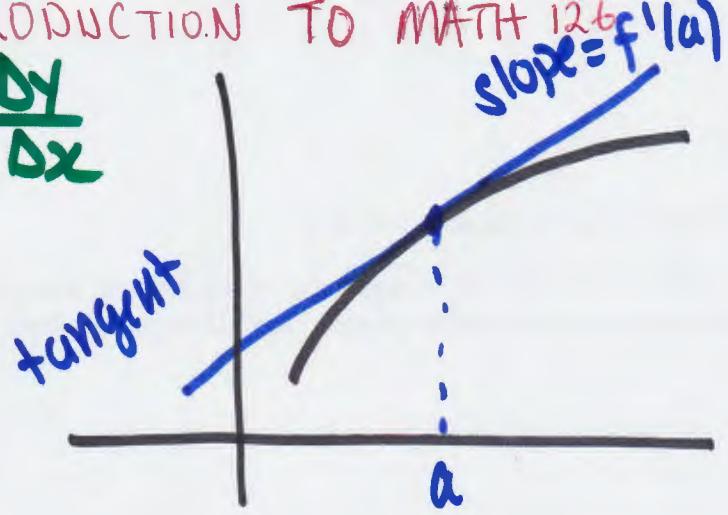
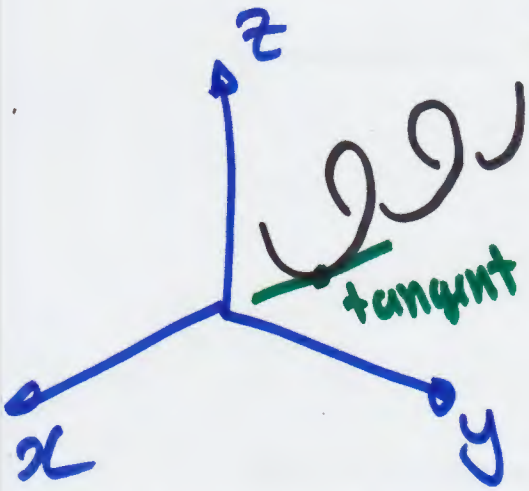
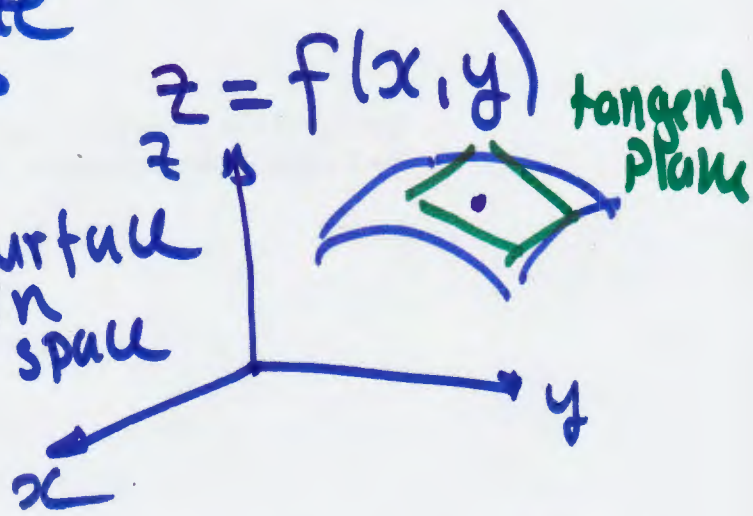
INTRODUCTION TO MATH 126

Slope = $\frac{\Delta y}{\Delta x}$



Generalize





(Ch. 13) curve in space

slope ??

replace with DIRECTION

idea: $\frac{\Delta z}{\Delta x}$ $\frac{\Delta z}{\Delta y}$

→ partial derivatives (Chp. 14)

