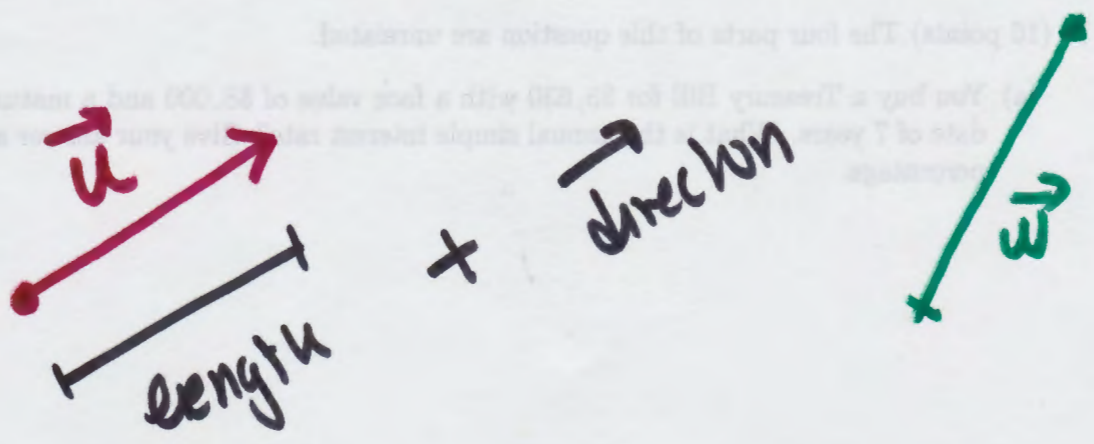
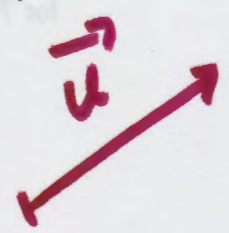


# 12.2 Vectors



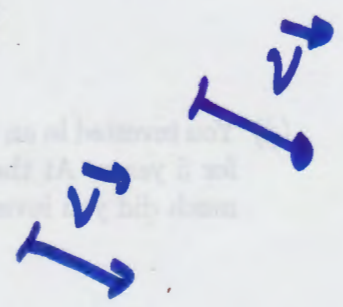
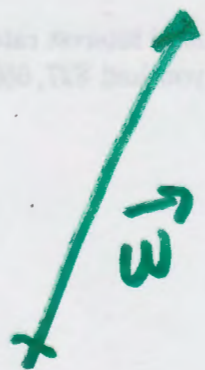
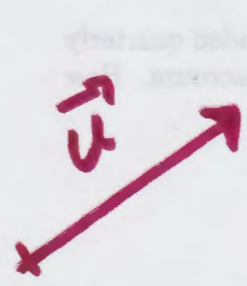
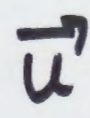
A vector is a quantity with length (norm, size) and direction.



