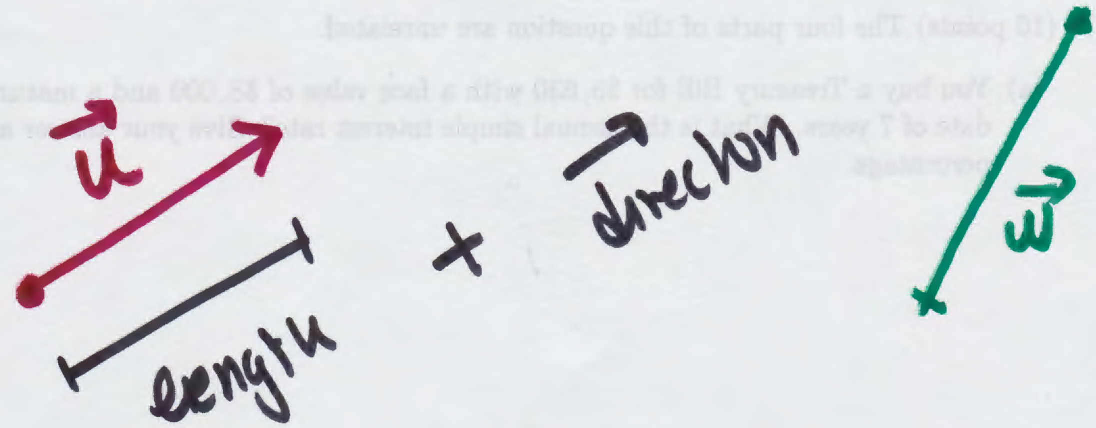
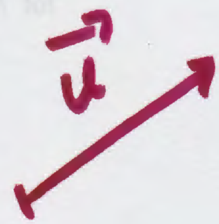
# 12.2 Vectors

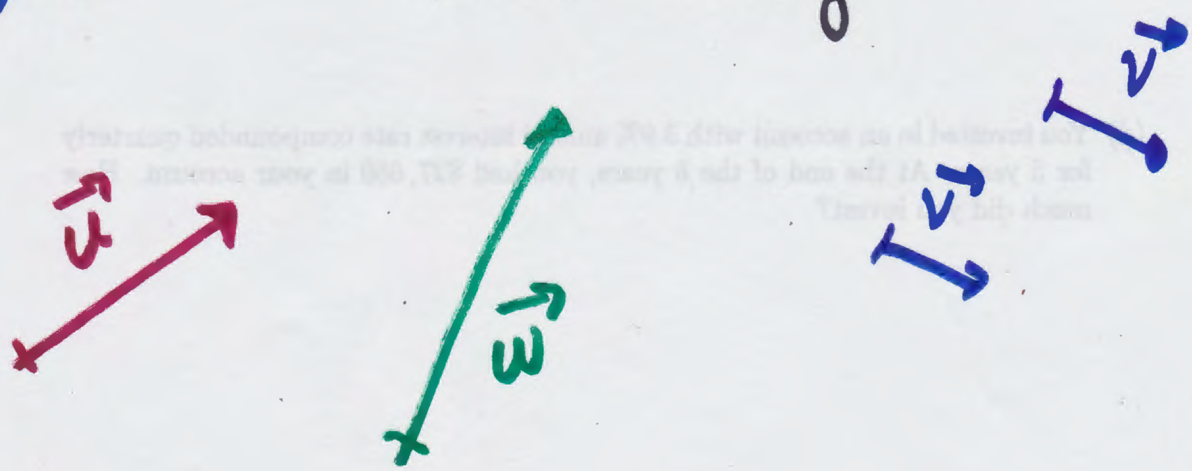



A vector is a quantity with length (norm, size) and direction.