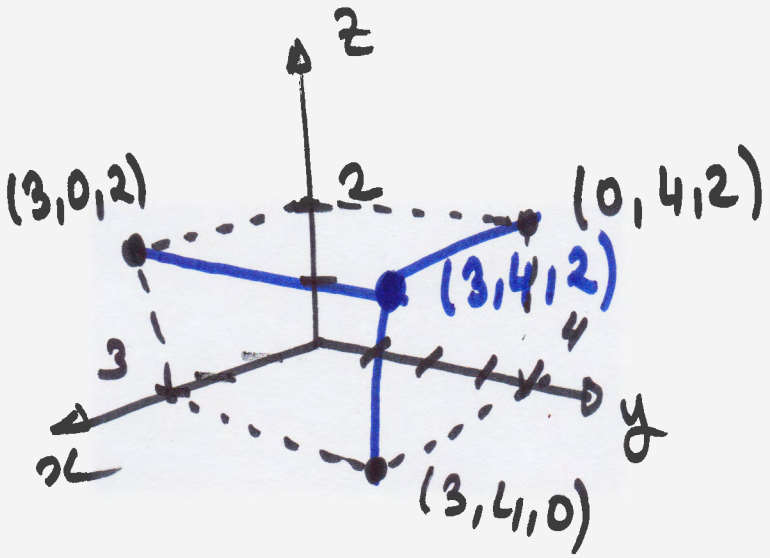
VECTORS
Chapter 12

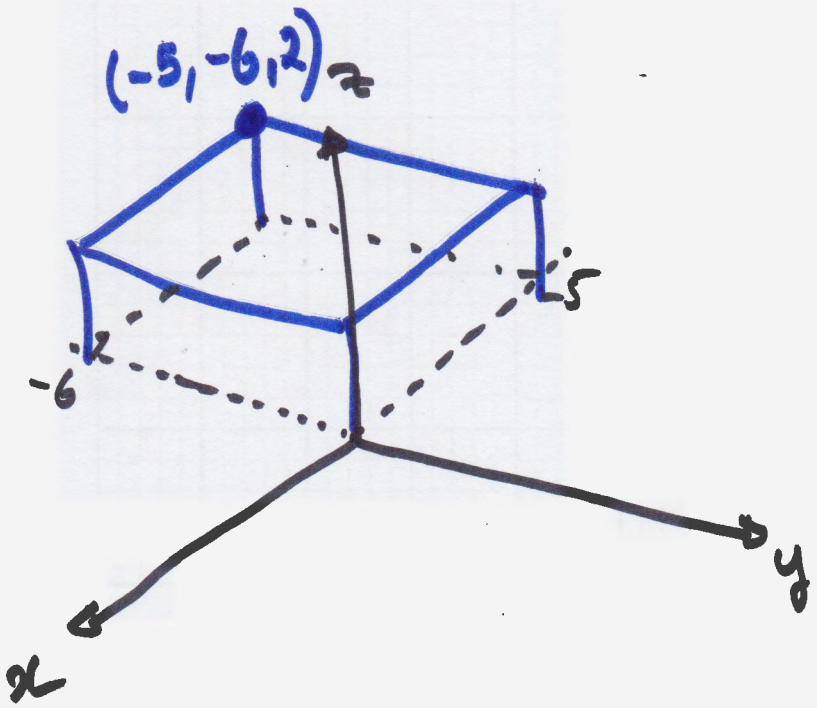
Chapter 15 – double integrals

Finish with Taylor Polynomials