notation: in print - bold letters

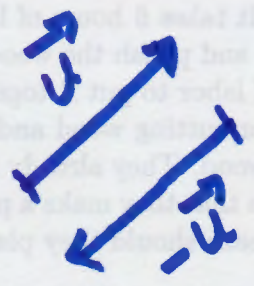


in handwriting

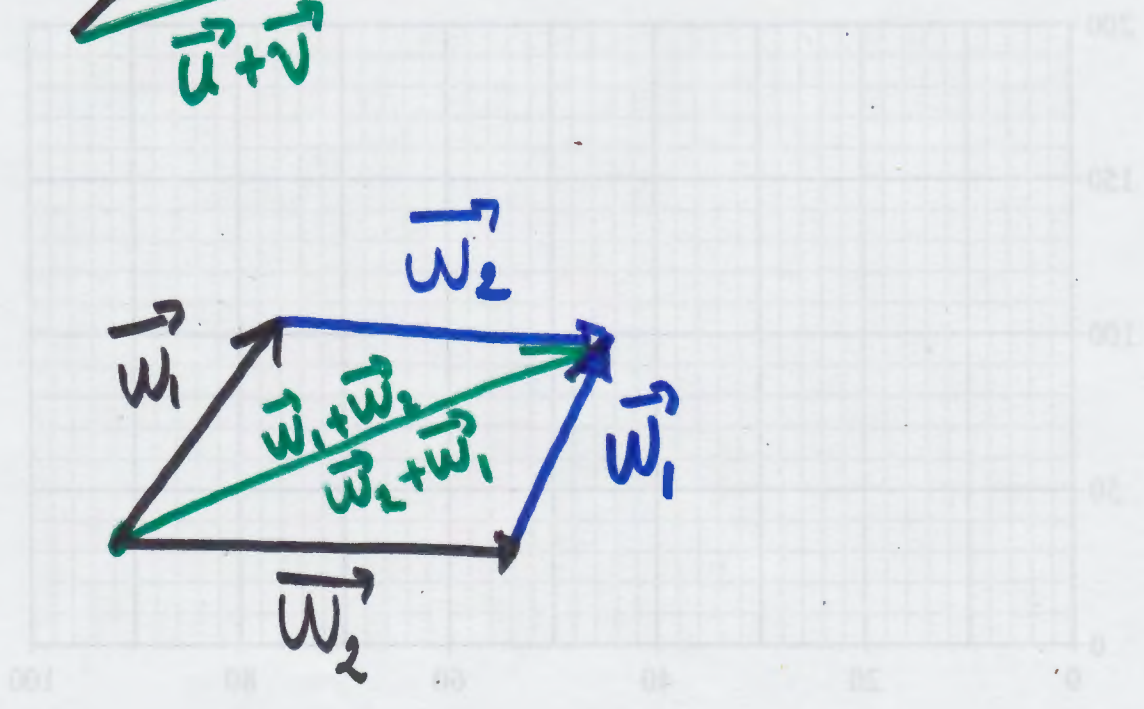
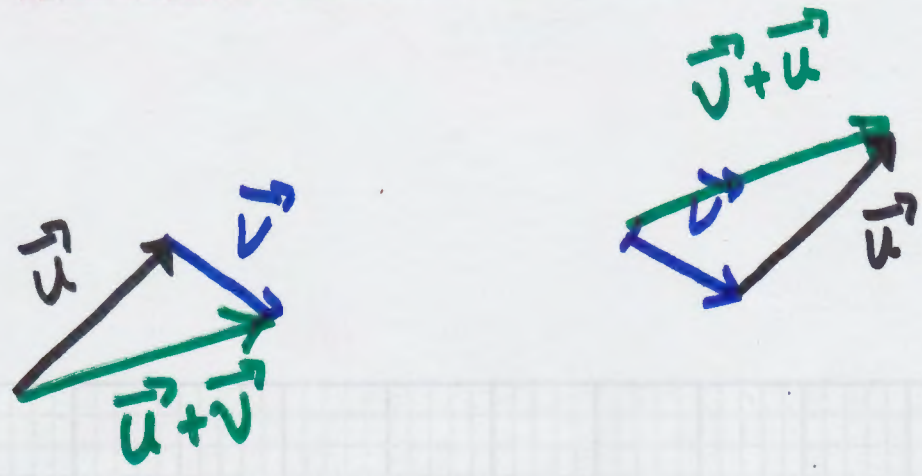


# opposite of a vector

8. (13 points) A furniture factory produces desks and tables. It takes 3 hours to cut the parts of a desk and 2 hours to put them together and 2 hours to paint the desk. It takes 2 hours to cut the parts of a table and 2 hours of labor for painting the table together and 2 hours of labor for putting pieces together and polishing the wood. The factory has 360 hours of labor for cutting and 240 hours of labor for putting pieces together and polishing the wood. Given that the profit is \$100 on each desk and \$120 on each table, how many of each should the factory produce to maximize their profit?