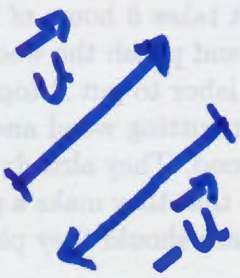



notation: in print – bold letters **u**

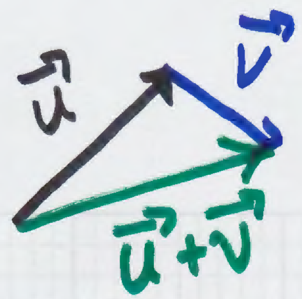

in handwriting $\vec{u}$

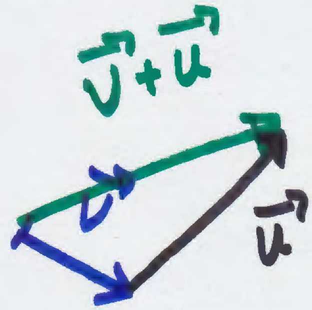


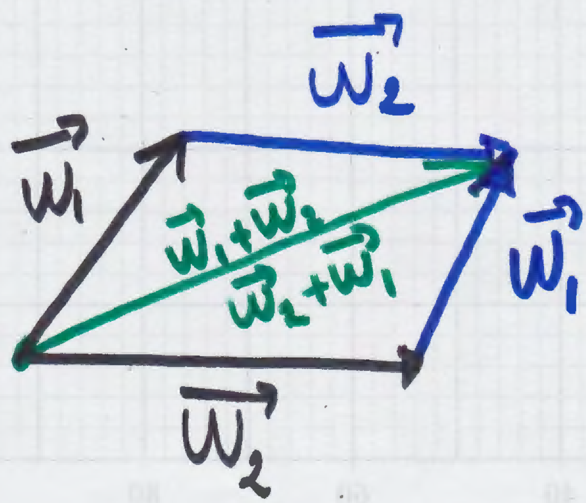
# Opposite of a vector




# Addition of vectors

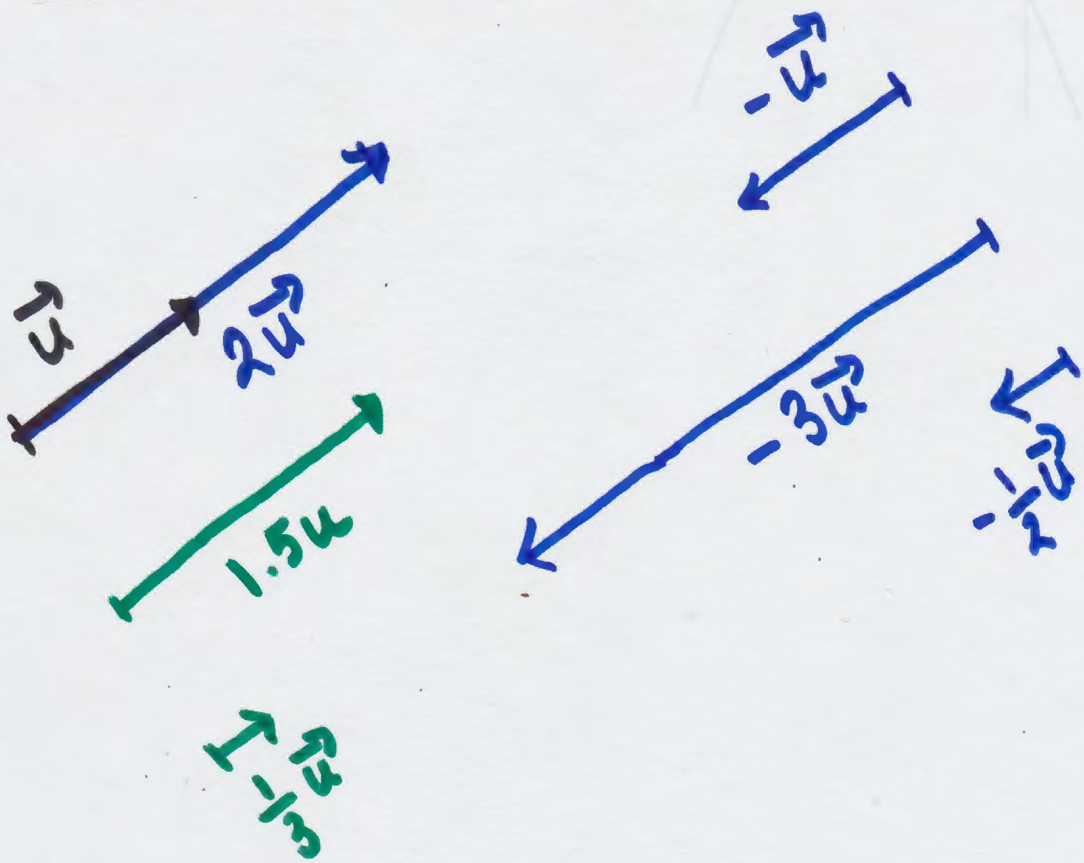