Chapter 12 - Geometry of Space + Vectors

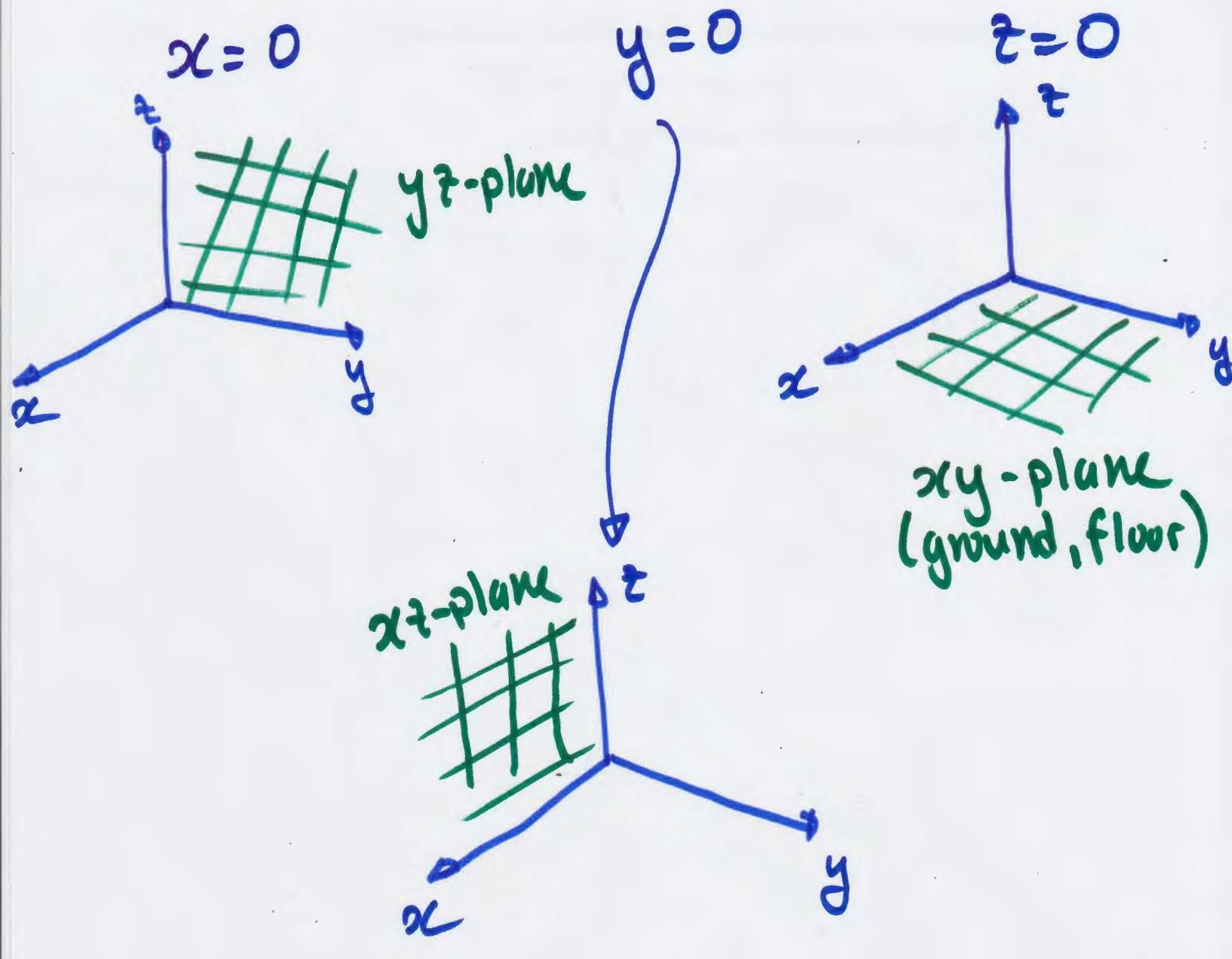
12.1 3D coordinate space





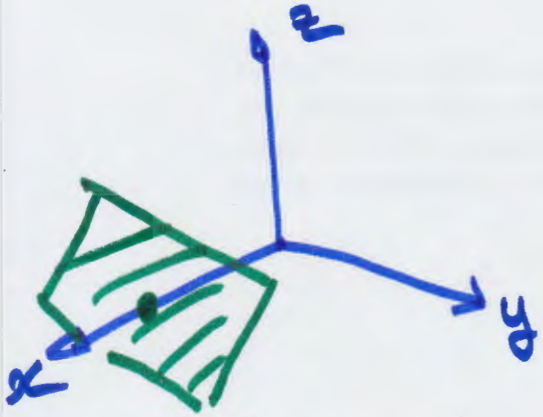
(-5,-6,2)