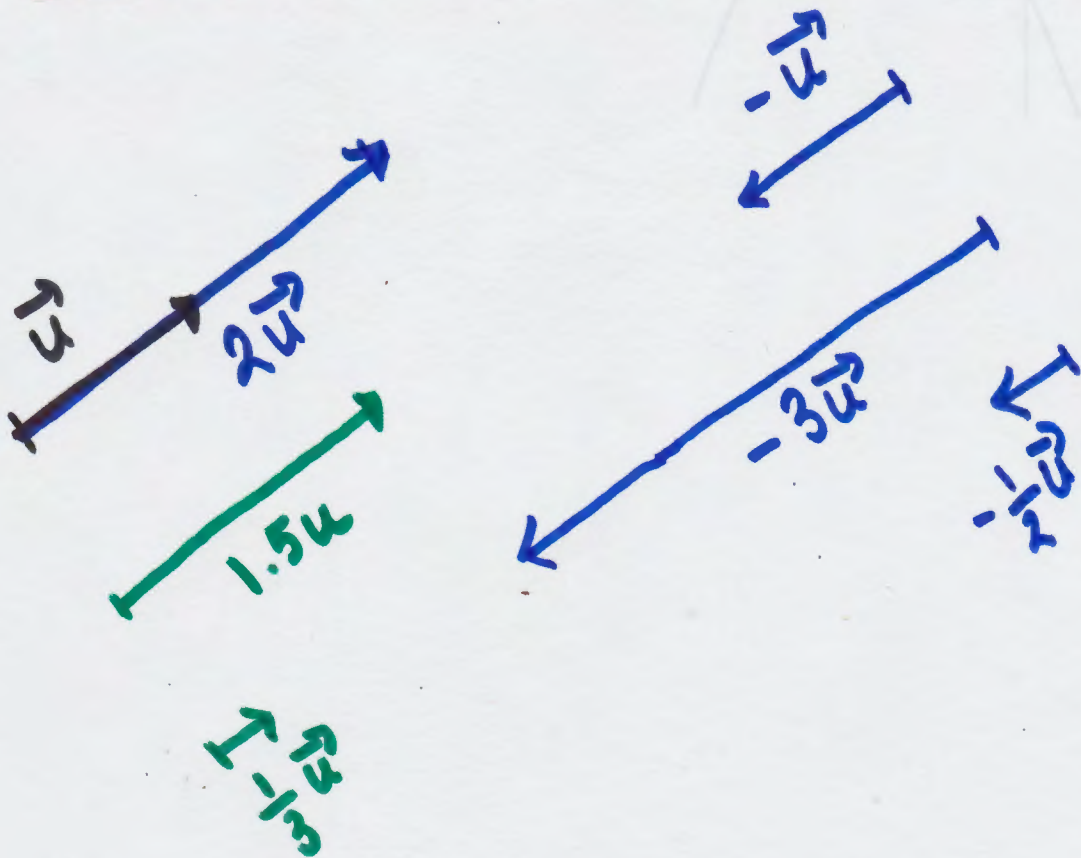


# Addition of vectors

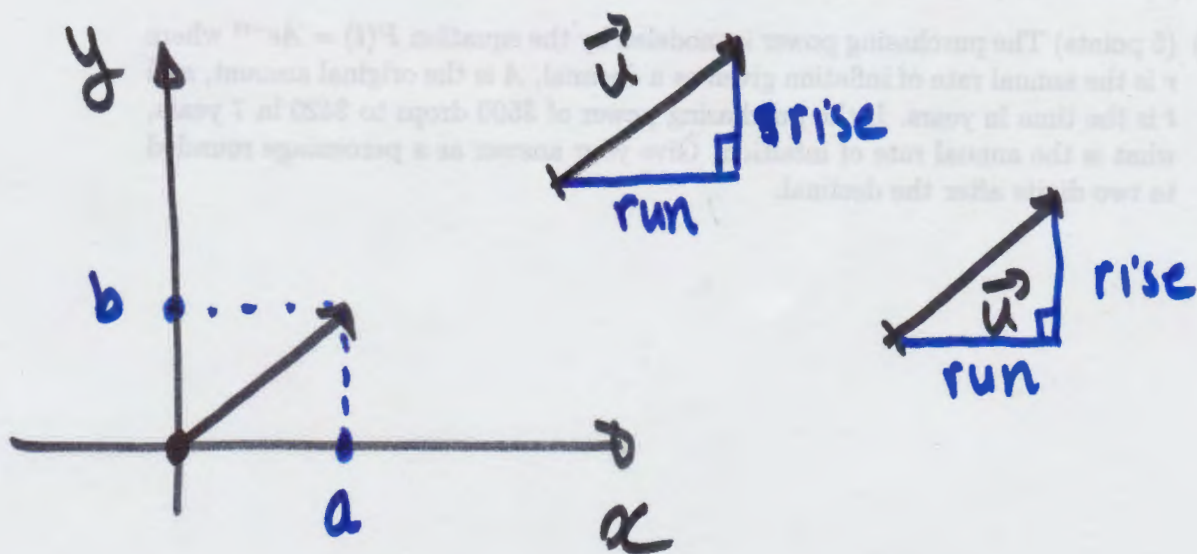


In the vector world,  
numbers (2, 0,  $\pi$ , 1.7, etc.)  
are called scalars.

multiplication by a scalar



