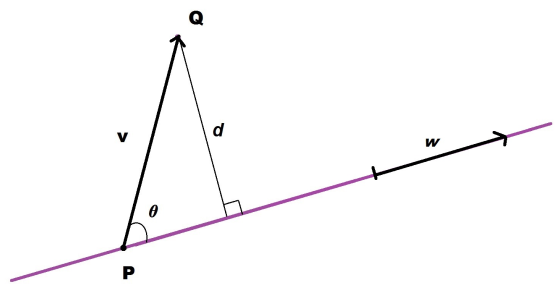
Computing Distances in Space

Points, lines and planes are the simplest objects in space. This worksheet takes you through computing the distance formulas through use of vectors.

Distance from a Point to a Line



1. Compute the distance d in terms of the angle θ and the length of the vector $\mathbf{v}$. Note that the line segment of length d intersects the line at a right angle.

2. Compute the angle θ using the vectors $\mathbf{v}$ and $\mathbf{w}$.

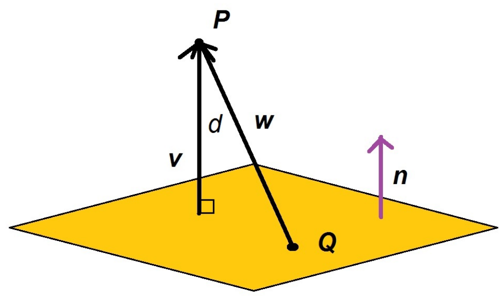
3. Now put the first two parts together to get the distance in terms of the vectors $\mathbf{v}$ and $\mathbf{w}$.

4. Would it make a difference if the vector $\mathbf{w}$ was reversed? If its length changed? Why or why not?

5. If the point is given as $Q(1, 2, 3)$ and the line is $\mathbf{l} = \langle 3 - 4t, 7t, 1 + 4t \rangle$, find a point P on the line, write the vectors $\mathbf{v}$ and $\mathbf{w}$ and compute the distance from the point Q to the line.

Distance from a Point to a Line

1. What is the distance d in terms of the vector $\mathbf{v}$.



2. Now compute the vector $\mathbf{v}$ in terms of $\mathbf{w}$ and $\mathbf{n}$. Hint: You can move $\mathbf{n}$ so it starts at the same place as $\mathbf{v}$.

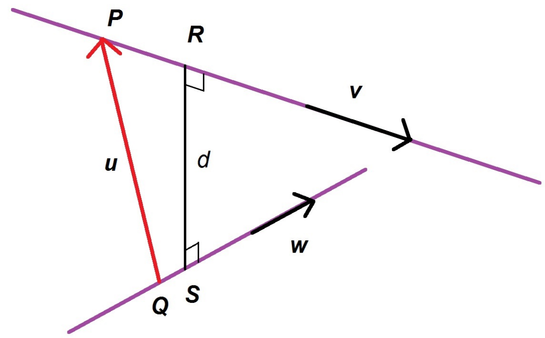
3. Now compute the distance between the point P and the plane.

3. Would it make a difference if the vector $\mathbf{n}$ was replaced by a scalar multiple of itself like $c\mathbf{n}$? Why or why not?

4. If the point is given as $P(2, 0, -5)$ and the plane is $2x - 5y + z = -5$, find a point Q on the plane, write the vectors $\mathbf{n}$,$\mathbf{v}$, and $\mathbf{w}$ to compute the distance from the point P to the plane.

BONUS If a point is give $P(x_0, y_0, z_0)$ and a plane is given by $Ax + By + Cz = D$, compute the distance from the point to the plane and compare your answer with the formula in the textbook. (You can do this on a separate piece of paper if you have time left.)

Distance between two skew lines



1. Start by coming up with a vector parallel to the line segment which measures d. Note the right angles at the points R and S. Add your answer to the picture. Does it point up or down?

2. Project the vector $\mathbf{u}$ onto the vector you found above.

3. How is your answer above related to the distance d? Compute the distance d.

4. If the two lines are given by $\mathbf{l}_1 = \langle 3-t, 1+2t, -3+5t\rangle$ and $\mathbf{l}_2 = \langle -2t, 4+3t, 5\rangle$, find vectors $\mathbf{v}$ and $\mathbf{w}$ and two points on the lines P and Q. Now compute the distance between the two lines.

The following can be computed using the formulas you have already worked out. Which formula do you use for the following and how?

1. Distance between two parallel lines

2. Distance between two parallel planes

3. Distance between a line and a plane, where the line is parallel to the plane