Basic Equations



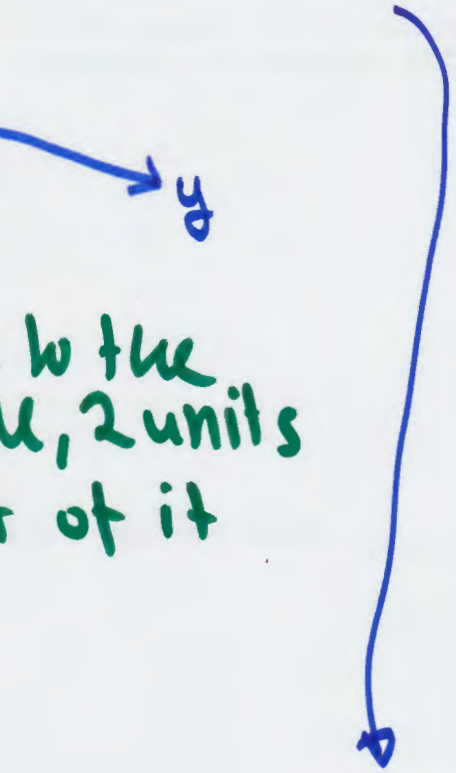
These are called the coordinate planes

$x = 2$

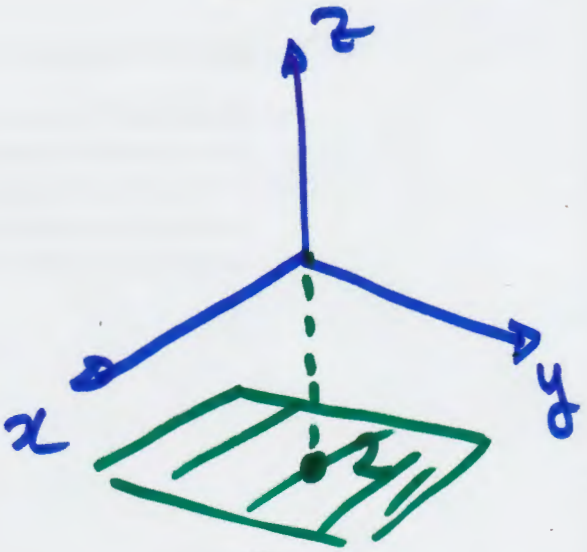


parallel to the yz-plane, 2 units in front of it

$y = 1$

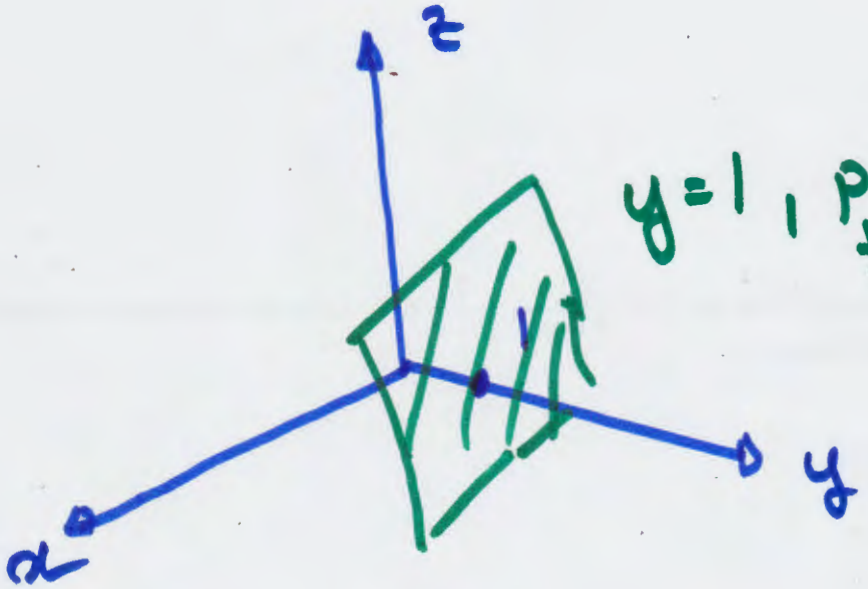


$z = -2$



$z = 2$