# Representing Vectors

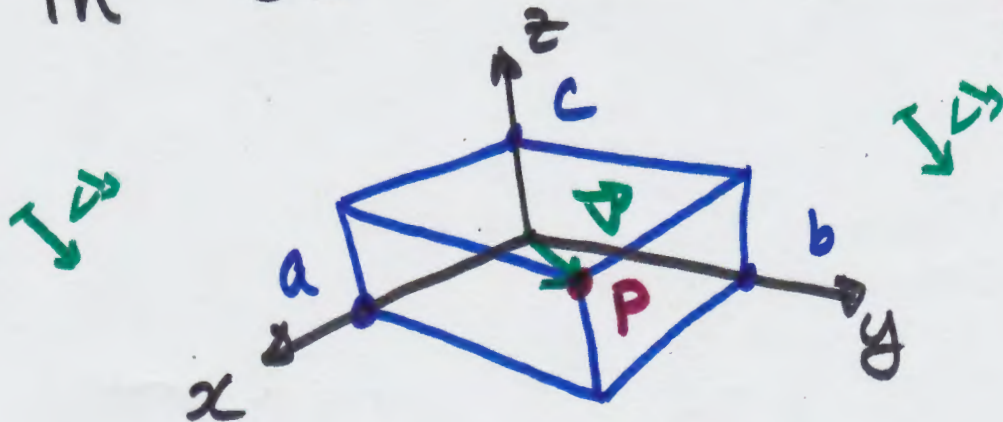


$$\vec{u} = \langle a, b \rangle$$

If we place the vector so that it starts at the origin, its tip points to the point  $(a, b)$

