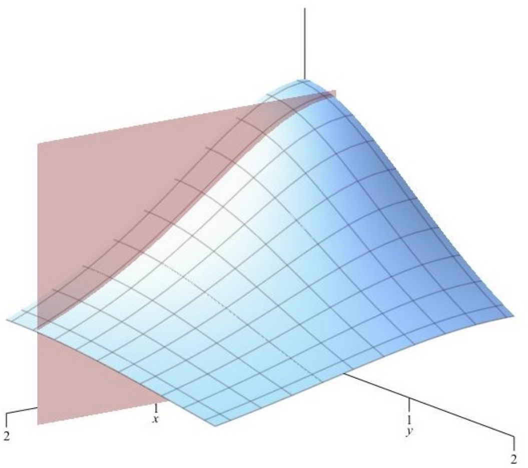
Partial Derivatives and Slopes

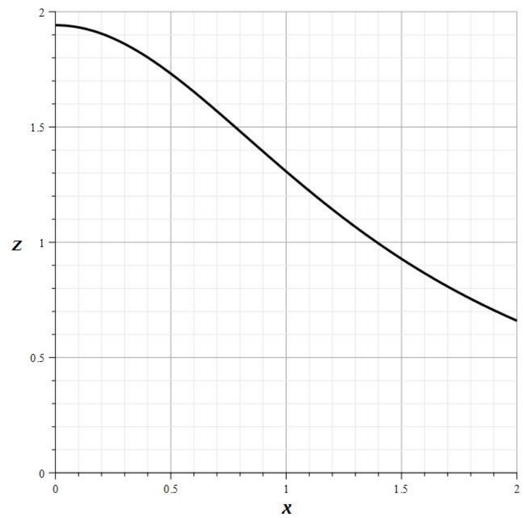
The questions below use the function

$$f(x, y) = \frac{12}{6 + 3x^2 + 2y^2}.$$

1. The first example is about understanding $f_x(a, b)$ as a slope. You will compute $f_x(0.5, 0.3)$.



The surface *z=f(x,y)* and the plane *y=0.3*



z=f(x,0.3)

   (a) Graph the line (as best as you can) on the plane $y = 0.3$, tangent to the curve at $x = 0.5$ on the picture on the left. The slope of this line computed as $\frac{\Delta z}{\Delta x}$ is $f_x(0.5, 0.3)$.

   (b) The curve on the right is the same as the curve of intersection of the surface $z = f(x, y)$ and the plane $y = 0.3$ on the left. You can think about the xz- axes on the right as the plane $y = 0.3$ on the left. Graph the line tangent to the curve at $x = 0.5$ on the picture on the right. Compute its slope (approximately) from the graph.

   (c) Now let $g(x) = f(x, 0.3)$ and simplify. This is the graph you see on the right. Compute $g'(x)$ and $g'(0.5)$.

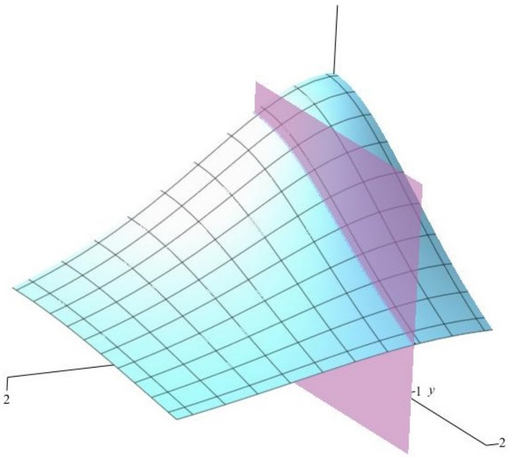
   (d) Compute $f_x(x, y)$ and $f_x(0.5, 0.3)$.

   (e) Now, using that same slope you computed three times in parts (b)-(d), write a vector that is in the same direction as the tangent line on the right. Modify that vector by adding a y-component to write a vector parallel to the tangent line you drew on the left.

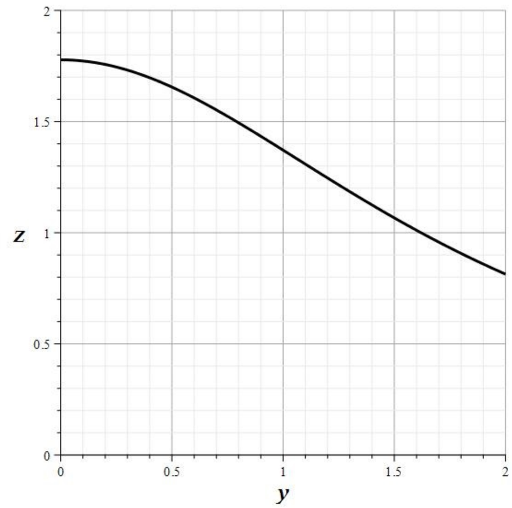
Again, the function is

$$f(x, y) = \frac{12}{6 + 3x^2 + 2y^2}.$$

2. The second example is about understanding $f_y(a, b)$ as a slope. You will compute $f_y(0.5, 0.3)$.



The surface *z=f(x,y)* and the plane *x=0.5*



z=f(0.5,y)

(a) Graph the line on the plane $x = 0.5$, tangent to the curve at $(0.5, 0.3, f(0.5, 0.3)$ on the picture on the left. The slope of this line computed as $\frac{\Delta z}{\Delta y}$ is $f_y(0.5, 0.3)$.