parallel to the xy-plane, 2 units below it



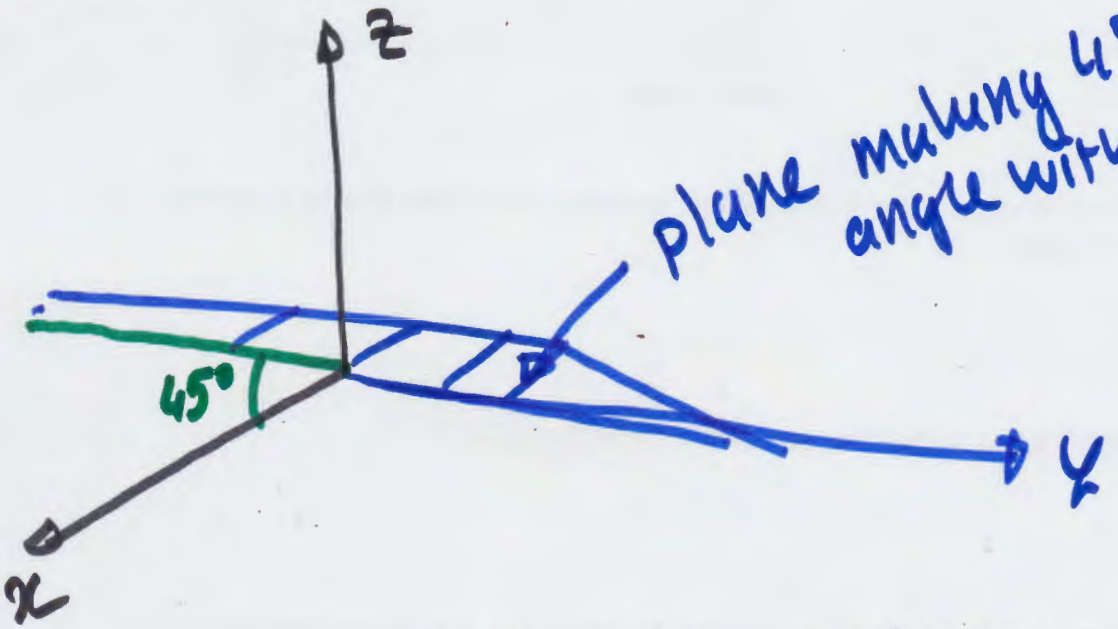
$y = 1$, parallel to the xz-plane

$x = z$ or $x + y = 3$ OR $2x - 5y + 3z = 11$ later in 12.4

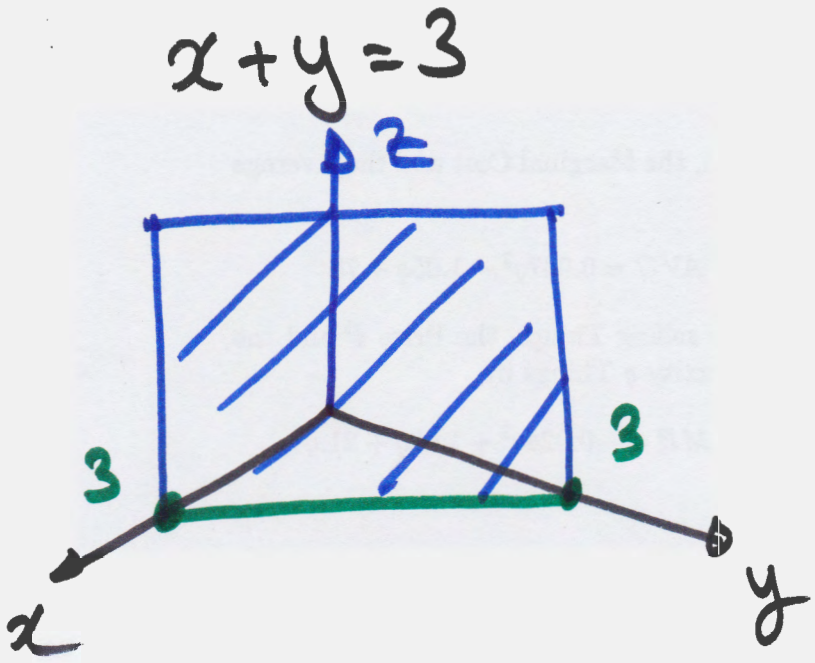
one variable is missing in these equations