In 3D - same idea

$P(a, b, c)$



$$\vec{v} = \langle a, b, c \rangle$$

$\vec{v} = \langle a, b, c \rangle$   
↑ ↑ ↑  
components

$P(a, b, c)$   
↑ ↑ ↑  
coordinates

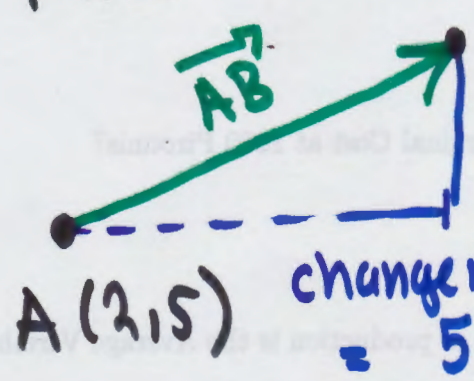
### The zero vector

in 2D  
 $\langle 0, 0 \rangle$

in 3D  
 $\langle 0, 0, 0 \rangle$

example: Find the ~~the~~ vector

$\vec{AB}$

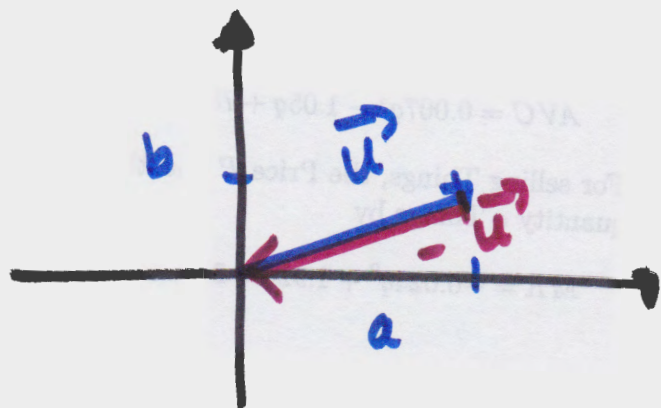


change in y = rise =  $6 - 5 = 1$

change in x = run  
 $= 5 - 2 = 3$

$$\vec{AB} = \langle 5 - 2, 6 - 5 \rangle = \langle 3, 1 \rangle$$

## opposite of a vector



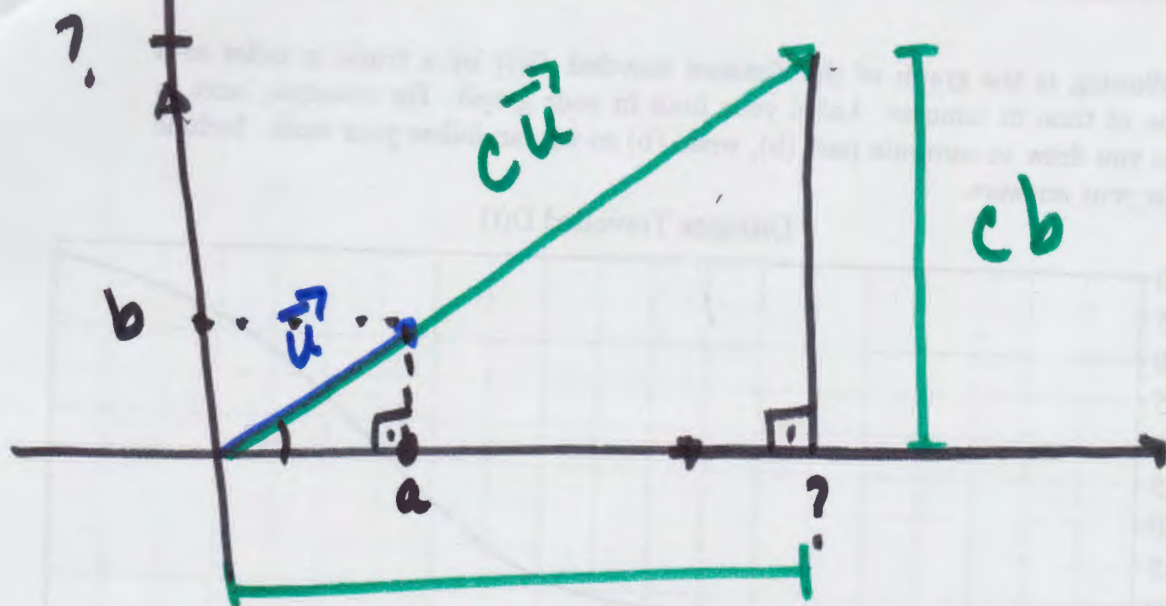
$$-\vec{u} = \langle -a, -b \rangle$$

same idea in 3D:

$$\vec{w} = \langle a, b, c \rangle$$

$$-\vec{w} = \langle -a, -b, -c \rangle$$

## multiplication by a scalar



$$\vec{u} = \langle a, b \rangle$$

$ca$

$cb$

Similar right triangles.

Hypotenuse was scaled by a factor of  $c$ . The same is true for the legs.