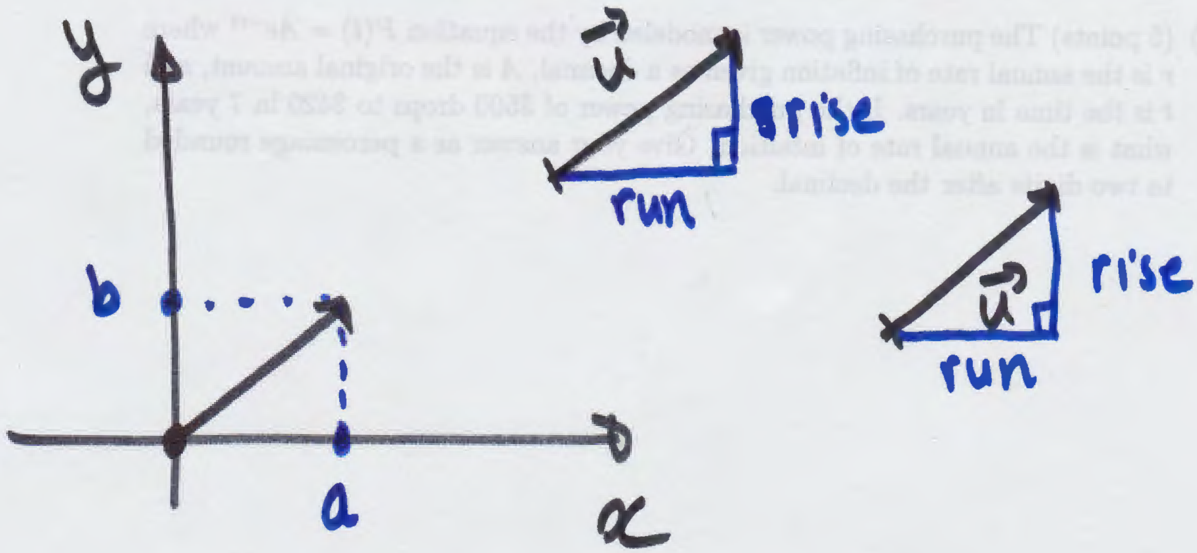



In the vector world, numbers (2, 0, $\pi$, 1.7, etc.) are called <u>scalars</u>.

## multiplication by a scalar



# Representing Vectors

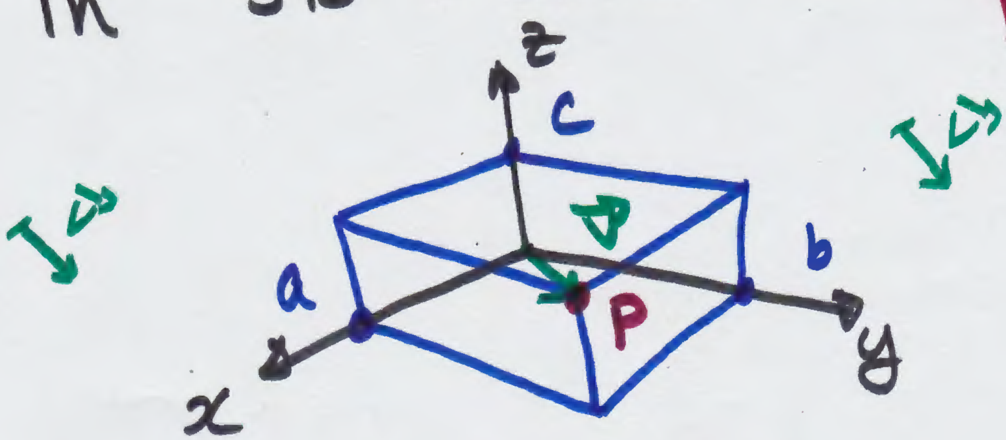



$\vec{u} = \langle a,b \rangle$

If we place the vector so that it starts at the origin, its tip points to the point $(a,b)$

