You can follow the steps below to show that the curvature formula

$$\kappa = \frac{|\mathbf{r}' \times \mathbf{r}''|}{|\mathbf{r}'|^3}$$

follows from $\kappa = \frac{\left|\frac{d\mathbf{T}}{dt}\right|}{|\mathbf{r}'(t)|}$, which in turn follows from the definition $\kappa = \left|\frac{d\mathbf{T}}{ds}\right|$ using the chain rule $\frac{d\mathbf{T}}{ds} = \frac{\frac{d\mathbf{T}}{dt}}{\frac{ds}{dt}}$ and the Fundamental Theorem of Calculus

$$\frac{ds}{dt} = \frac{d}{dt}\int_0^t |\mathbf{r}'(u)|\,du = |\mathbf{r}'(t)|$$

Here are the steps:

Since

$$\kappa = \frac{\left|\frac{d\mathbf{T}}{dt}\right|}{|\mathbf{r}'(t)|}$$

all we need to show is

$$\left|\frac{d\mathbf{T}}{dt}\right| = \frac{|\mathbf{r}' \times \mathbf{r}''|}{|\mathbf{r}'|^2}$$

(I omitted the t variable so the formulas look cleaner below, but you can, if you want, use $\mathbf{r}(t), \mathbf{T}(t), \mathbf{v}(t)$, etc.)

1. First, show that for any vector function $\mathbf{v}(t)$ we have

   $$\frac{d}{dt}|\mathbf{v}| = \frac{\mathbf{v} \cdot \mathbf{v}'}{|\mathbf{v}|}.$$

   To do that, you can either

   (a) Use $|\mathbf{v}| = (\mathbf{v} \cdot \mathbf{v})^{1/2}$ and the Product Rule (with dot product of vectors) and the chain rule OR

   (b) Write $\mathbf{v} =< f, g, h >$ and calculate both sides to check they are the same.

2. Use

   $$\mathbf{T} = \frac{1}{|\mathbf{r}'|}\mathbf{r}'$$

   and the product rule (for multiplication of a vector by a scalar) to show that

   $$\frac{d\mathbf{T}}{dt} = -\frac{1}{|\mathbf{r}'|^3}[(\mathbf{r}' \cdot \mathbf{r}'')\mathbf{r}' - (\mathbf{r}' \cdot \mathbf{r}')\mathbf{r}''].$$

   At some point you will need 1. above.

3. Now use (6, Theorem 11, Section 12.4), and the fact that the cross product of two vectors is orthogonal to both together with $|\mathbf{a} \times \mathbf{b}| = |\mathbf{a}||\mathbf{b}|\sin\theta$ to show that

   $$\left|\frac{d\mathbf{T}}{dt}\right| = \frac{|\mathbf{r}' \times \mathbf{r}''|}{|\mathbf{r}'|^2}.$$