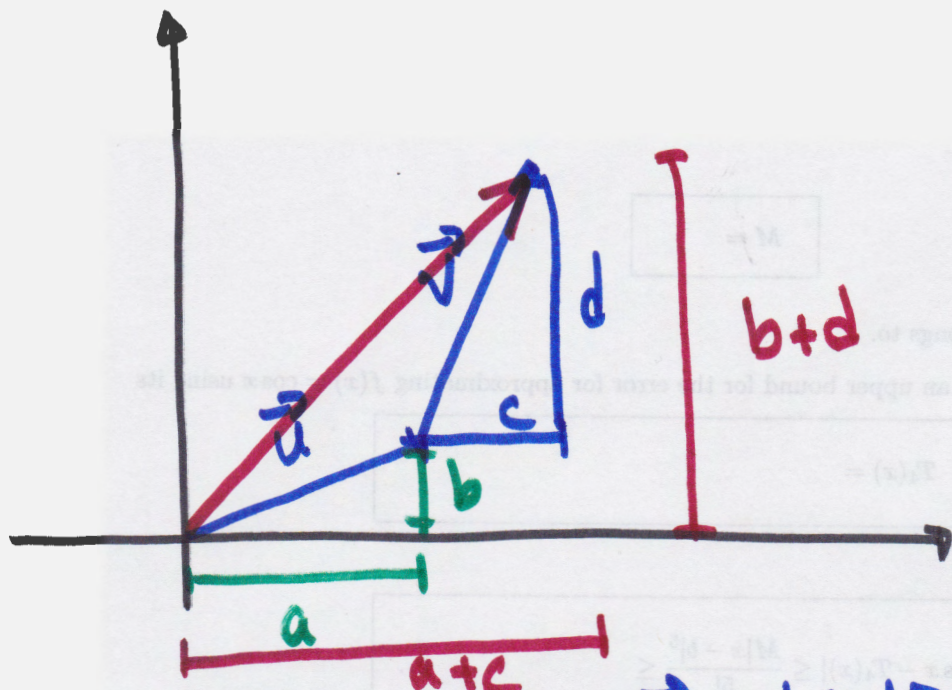
$$c \langle a, b \rangle = \langle ca, cb \rangle$$

Same in 3D

$$d \langle a, b, c \rangle = \langle da, db, dc \rangle$$

ex:  $5 \langle 1, 0, -7 \rangle = \langle 5, 0, -35 \rangle$

# Addition of Vectors



$$\vec{u} = \langle a, b \rangle$$

$$\vec{v} = \langle c, d \rangle$$

$$\vec{u} + \vec{v} = \langle a+c, b+d \rangle$$

Same idea in 3D

$$\begin{aligned} &\langle a_1, a_2, a_3 \rangle + \langle b_1, b_2, b_3 \rangle \\ &= \langle a_1 + b_1, a_2 + b_2, a_3 + b_3 \rangle \end{aligned}$$



examples:

$$\textcircled{1} \quad - \langle 2, 3, -7 \rangle = \langle -2, -3, 7 \rangle$$

$$\textcircled{2} \quad \frac{1}{2} \langle 6, 0, 5 \rangle = \langle 3, 0, \frac{5}{2} \rangle$$

$$\textcircled{3} \quad \langle 5, -1, 0 \rangle + \langle 2, 13, 17 \rangle \\ = \langle 7, 12, 17 \rangle$$

$\textcircled{4}$  linear ~~compi~~ combination

$$2 \langle 1, 2, 3 \rangle - 5 \langle 0, -1, 1 \rangle$$

$$= \langle 2 \cdot 1 - 5 \cdot 0, 2 \cdot 2 - 5(-1), 2 \cdot 3 - 5(1) \rangle$$

$$= \langle 2, 9, 1 \rangle$$

# Alternative Notation

example :  $\langle 2, 5, -7 \rangle$

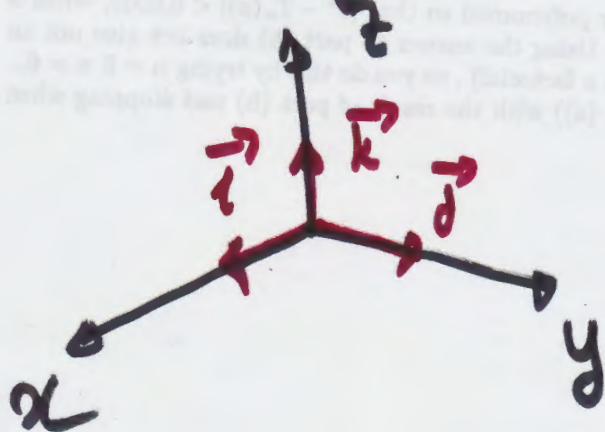
$$= \langle 2, 0, 0 \rangle + \langle 0, 5, 0 \rangle + \langle 0, 0, -7 \rangle$$

$$= 2 \underbrace{\langle 1, 0, 0 \rangle}_{\vec{i}} + 5 \underbrace{\langle 0, 1, 0 \rangle}_{\vec{j}} - 7 \underbrace{\langle 0, 0, 1 \rangle}_{\vec{k}}$$

