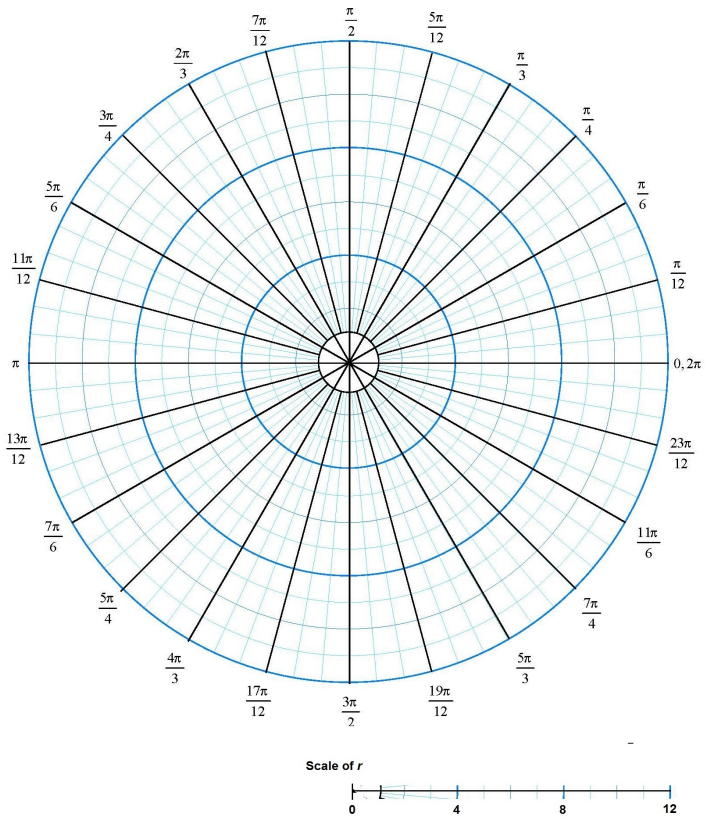
Graphing Polar Curves

The aim of this worksheet is to help you familiarize with the polar coordinate system. In particular, how the angle θ increases counter-clockwise and how the radius r increases going away from the origin. In the first examples, you can make a table of values and plot them. As you get more comfortable, you start thinking whether $|r|$ is increasing (spiraling outward) or decreasing (sprinkling inward). You also have to keep track of the sign of r.

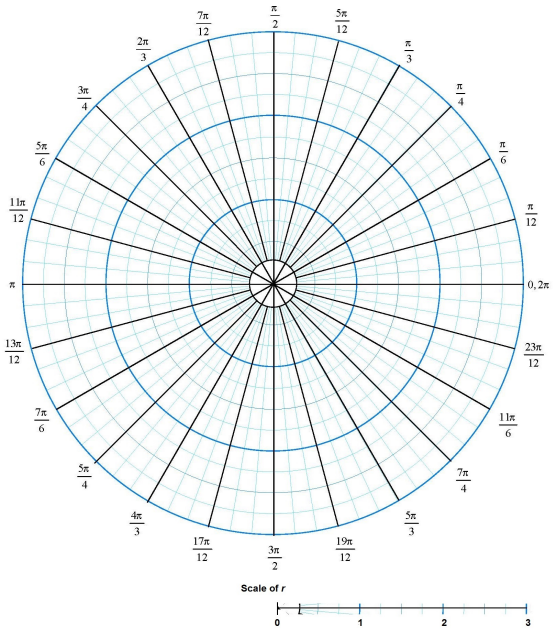
1. Graph $r = \theta$ by filling out the table using your calculator. Each circular tick corresponds to 1 unit on the scale of r as shown.

θ	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π	$\frac{5\pi}{2}$	3π	$\frac{7\pi}{2}$	4π
$r = \theta$									



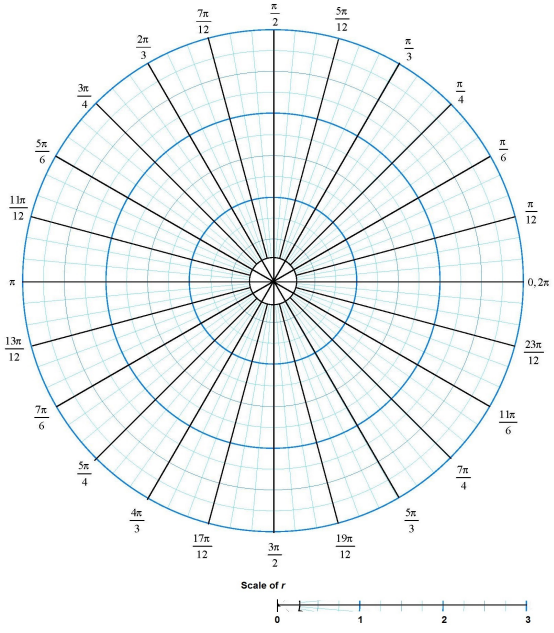
2. Graph $r = 1 + \sin\theta$ by filling out this table of values and plotting on the graph. The graph is scaled so that the radius of the complete circle is 3 units.

