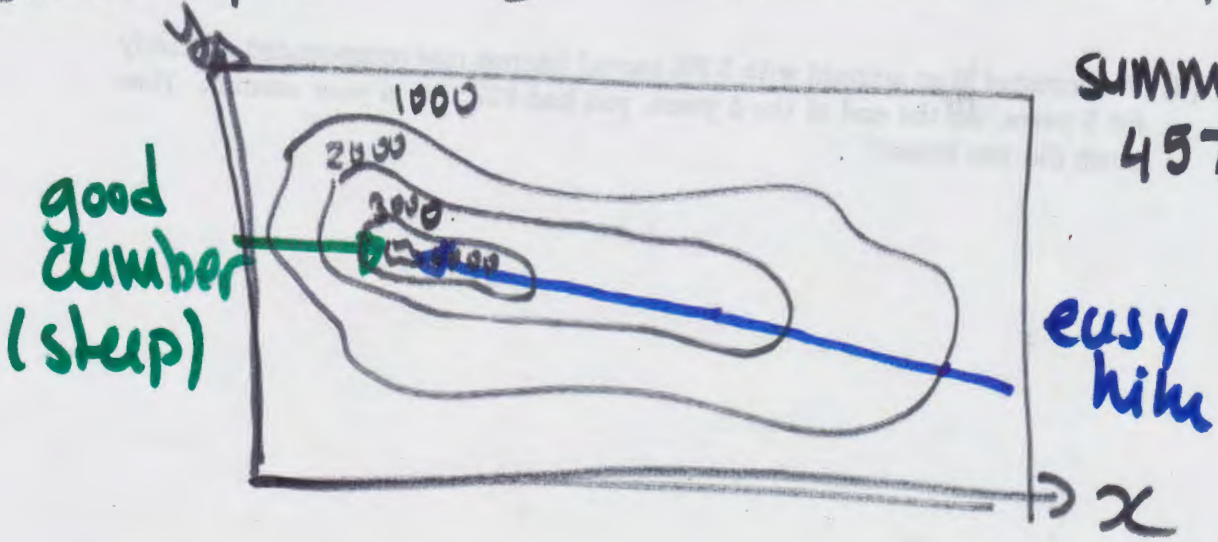
Chapter 14 - Functions of two (or more) variables & Partial Derivatives

- functions $f(x,y)$
- partial derivatives
- tangent planes → linear approximation
- critical points → optimization

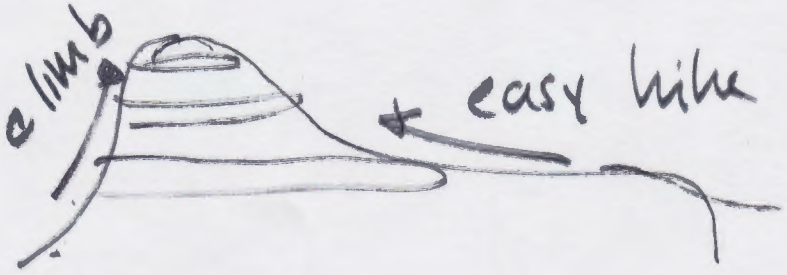
14.1 Functions of two (or more) variables

example: ① trail map



$f(x,y)$ - altitude

elevations for the contours are always equally spaced.



ex ② Formulas

Linear

$g(x,y) = x + 3y$

Cubic

$f(x,y) = x^2 + 2xy + y^3$

$f(0,1) = 0^2 + 2 \cdot 0 \cdot 1 + 1^3 = 1$

$f(2,3) = 4 + 12 + 9 = 25$

$$h(x,y) = \frac{\cos(xy) + \ln(1+x^2)}{\tan^{-1}(x^3 + 5e^x)}$$

ex (3) Body Mass Index

BMI

Domain:

w: (mass) in kg — $w > 0$

h: height, m — $h > 0$

$$B(h,w) = \frac{w}{h^2}$$

(4) $f(x,y) = \sqrt{9 - x^2 - y^2}$ Formula

To find domain, you remember that we do not

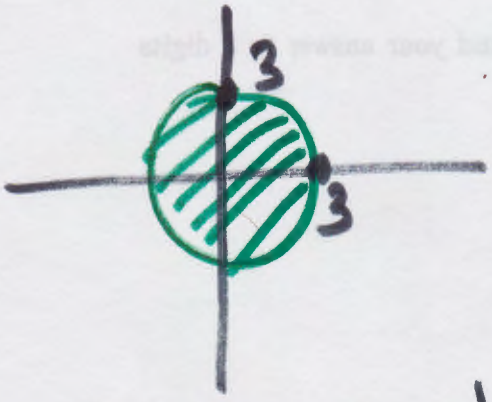
* we don't divide by 0
* we don't $\sqrt{\ }$ of negative number
* we don't take logarithms (in particular end of 0 or negative numbers.

So in my example

$$9 - x^2 - y^2 \geq 0$$

$$9 \geq x^2 + y^2$$

disk with center (0,0) + radius 3



Domain of f(x,y)

Domain: all (x,y)'s valid
for (the formula) of f(x,y)
It's part of xy-plane.

