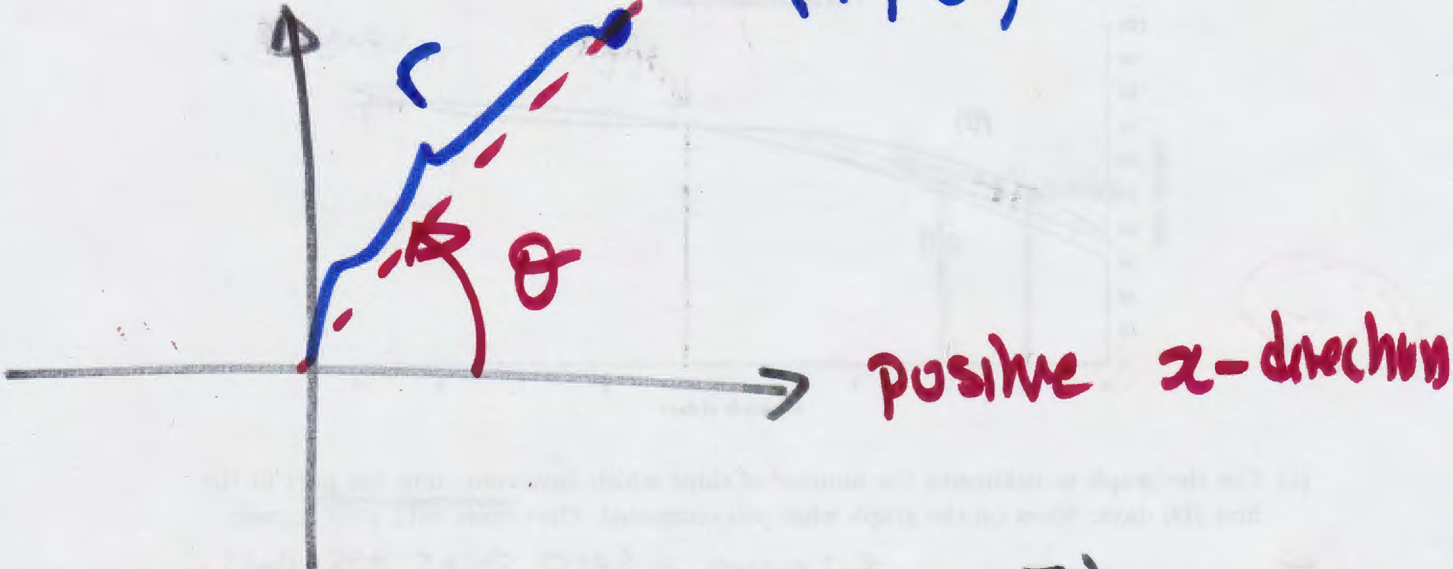
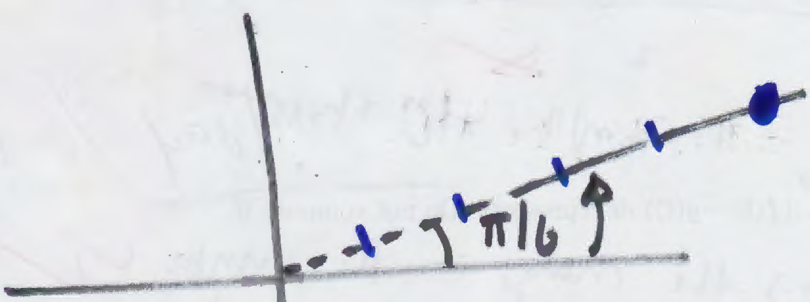
10.3 Polar Coordinates

we need this as a "technique" in double integrals

(r, θ)