(b) Graph the line tangent to the curve at $y = 0.3$ on the picture on the right. Compute its slope (approximately) from the graph.

(c) Now let $g(y) = f(0.5, y)$ and simplify. Compute $g'(y)$ and $g'(0.3)$.

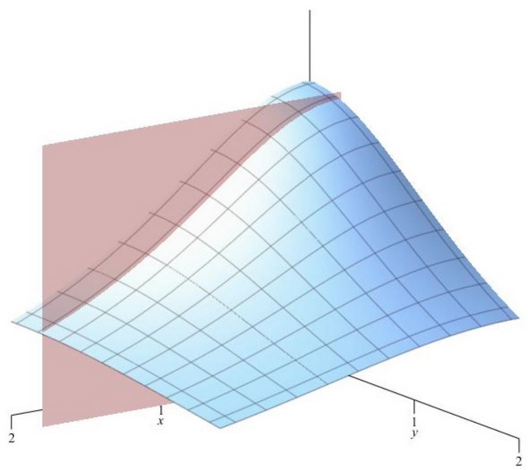
(d) Compute $f_y(x, y)$ and $f_y(0.5, 0.3)$.

(e) Now, using that same slope you computed three times in parts (b)-(d), write a vector that is in the same direction as the tangent line on the right. Modify that vector by adding an x-component to write a vector parallel to the tangent line you drew on the left.

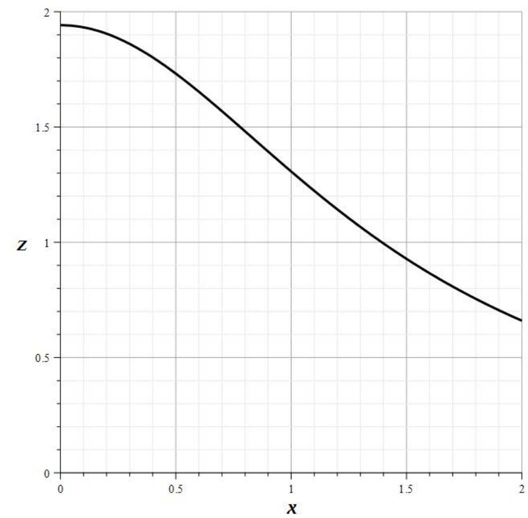
Copy your derivative from Question 1:

$f_x(x, y) =$

3. f_{xx} tells how f_x changes when y is (still) kept constant and x increases. Therefore, it measures how the slopes change.



The surface *z=f(x,y)* and the plane *y=0.3*



z=f(x,0.3)

(a) On the left, draw the lines on the plane $y = 0.3$ which are tangent to the curve of intersection at $(0.2, 0.3, f(0.2, 0.3))$ and $(0.5, 0.3, f(0.5, 0.3))$. These will be approximate as you cannot clearly see the x values.

(b) On the right, draw the lines which are tangent to the curve at $x = 0.2$ and $x = 0.5$. These are the same lines but easier to see, draw and work with.

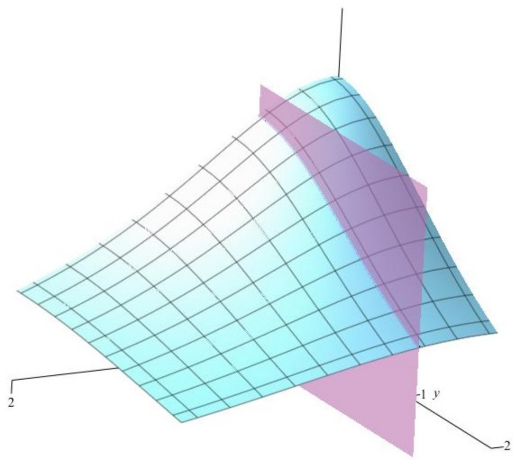
(c) By just looking, can you tell if the slope increased or decreased from $x = 0.2$ to $x = 0.5$?

(d) Now compute $f_{xx}(x, y)$ and $f_{xx}(0.2, 0.3)$ and compare the sign of $f_{xx}(0.2, 0.3)$ with your answer above.