Representing functions $f(x,y)$

I. Graph in 3D $z = f(x,y)$

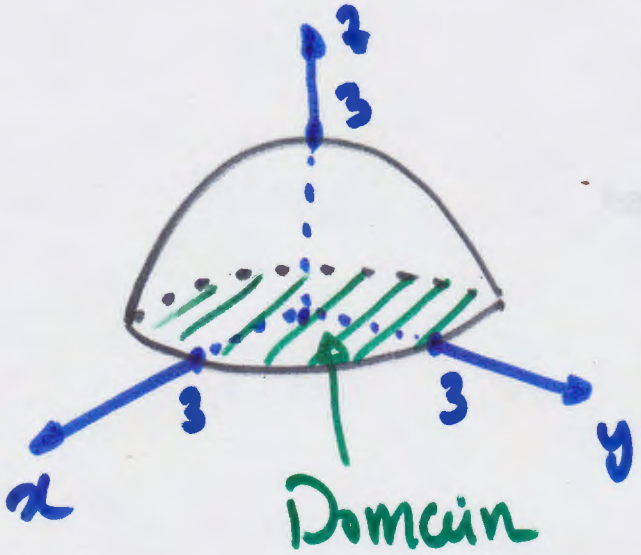
ex: $f(x,y) = \sqrt{9 - x^2 - y^2}$

graph $z = \sqrt{9 - x^2 - y^2}$ ← $z \geq 0$ hemisphere

$\rightarrow z^2 = 9 - x^2 - y^2$

$\rightarrow x^2 + y^2 + z^2 = 9$

sphere with center $(0,0,0)$ radius 3

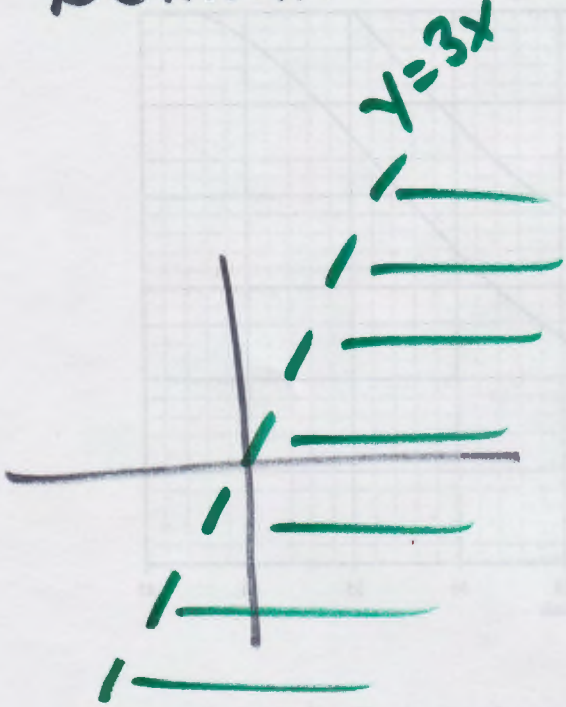


hemisphere

ex: $f(x,y) = \ln(3x - y)$

Domain: $3x - y > 0$

$3x > y$



ex: $f(x,y) = \dfrac{\sin(xy) + ye^{x}}{\sqrt{1+y^2+x^2}}$

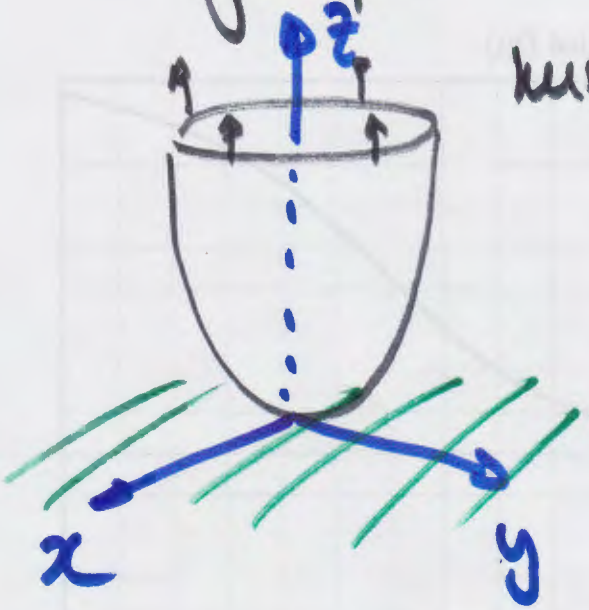
Domain: All pairs (x,y)

i.e. all of xy-plane.

ex: $f(x,y) = x^2 + y^2$

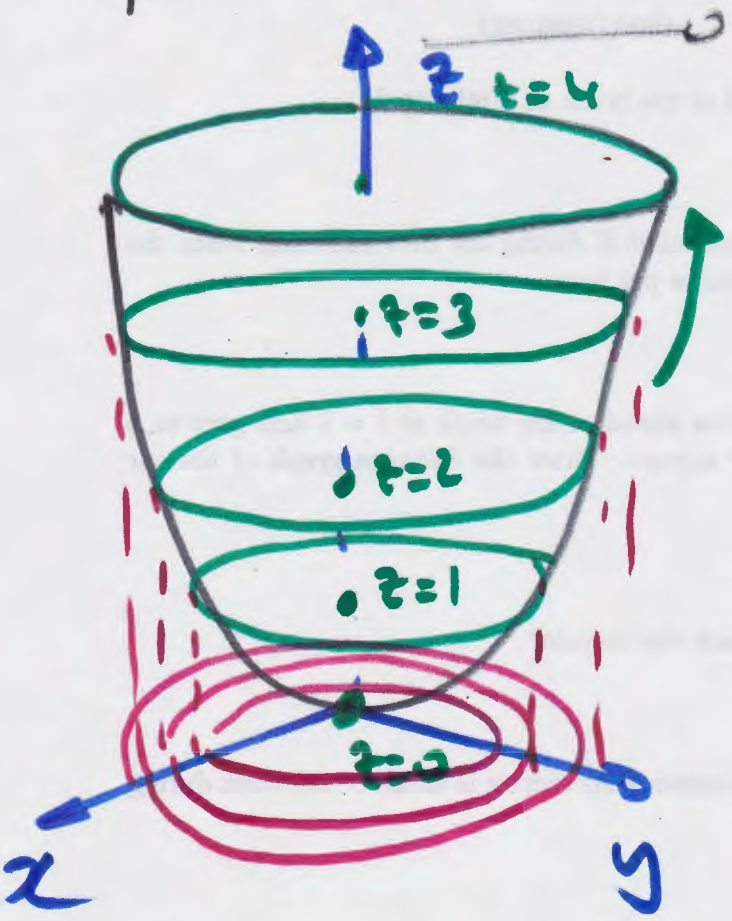
graph $z = x^2 + y^2$ (12.6 paraboloid)

keeps going



Domain: all (x,y)

paraboloid



Fix $z = 0,1,2,3,4$
(equally spaced)