Trick for sketching or visualizing equations where one variable is missing

STEP 1 - Ignore the missing variable

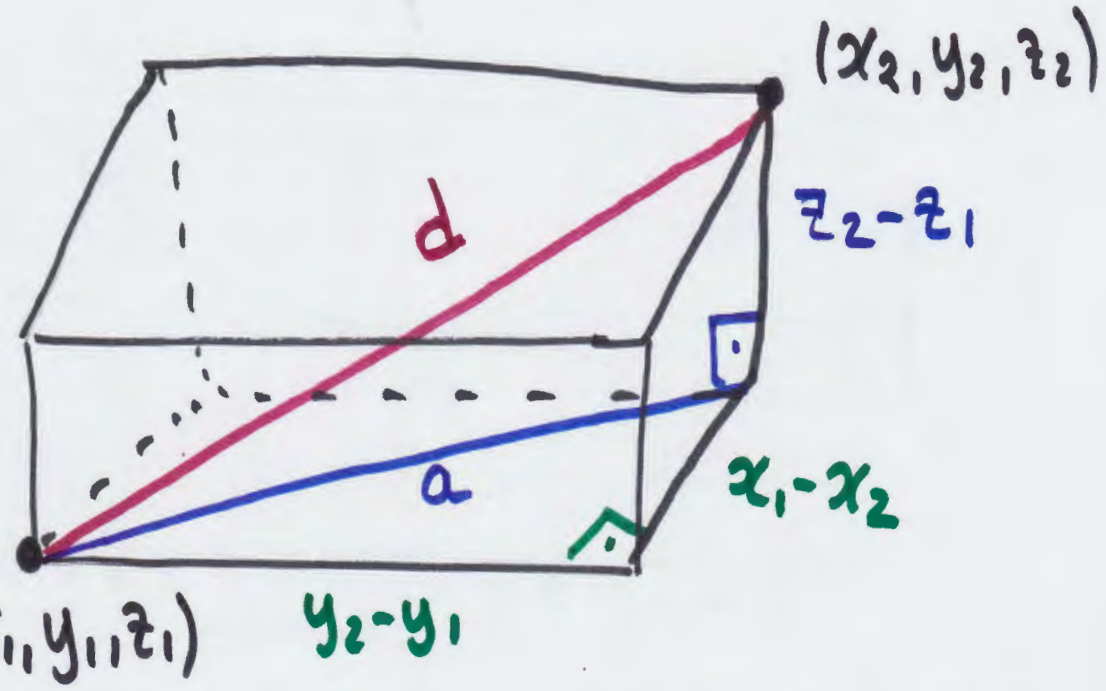


STEP 2 - Add the missing variable axis & we "pull" the graph we drew in that direction



we can use the same trick with non-linear equations (12.6)

Distance Formula



Pythagorean Theorem:

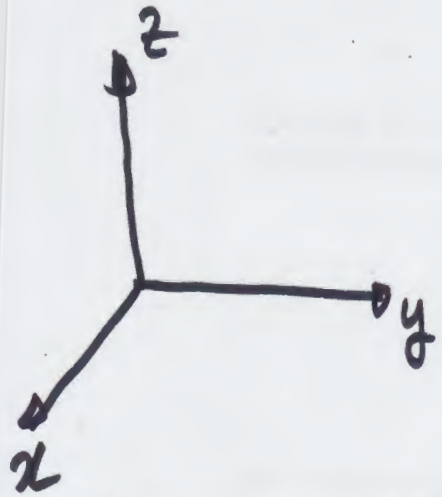
$$d^2 = a^2 + (z_2 - z_1)^2$$

Pythagorean Theorem

$$a^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$

$$d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

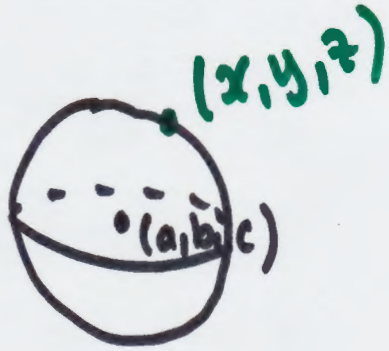


In my picture: $y_2 > y_1$

$z_2 > z_1$

$x_2 < x_1$

Equation of the sphere with center (a,b,c) and radius r



(distance)^2 from an arbitrary point (x,y,z) to the center (a,b,c) is always $(r)^2$

$$r^2 = (x-a)^2 + (y-b)^2 + (z-c)^2$$

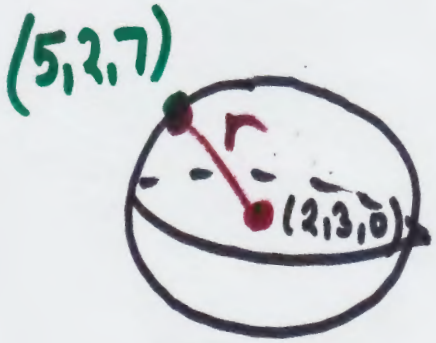
A sphere is an example of a surface.

examples:

① write down the equation of the sphere with center $(1,2,5)$ and radius 3.

$$(x-1)^2+(y-2)^2+(z-5)^2=9$$

② write down the equation of the sphere with center $(2,3,0)$ which passes through the point $(5,2,7)$



$$r=\sqrt{(5-2)^2+(2-3)^2+(7-0)^2}$$

$$=\sqrt{59}$$

$$(x-2)^2+(y-3)^2+z^2=59$$

③ Describe the sphere (center + radius) given by

$$x^2 - 3x + y^2 + z^2 - 2z = 2.$$

complete squares:

$$\left(x^2 - 3x + \frac{9}{4}\right) + y^2 + \left(z^2 - 2z + 1\right) = 2 + 1 + \frac{9}{4}$$

$$\left(\frac{-3}{2}\right)^2 = \frac{9}{4} \qquad \left(\frac{-2}{2}\right)^2 = 1$$

$$\left(x - \frac{3}{2}\right)^2 + y^2 + (z-1)^2 = \frac{21}{4}$$

center $\left(\frac{3}{2}, 0, 1\right)$

radius $\frac{\sqrt{21}}{2}$