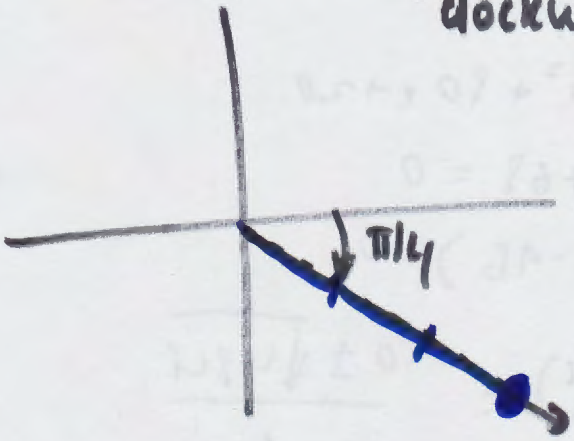


example: $(\cancel{\tfrac{\pi}{6}, 5})$ $(5, \tfrac{\pi}{6})$



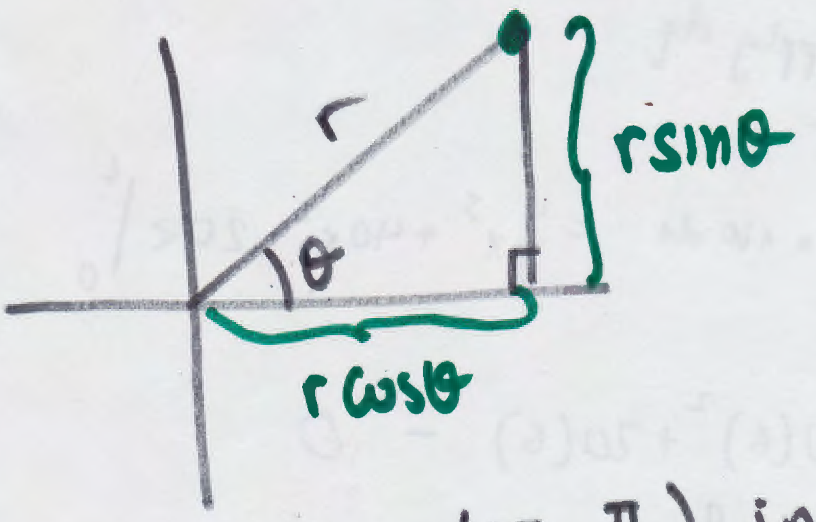
ex: $(3, -\frac{\pi}{4})$

clockwise



Switching from Polar to Cartesian (x,y)

$(x,y) = (r\cos\theta, r\sin\theta)$



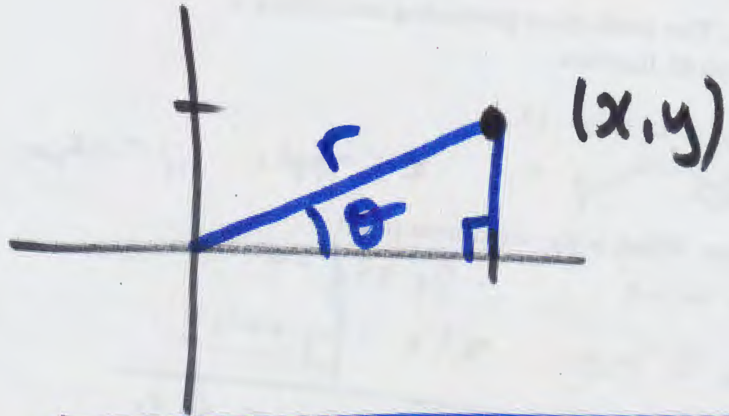
ex: Write $(15, \frac{\pi}{6})$ in Cartesian:

$x = 15\cos\frac{\pi}{6} = 15 \cdot \frac{\sqrt{3}}{2}$

$y = 15\sin\frac{\pi}{6} = 15 \cdot \frac{1}{2}$

$\left(\frac{15\sqrt{3}}{2}, \frac{15}{2}\right)$

Switching from Cartesian → Polar

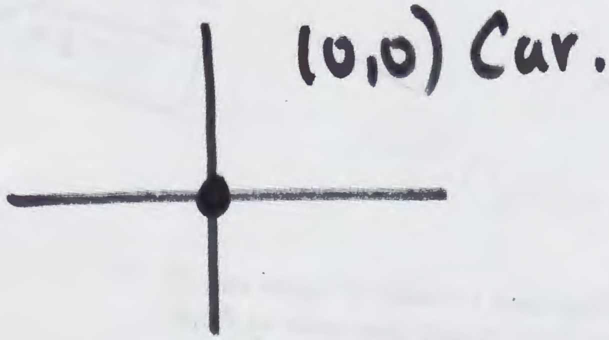


$r^2 = x^2 + y^2$

$\boxed{r = \sqrt{x^2 + y^2}}$

$\boxed{\tan\theta = \frac{y}{x} \quad , \quad x \neq 0}$

If $x = 0$

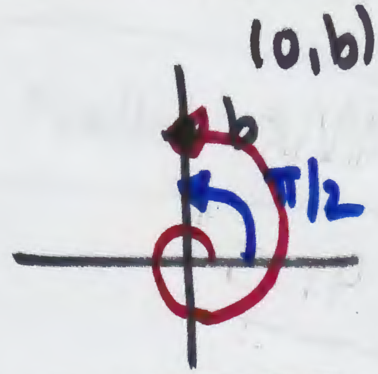


$\boxed{(0,0) \text{ Polar}}$

$(0, \frac{\pi}{4})$

$(0, 17\pi)$

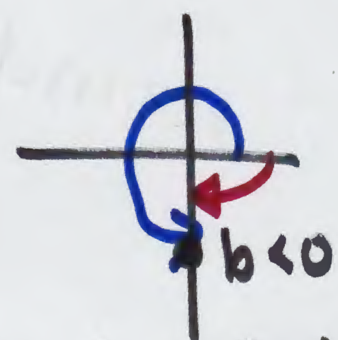
$(0, \alpha)$ is the origin in polar coordinates



$\boxed{(b, \frac{\pi}{2})}$

$(b, 2\pi + \frac{\pi}{2})$

$(b, 2\pi k + \frac{\pi}{2})$



$(-b, \frac{3\pi}{2})$

$(-b, -\frac{\pi}{2})$

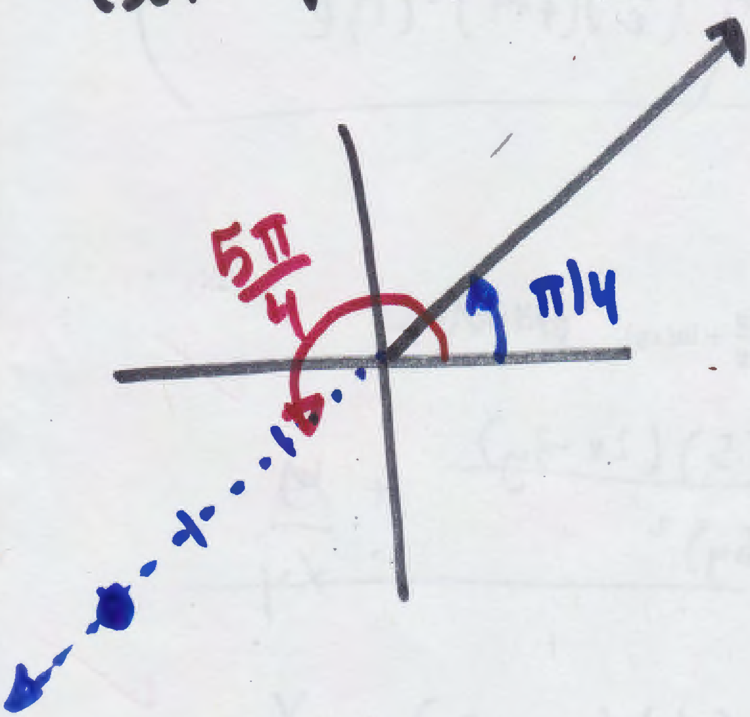
The point (r, θ)

all the $(r, \theta + 2\pi k)$, k integer

are the same.

We can also have negative r's:

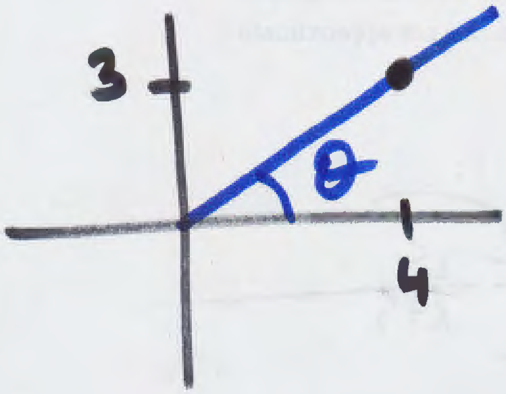
ex: $(-3, \pi/4)$



same as

$(3, \frac{5\pi}{4})$

ex① Convert (4,3) to Polar



$r^2 = 3^2 + 4^2 = 5^2$

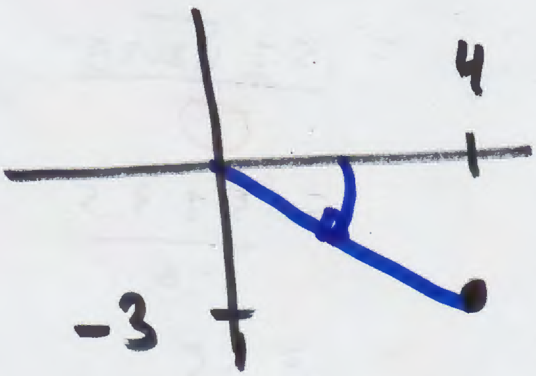
$r = 5$

$\tan\theta = \frac{3}{4} = 0.75$

$\theta = \tan^{-1}(0.75) \simeq 0.64$

$\left(5, \tan^{-1}\left(\frac{3}{4}\right)\right)$

ex② Convert (4,−3) to Polar



$r^2 = 4^2 + 3^2 = 5^2$

$r = 5$

$\tan\theta = \frac{-3}{4}$

$\theta = \tan^{-1}\left(\frac{-3}{4}\right)$

$= -\tan^{-1}\left(\frac{3}{4}\right)$

$\left(5, \tan^{-1}\left(\frac{-3}{4}\right)\right)$

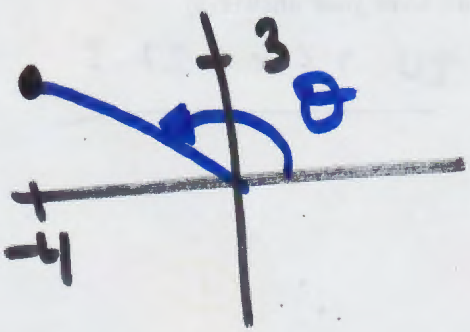
If $A \geq 0$, $0 \leq \tan^{-1} A < \frac{\pi}{2}$

If $A < 0$, $-\frac{\pi}{2} < \tan^{-1} A < 0$

Note: If your point is in the 1st or 4th quadrant then $\theta = \tan^{-1}\left(\frac{y}{x}\right)$.

③ Convert $(-4, 3)$ to Polar

$r = 5$

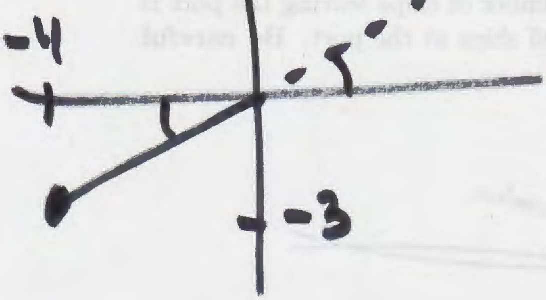


$\tan\theta = \frac{3}{-4} = \frac{-3}{4}$

$\theta = \pi - \tan^{-1}\left(\frac{3}{4}\right)$

$= \pi + \tan^{-1}\left(-\frac{3}{4}\right)$

(4) Convert $(-4,-3)$ to Polar



$$\tan\theta = \frac{-3}{-4} = \frac{3}{4}$$

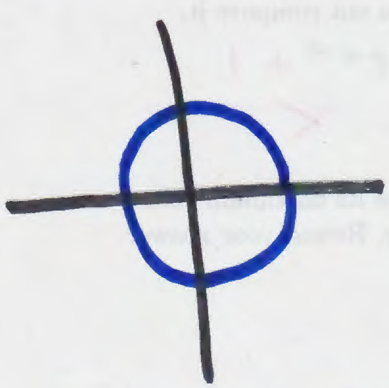
$$\theta = \pi + \tan^{-1}\left(\frac{3}{4}\right)$$

If your point is in the 2nd or 3rd quadrant $\theta = \pi + \tan^{-1}\left(\frac{y}{x}\right)$