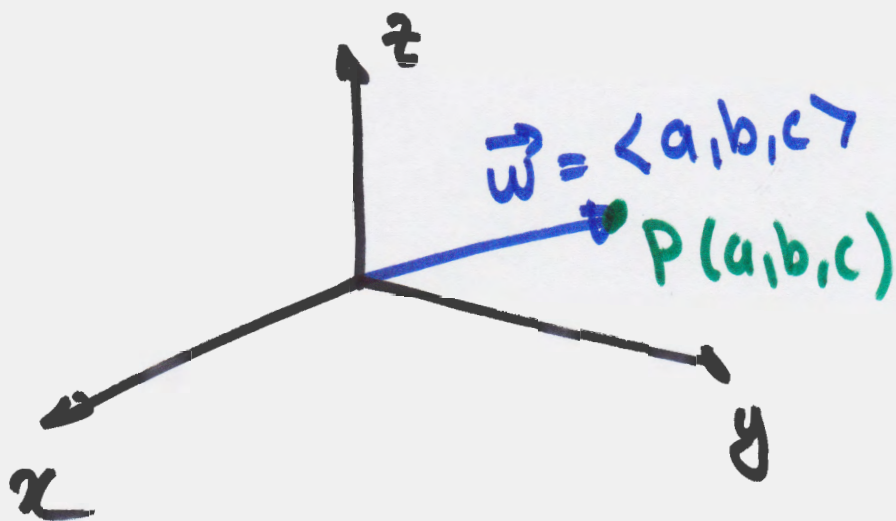
$$= 2\vec{i} + 5\vec{j} - 7\vec{k} \quad (\hat{i}, \hat{j}, \hat{k})$$

$\vec{i}, \vec{j}, \vec{k}$  are unit vectors

i.e. they have length 1.



# Length of ~~of~~ a vector

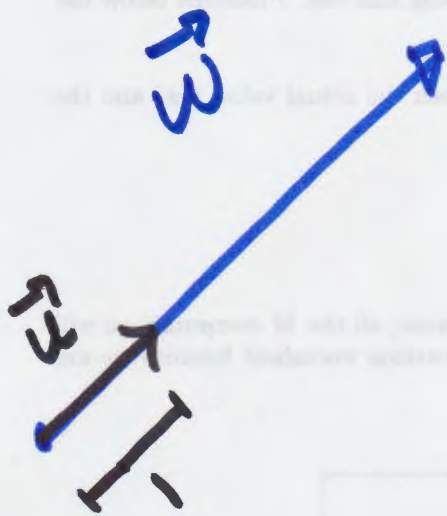


Length of  $\vec{w}$  is the distance from the origin to  $P(a, b, c)$

$$|\vec{w}| = \sqrt{a^2 + b^2 + c^2}$$

$|-2|$  : absolute value

$|\langle -2, 1 \rangle|$  : length / norm



$\vec{u}$  is the unit vector in the same direction as  $\vec{w}$

To get  $\vec{u}$  from  $\vec{w}$  we have to scale  $\vec{w}$ :

$$\vec{u} = \frac{1}{|\vec{w}|} \vec{w}$$

vector                      scalar                      vector

example:  $\vec{w} = \langle 2, 7, 1 \rangle$

$$|\vec{w}| = \sqrt{4 + 49 + 1} = \sqrt{54}$$

$$\vec{u} = \frac{1}{\sqrt{54}} \langle 2, 7, 1 \rangle = \left\langle \frac{2}{\sqrt{54}}, \frac{7}{\sqrt{54}}, \frac{1}{\sqrt{54}} \right\rangle$$

check:  $|\vec{u}| = 1.$

example: Write ~~a~~ the vector  $\vec{v}$  which has length 3, which points in the same direction as  $\langle 1, 2 \rangle$ .