In 3D - same idea

$P(a,b,c)$




$\vec{v} = \langle a,b,c \rangle$

$\vec{V} = \langle a, b, c \rangle$ $\qquad$ $P(a, b, c)$

components $\qquad$ coordinates

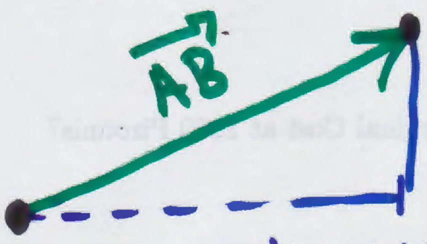
## The <u>zero vector</u>

in 2D $\qquad$ in 3D

$\langle 0, 0 \rangle$ $\qquad$ $\langle 0, 0, 0 \rangle$

example: Find the vector $\overrightarrow{AB}$

B(5, 6)

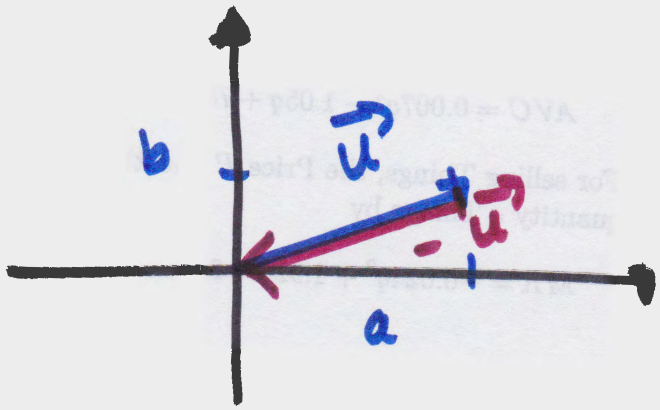



change in y = rise = 6 - 5 = 1

A(2, 5) $\qquad$ change in x = run = 5 - 2 = 3

$$\overrightarrow{AB} = \langle 5-2,\ 6-5 \rangle = \langle 3, 1 \rangle$$

# opposite of a vector



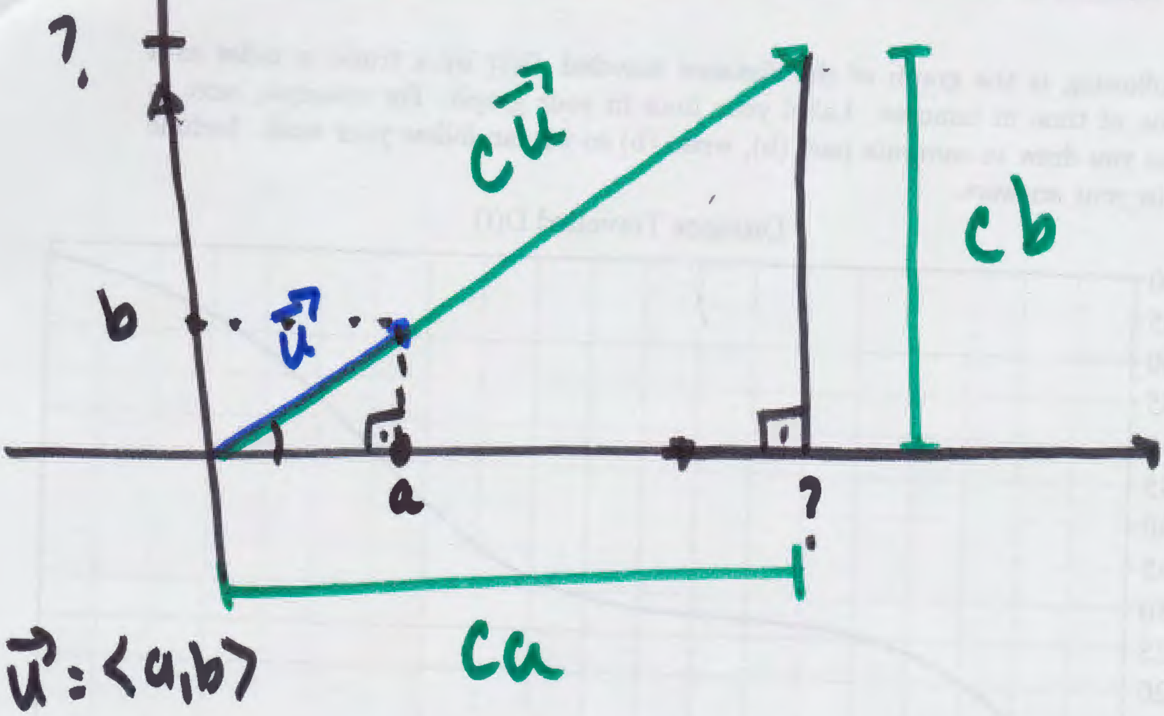

$-\vec{u} = \langle -a, -b \rangle$

same idea in 3D:

$\vec{w} = \langle a, b, c \rangle$

$-\vec{w} = \langle -a, -b, -c \rangle$

# multiplication by a scalar




$\vec{u} = \langle a,b \rangle$

similar right triangles.
Hypotenuse was scaled by a factor of c. The same is true for the legs.