Polar Curves

Think $r = f(\theta)$

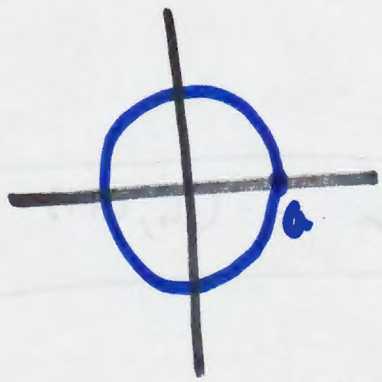
examples (1) $r = 1$



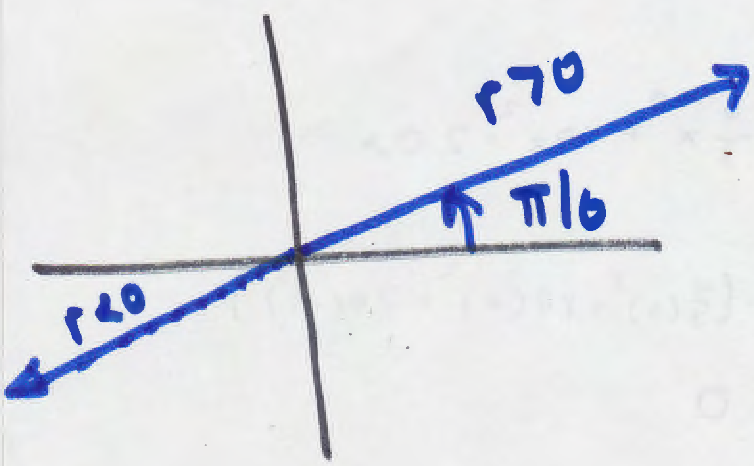
unit circle

In general, $r = a$ $(a > 0)$ is the circle with center $(0,0)$ and radius a.

IMPORTANT!



ex② $\theta = \frac{\pi}{6}$



A line through the origin with slope $\tan\frac{\pi}{6} = \sqrt{3}$.

$r = \theta$

ex: $r = 2\sin\theta$

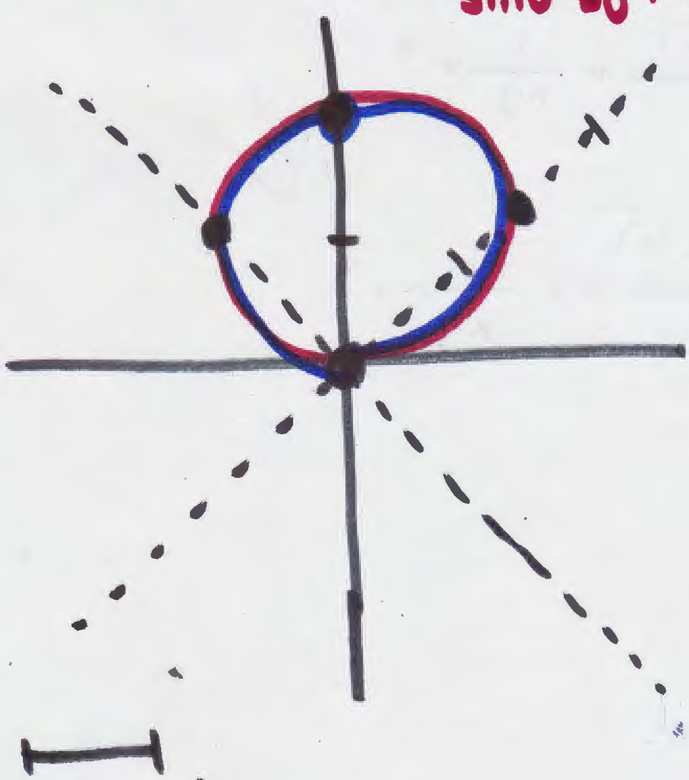
$|r| \nearrow 2$ spiral out

$\sin\theta \searrow -1 \quad r \searrow -2$

$\sin\theta \nearrow \quad r \nearrow$ spiral out

θ	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π	$\frac{5\pi}{4}$	$\frac{3\pi}{2}$	$\frac{7\pi}{4}$	2π
r	0	$\sqrt{2}$	2	$\sqrt{2}$	0	$-\sqrt{2}$	-2	$-\sqrt{2}$	0
	origin		same		origin		same		origin

$\sin\theta \searrow 0 \quad r \searrow 0$ spiral in



1 unit

Convert $r=2\sin\theta$ to cartesian

recall: $r^2=x^2+y^2$

$x=r\cos\theta$

$\boxed{y=r\sin\theta}$

$$r^2=2r\sin\theta$$

$$x^2+y^2=2y$$

$$x^2+y^2-2y=0$$

$$x^2+(y^2-2y+1)=0+1$$

$$x^2+(y-1)^2=1^2$$

circle with radius 1 and center (0,1)

In general, $r = 2a\sin\theta$, $a > 0$ we have a circle with center $(0, a)$ and radius a.

IMPORTANT!

We also have $r = 2a\cos\theta$, $a > 0$ we have a circle with center $(a, 0)$ and radius a.

IMPORTANT!