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{7\pi}{4}$	$\frac{11\pi}{6}$	2π
$r = 1 + \sin\theta$																	



3. Graph $r = 1 + \cos\theta$ by using the table and information below. The scaling is the same as above. When r increases, it spirals away from the origin. When r decreases, it spirals towards the origin.

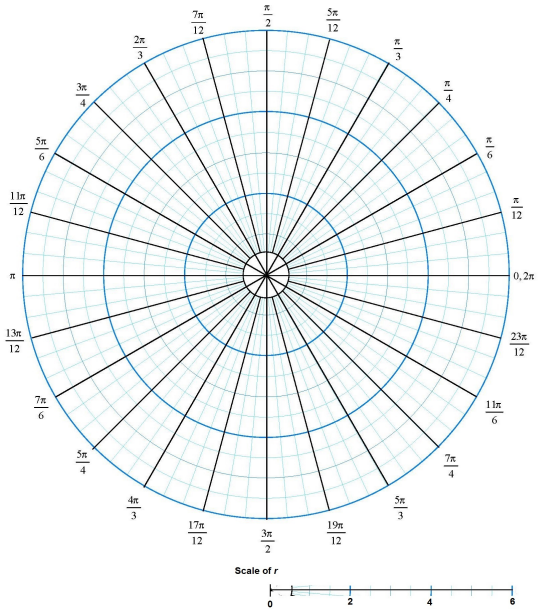
θ	0		$\frac{\pi}{2}$		π		$\frac{3\pi}{2}$		2π
$r = 1 + \cos\theta$	2	decreasing	1	decreasing	0	increasing	1	increasing	2



Both graphs have the same shape with different orientations. They have symmetry with respect to the x or y axes.

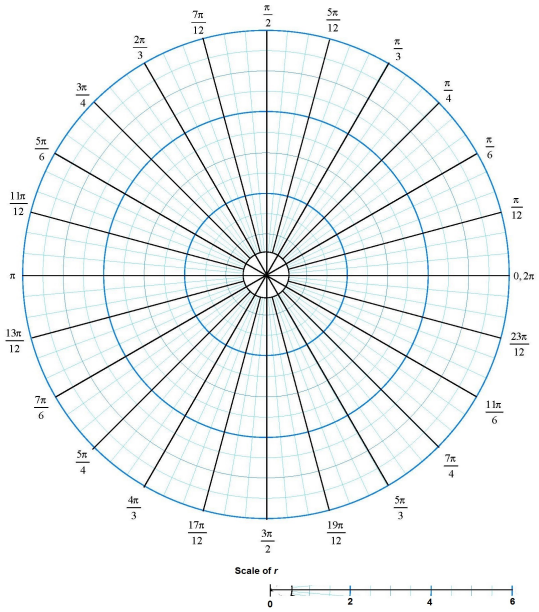
4. Graph $r = 4 + 2\cos\theta$ below by completing the following table in a way similar to the previous graph. For the values in between multiples of $\pi/2$, make a note of whether r is increasing or decreasing and then use that information to graph the polar curve. The graph is scaled so the whole circle has radius 6, the maximum possible value for r.

θ	0		$\frac{\pi}{2}$		π		$\frac{3\pi}{2}$		2π
$r = 4 + 2\cos\theta$									



5. Graph $r = 2 + 4\sin\theta$ below by completing the following table . Compute the r values for the given θ. For the values in between, just make a note of whether $|r|$ is increasing (spiraling out) or decreasing (spiraling in) and then graph the polar curve. Note that r takes **negative values** between $\frac{7\pi}{6}$ and $\frac{11\pi}{6}$.

θ	0		$\frac{\pi}{2}$		π		$\frac{7\pi}{6}$		$\frac{3\pi}{2}$		$\frac{11\pi}{6}$		2π
$2 + 4\cos\theta$													



The graphs in Questons 2-5 are from the family of cardioids. They have equations of the form $r = a + b\cos\theta$ or $r = a + b\sin\theta$, with $a, b > 0$. You get one of the three shapes you drew depending on whether $a > b$ (Question 4), $b < a$ (Question 5) or $b = a$ (Questions 2 and 3). The other popular family of polar curves are the roses with equations $r = a\cos(n\theta)$ or $r = a\sin(n\theta)$ where $n > 1$ is a positive integer. You can use the polar graphs below to draw some examples from the book or sketch your homework problems. Scale the r as appropriate.