$$|\langle 1, 2 \rangle| = \sqrt{1+2} = \sqrt{5}$$

$$\vec{u} = \frac{1}{\sqrt{5}} \langle 1, 2 \rangle$$

$$\vec{v} = 3 \cdot \frac{1}{\sqrt{5}} \langle 1, 2 \rangle$$

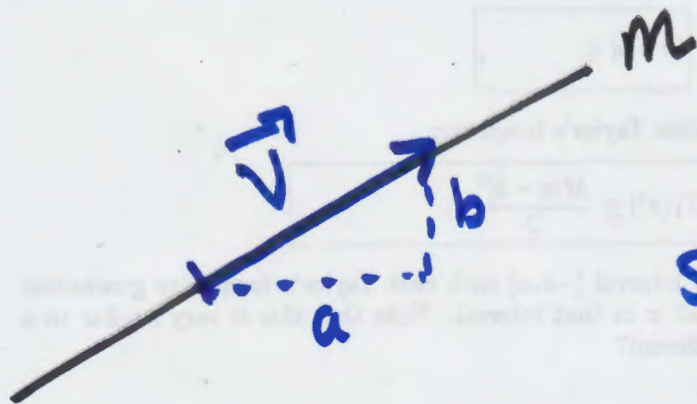
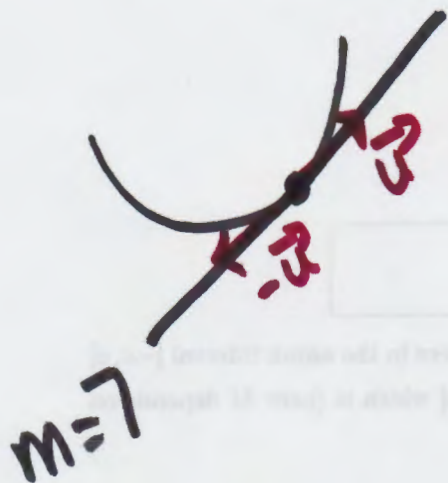
$$= \left\langle \frac{3}{\sqrt{5}}, \frac{6}{\sqrt{5}} \right\rangle$$

example: Find a unit vector parallel to the tangent line to  $y = x^2 + 3x + 1$  at  $x = 2$ .

slope:

$$y' = 2x + 3$$

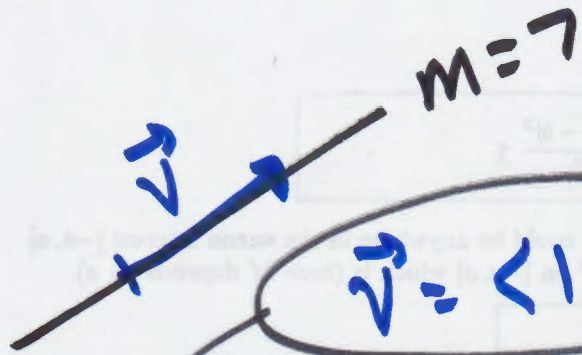
$$y'(2) = 2 \cdot 2 + 3 = 7$$



$\vec{v} = \langle a, b \rangle$   
such that  $\frac{b}{a} = m$   
 $\rightarrow b = ma$

$$\vec{v} = \langle 1, m \rangle$$

$$\vec{v} = \langle 5, 5m \rangle \dots$$



$$\text{(or } \vec{v} = \langle 2, 14 \rangle$$

$$\text{or } \vec{v} = \langle -3, -21 \rangle)$$

$$\rightarrow \vec{u} = \frac{1}{|\vec{v}|} \vec{v}$$

$$= \frac{1}{\sqrt{1+49}} \langle 1, 7 \rangle$$

$$= \left\langle \frac{1}{\sqrt{50}}, \frac{7}{\sqrt{50}} \right\rangle$$

$$\text{OR } -\vec{u} = \left\langle -\frac{1}{\sqrt{50}}, -\frac{7}{\sqrt{50}} \right\rangle$$

both answer the question.