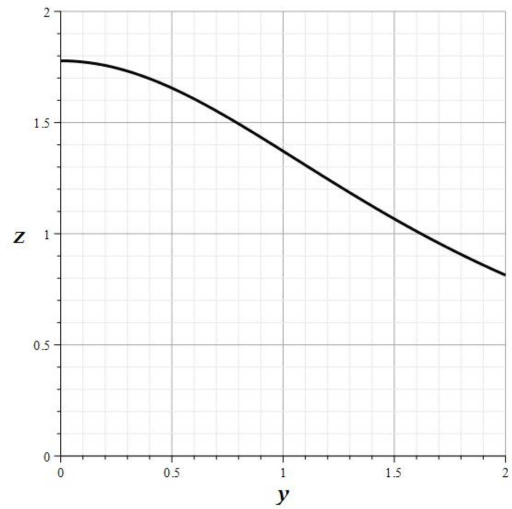
Copy your derivative from Question 1:

$f_y(x, y) =$

4. f_{yy} tells how f_y changes when x is (still) kept constant and y increases just a little bit. Therefore, it measures how the slopes change.



The surface *z=f(x,y)* and the plane *x=0.5*



z=f(0.5,y)

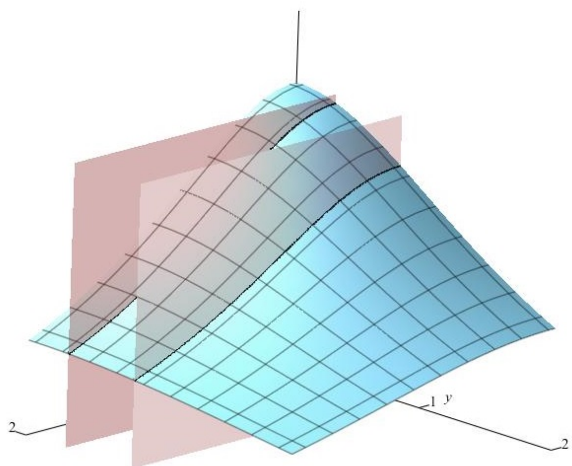
(a) On the left, draw the lines on the plane $x = 0.3$ which are tangent to the curve of intersection at $y = 0.2$ and 0.5. These will be approximate as you cannot clearly see the y values.

(b) On the right, draw the lines which are tangent to the curve at $y = 0.2$ and $y = 0.5$. These are easier to see and work with.

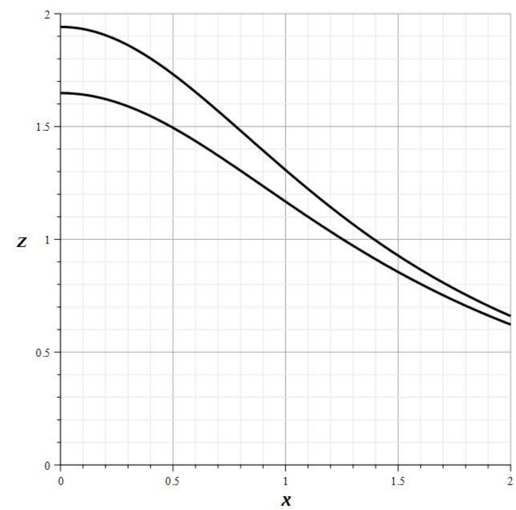
(c) By just looking, can you tell if the slope increased or decreased from $y = 0.2$ to $y = 0.5$?

(d) Now compute $f_{yy}(x, y)$ and $f_{yy}(0.5, 0.2)$ and compare the sign of $f_{yy}(0.5, 0.2)$ with your answer above.

5. f_{yx} is trickier to see. It shows how the slopes f_y change when x changes.



The surface z=f(x,y) and the planes y=0.3 and y=0.8



z=f(x,0.3) (top) and z=f(x,0.8) (bottom)

(a) Draw the the two lines on the planes $y = 0.2$ and $y = 0.8$ tangent to the surface (and hence to the two curves of intersection) at $x = 0.2$. These will be approximate drawings.

(b) Draw the the tangent lines to both curves at $x = 0.2$. Did the slope increase as you went from the top curve (which we got by setting $y = 0.3$) to the bottom (which we got by setting $y = 0.8$)? Do you think f_{xy} is positive or negative?

(c) Compute $f_{xy}(0.2, 0.3)$ and compare with your answer above.

Thinking about f_{xy} is left as an exercise. If you still have time left, do this question:

6. Use your answers to 1(d) and 2(d) to write the equation of the tangent plane to the surface $z = f(x, y)$ at the point $(0.5, 0.3, f(0.3, 0.5))$. The tangent plane is the plane that contains the two tangent lines you drew in Questions 1 and 2.