In Chapter 14 and Chapter 15 you will be working with functions of two variables and their graphs z = f(x, y) which are surfaces. It helps to have a good understanding of some of the equations you will see frequently. The first part of this worksheet is a matching exercise with equations, 3D surface graphs and contour graphs. You can try to match the equations to their surface graphs, equations to their contour graphs or the surface graphs to their contour graphs. And then compare your solutions to see if they match. The second part is for practicing drawing some of the basic surfaces you will see. Section 14.1 assignment on webassign is multiple choice, but, especially in Chapter 15, you will have to sketch or imagine these surfaces without the help of a computer graphing device.

Matching equations, surface graphs and contour graphs

In this section you will match equations with surface graphs (page 2) and contour graphs (page 3).

Equations

Things to look for:

- 1. Symmetry: What happens when you replace x by y, or x by -x, or y by -y?
- 2. Does it have circular symmetry an $x^2 + y^2$ term in the equation?
- 3. Does it take negative values or is it always above the xy-plane?
- 4. Is it one of the surfaces you have seen in Chapter 12? If yes, can you name it before looking at the pictures on the next page?

Here are the equations:

- 1. f(x,y) = |x| + |y|
- 2. $f(x,y) = x^2 y^2$
- 3. $f(x,y) = x^2 + y^2$
- 4. $f(x,y) = \sqrt{x^2 + y^2}$
- 5. $f(x,y) = x y^2$
- 6. $f(x,y) = \sqrt[4]{x^2 + y^2}$
- 7. $f(x,y) = x^2 + 5y^2$
- 8. $f(x,y) = (2x + 5y)^3$
- 9. f(x,y) = 2x + 5y

Here are their graphs z = f(x, y) as surfaces in space. The z axis points up in all pictures. The x and y axes have been scaled the same. The scaling of z may vary for a better picture. Look for symmetry and shapes of traces z = c.



Here are their contour graphs, sketches of f(x, y) = c on the plane for different values of c shown. Again, the x and y axes have been scaled the same. The axes have not been marked so focus on the types of curves and what happens to the distances between contours rather than specific values from the formulas. The closer the contours are together, the steeper the 3D graph will be. Think about how you would read a map.



Graphing common surfaces

This section is for practicing some drawing. All surfaces are from Section 12.6. Here are three that are common:



paraboloid

cone

(half of) a sphere

Replacing x, y or z by $\frac{x}{a}$, $\frac{y}{b}$ or $\frac{z}{c}$ does not change the basic shape causing only a stretch in the related axes. (Circular cross sections will become ellipses.) Sketch the following:



Replacing x, y or z by x - a, y - b or z - c causes a shift in the respective direction. Sketch the following:

$$f(x,y) = 1 + x^2 + 3y^2 \qquad \qquad f(x,y) = 3 + \sqrt{4x^2 + 5y^2} \qquad f(x,y) = \sqrt{25 - (x-1)^2 - (y-2)^2}$$

(half of) an ellipsoid

(elliptic) cone

(elliptic) paraboloid

More sketching practice. These are inverted.

$$f(x,y) = -x^2 - y^2 \qquad \qquad f(x,y) = -\sqrt{2x^2 + y^2} \qquad \qquad f(x,y) = -\sqrt{25 - x^2 - y^2}$$

(elliptic) paraboloid

(elliptic) cone

(half of) an ellipsoid

Inverted and shifted:

$$f(x,y) = 4 - x^2 - y^2$$

$$f(x,y) = 8 - \sqrt{4x^2 + y^2}$$

$$f(x,y) = 5 - \sqrt{25 - x^2 - y^2}$$

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Another familiar category are the generalized cylinders. In their equations one variable is missing:

$$f(x,y) = 2x^2$$

$$f(x,y) = 4y^3$$

$$f(x,y) = \cos x$$

(generalized) cylinder

(generalized) cylinder

(generalized) cylinder

Here are shifted or inverted versions:

$$f(x,y) = 7 - x^2$$
 $f(x,y) = 4y^3 + 1$ $f(x,y) = \cos x + 1$

(generalized) cylinder (generalized) cylinder (generalized) cylinder Finally, we have the planes. The first two below are not really of the form z = f(x, y).

$$x = 2 \qquad \qquad y = 3 \qquad \qquad f(x, y) = 1$$

plane parallel to yz-plane plane parallel to xz-plane plane parallel to xy-plane To graph an arbitrary plane, we need three points. We usually use the intercepts (x, 0, 0), (0, y, 0) and (0, 0, z):

$$f(x,y) = 12 - 2x - 3y$$
 $f(x,y) = 2 - \frac{x}{4} + \frac{y}{2}$ $f(x,y) = x - y + 3$

plane

plane

plane