we graph on the xy plane

contours

radii

$0 = x^2 + y^2$

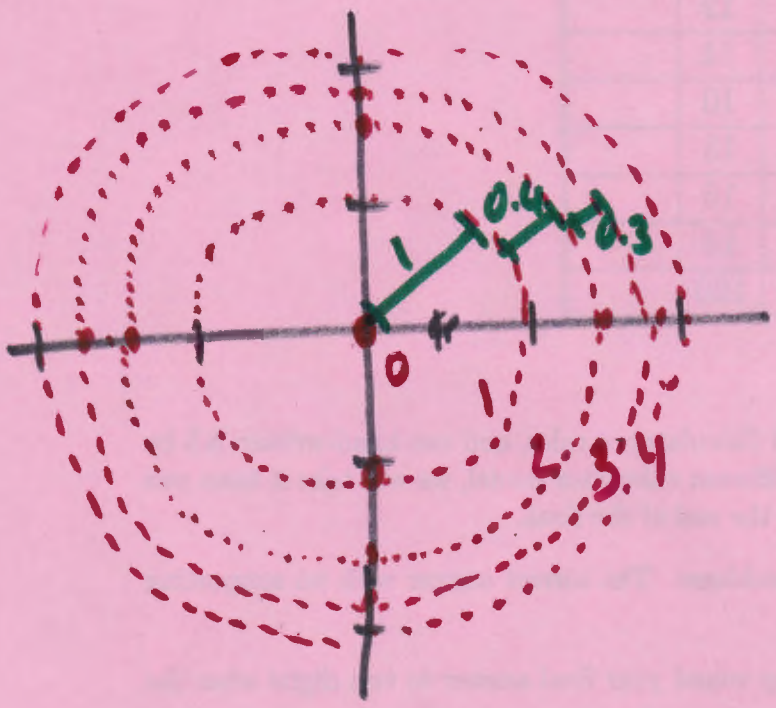
$1 = x^2 + y^2$

$1.4 \simeq \sqrt{2}$ $2 = x^2 + y^2$

$1.7 \simeq \sqrt{3}$ $3 = x^2 + y^2$

$2 = \sqrt{4}$ $4 = x^2 + y^2$

} circles center (0,0)

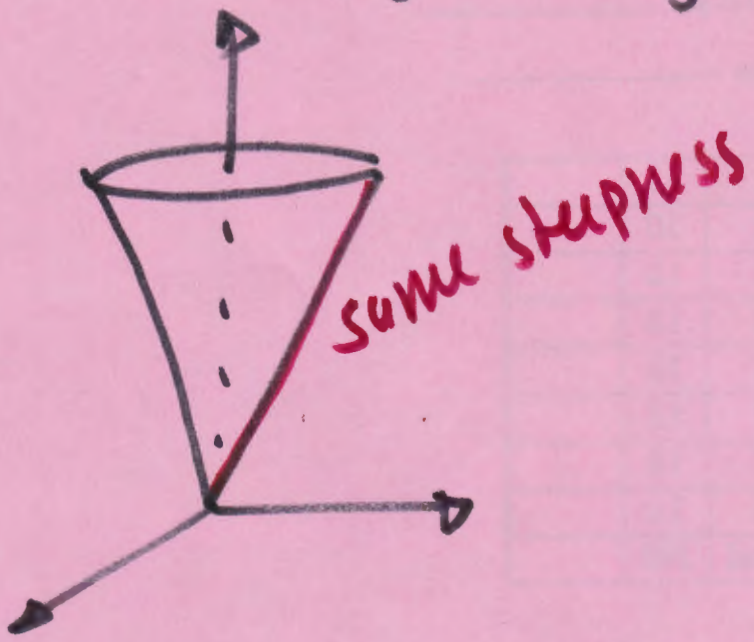


distance between contours is shrinking → the graph $z = f(x,y)$ is getting steeper