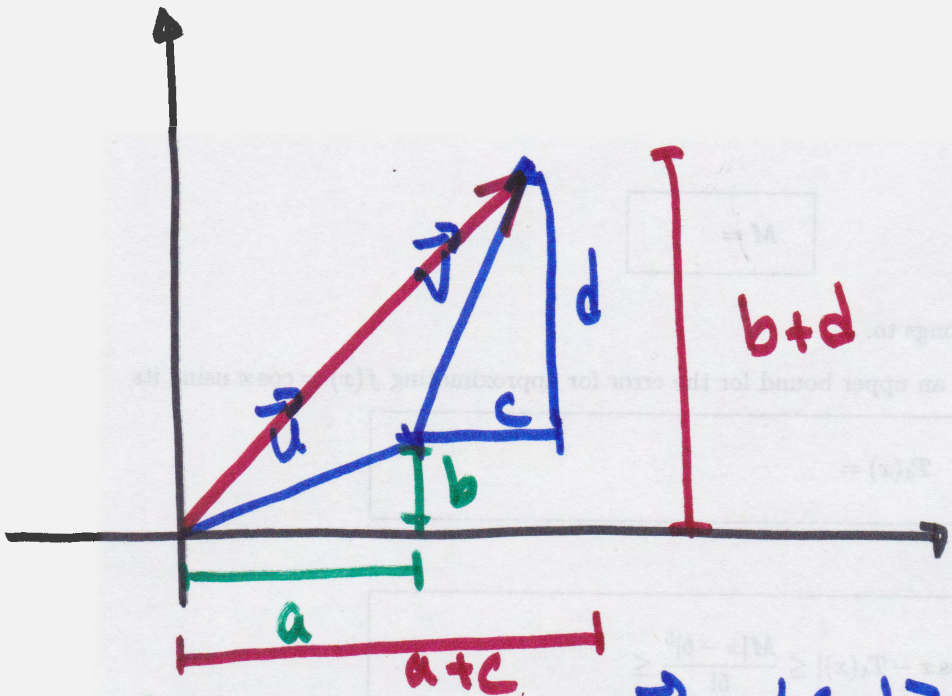
$$c\langle a,b\rangle = \langle ca, cb\rangle$$

Same in 3D

$$d\langle a,b,c\rangle = \langle da, db, dc\rangle$$

ex: $5\langle 1,0,-7\rangle = \langle 5,0,-35\rangle$

# Addition of Vectors




$\vec{u} = \langle a, b \rangle$ $\quad$ $\vec{v} = \langle c, d \rangle$

$$\vec{u} + \vec{v} = \langle a+c, b+d \rangle$$

Same idea in 3D

$$\langle a_1, a_2, a_3 \rangle + \langle b_1, b_2, b_3 \rangle$$
$$= \langle a_1 + b_1, a_2 + b_2, a_3 + b_3 \rangle$$

examples:

① $-\langle 2,3,-7\rangle = \langle -2,-3,7\rangle$

② $\frac{1}{2}\langle 6,0,5\rangle = \langle 3,0,\frac{5}{2}\rangle$

③ $\langle 5,-1,0\rangle + \langle 2,13,17\rangle$

$= \langle 7,12,17\rangle$

④ linear ~~compi~~ combination

$2\langle 1,2,3\rangle - 5\langle 0,-1,1\rangle$

$= \langle 2\cdot 1-5\cdot 0,\ 2\cdot 2-5(-1),\ 2\cdot 3-5(1)\rangle$

$= \langle 2,9,1\rangle$

# Alternative Notation

example: $\langle 2, 5, -7 \rangle$

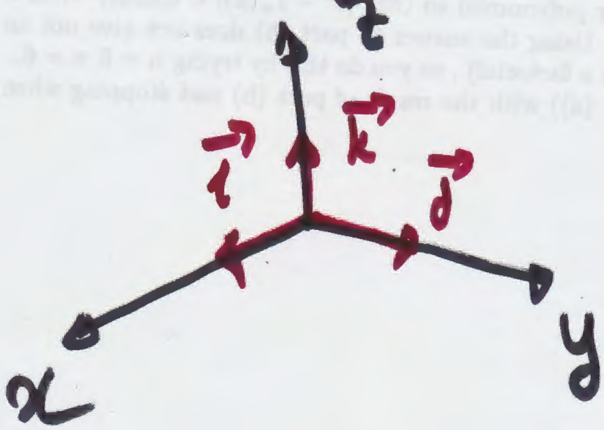
$$= \langle 2,0,0\rangle + \langle 0,5,0\rangle + \langle 0,0,-7\rangle$$

$$= 2\underbrace{\langle 1,0,0\rangle}_{\vec{i}} + 5\underbrace{\langle 0,1,0\rangle}_{\vec{j}} - 7\underbrace{\langle 0,0,1\rangle}_{\vec{k}}$$

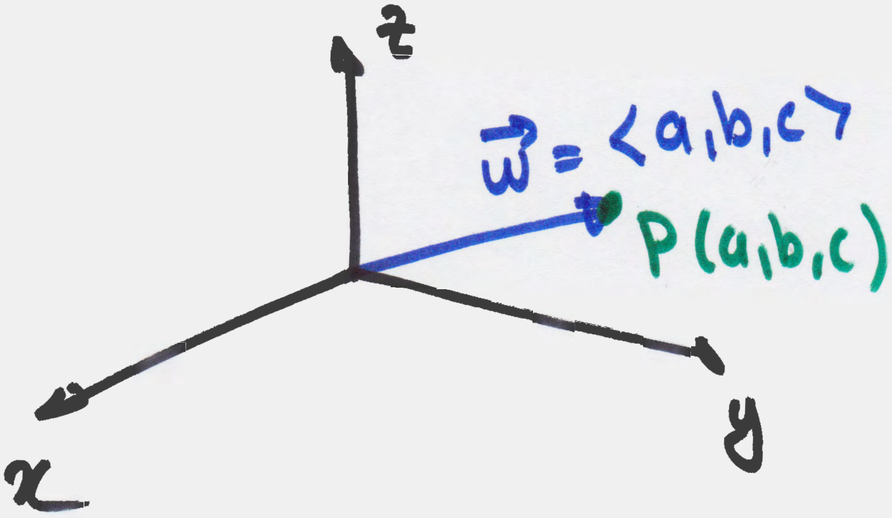
$$= 2\vec{i} + 5\vec{j} - 7\vec{k} \qquad (\hat{i}, \hat{j}, \hat{k})$$

$\vec{i}, \vec{j}, \vec{k}$ are unit vectors

i.e. they have length 1.



# Length of a vector

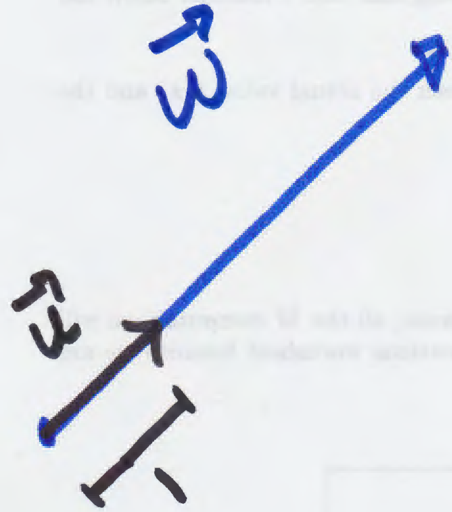



Length of $\vec{w}$ is the distance from the origin to P(a,b,c)

$$|\vec{w}| = \sqrt{a^2 + b^2 + c^2}$$

$|-2|$ : absolute value

$|\langle -2, 1 \rangle|$ : length/norm




$\vec{u}$ is the unit vector in the same direction as $\vec{w}$

To get $\vec{u}$ from $\vec{w}$ we have to scale $\vec{w}$:

$$\vec{u} = \underbrace{\frac{1}{|\vec{w}|}}_{\text{scalar}} \vec{w}$$

vector

vector

example: $\vec{w} = \langle 2, 7, 1 \rangle$

$$|\vec{w}| = \sqrt{4+49+1} = \sqrt{54}$$

$$\vec{u} = \frac{1}{\sqrt{54}} \langle 2, 7, 1 \rangle = \left\langle \frac{2}{\sqrt{54}}, \frac{7}{\sqrt{54}}, \frac{1}{\sqrt{54}} \right\rangle$$

Check: $|\vec{u}| = 1$.

example: Write the vector $\vec{v}$ which has length 3, which points in the same direction as $\langle 1, 2 \rangle$.


$$|\langle 1, 2 \rangle| = \sqrt{1+2} = \sqrt{5}$$

$$\vec{u} = \frac{1}{\sqrt{5}} \langle 1, 2 \rangle$$

$$\vec{v} = 3 \cdot \frac{1}{\sqrt{5}} \langle 1, 2 \rangle$$

$$= \left\langle \frac{3}{\sqrt{5}}, \frac{6}{\sqrt{5}} \right\rangle$$

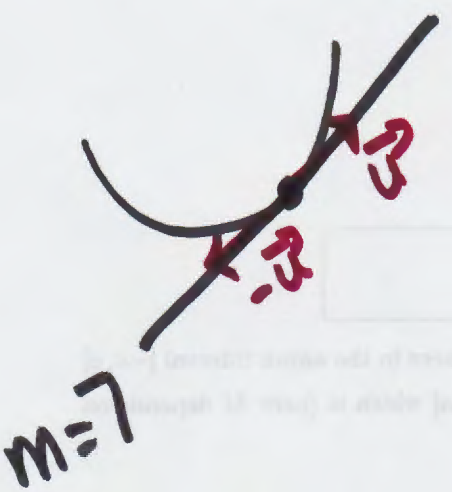
example: Find a unit vector parallel to the tangent line to

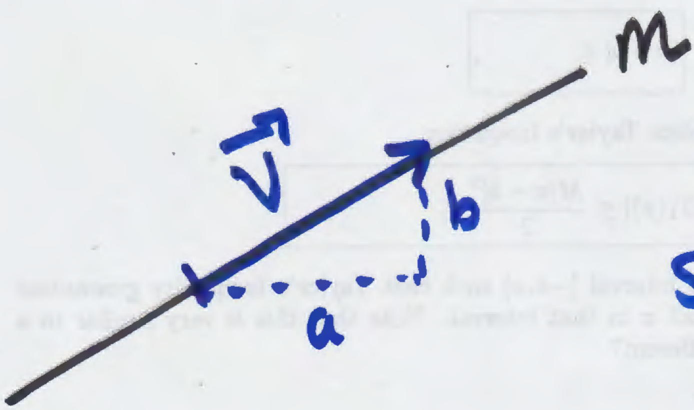
$y = x^2 + 3x + 1$ at $x = 2$.

slope:

$y' = 2x + 3$

$y'(2) = 2 \cdot 2 + 3 = 7$



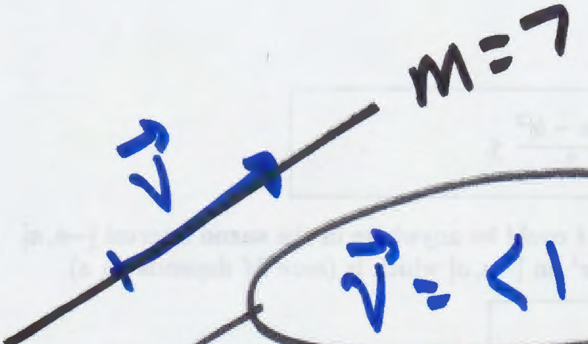

$\vec{v} = \langle a, b \rangle$

such that $\frac{b}{a} = m$

$\Rightarrow b = ma$

$\vec{v} = \langle 1, m \rangle$

$\vec{v} = \langle 5, 5m \rangle \ldots$




$\vec{v} = \langle 1, 7 \rangle$

(or $\vec{v} = \langle 2, 14 \rangle$

or $\vec{v} = \langle -3, -21 \rangle$ )

$$\vec{u} = \frac{1}{|\vec{v}|}\vec{v}$$

$$= \frac{1}{\sqrt{1+49}} \langle 1, 7 \rangle$$

$$= \left\langle \frac{1}{\sqrt{50}}, \frac{7}{\sqrt{50}} \right\rangle$$

OR $-\vec{u} = \left\langle \frac{-1}{\sqrt{50}}, \frac{-7}{\sqrt{50}} \right\rangle$

both answer the question.