$x = f(x,y) = \sqrt{x^2+y^2}$

$z = \sqrt{x^2+y^2}$, $z \geq 0$ single cone

$\Rightarrow z^2 = x^2+y^2$ double cone



Contour map $z = 0,1,2,3,4$

$0 = \sqrt{x^2+y^2}$

$1 = \sqrt{x^2+y^2}$

$2 = \sqrt{x^2+y^2}$

$3 = \sqrt{x^2+y^2}$

$4 = \sqrt{x^2+y^2}$

square → but 4 sides

point (0,0) radi

1

circles 2

center (0,0) 3

4

equally spaced
→ not exactly steeper

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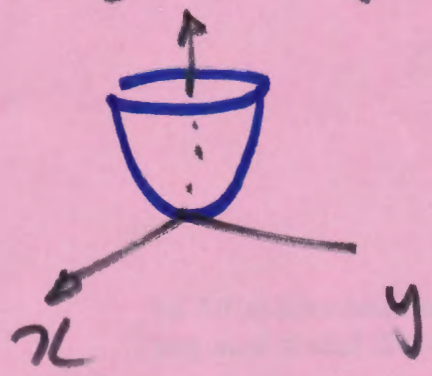
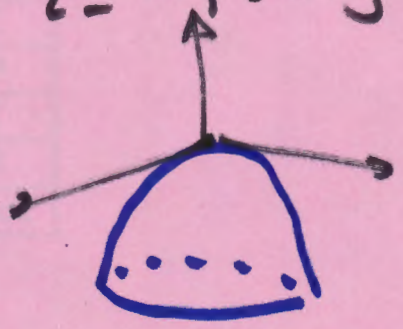
more on graphing

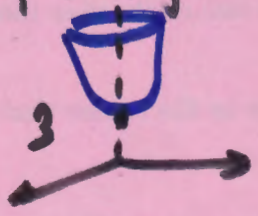
$z = f(x,y)$



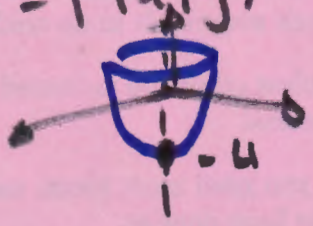
$z = -f(x,y)$



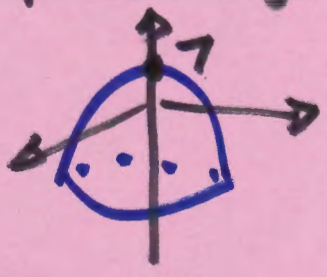
$z = f(x,y) + 3$



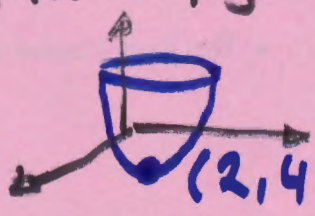
$z = f(x,y) - 4$



$z = 7 - f(x,y)$



$z = f(x-2, y-4)$



Linear Functions

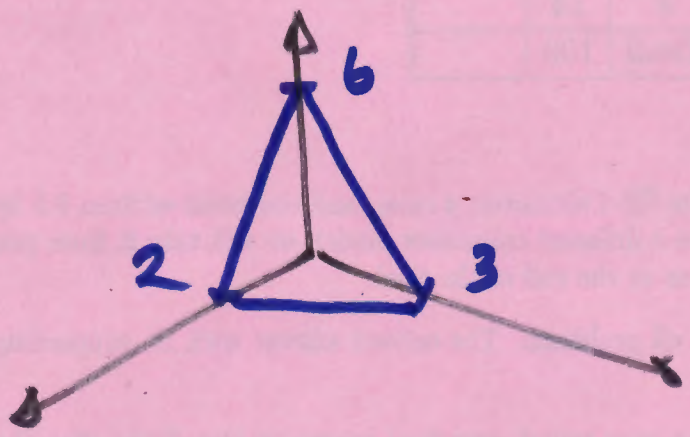
$f(x,y) = A + Bx + Cy$

graph $z = f(x,y)$ is a <u>plane</u>

ex: $z = 6 - 2x - 3y$

we sketch planes using 3 points

(0,0,6) (0,2,0) (3,0,0)



$-2 = 6 - 2x - 3y$

$-1 = 6 - 2x - 3y$

⋮

5 lines

contours $z = -2, -1, 0, 1, 2$

ALL <u>LINES</u>

check the all have the <u>SAME SLOPE</u>

check they're equally spaced.