Planning your Midterm II review

Functions of Two (or more) Variables - Chapter 14

1. (Section 14.1) Graphs of $z = f(x, y)$ in space. If it is a plane, for example $z = 2x + 3y - 4$, you are expected to visualize it. Three points determine a plane so you can graph the points $(x, 0, 0)$, $(0, y, 0)$, and $(0, 0, z)$ satisfying the plane equation and draw the triangle which is part of the plane. If x or y is missing from the equation, it is a generalized cylinder from Section 12.6. Or it may be one of the equations form the table in Section 12.6, for example $z = \sqrt{x^2 + y^2}$ is a cone. For more general graphs, you should be able to match the functions with their graphs. Look for symmetry and traces $z =$ constant.

2. (Section 14.1) Finding the domain of a function $z = f(x, y)$ by following the three rules: Do not divide by zero, do not take square roots of negative numbers, do not take logs (or lns) of non-positive numbers.

3. (Section 14.3) Partial derivatives computation, first and second derivatives. **Practice this a lot!** All rules of differentiation are valid. You should be also able to do this for functions of three variables $w = f(x, y, z)$. Here when you take f_x, you treat both y and z like constants. Also, you should be able to do implicit differentiation. For example computing z_x and z_y form $zx + xy + yz = xyz$.

4. (Section 14.3) Interpretation of partial derivative as rate of change in one direction (x or y) as the other is kept constant. You did a worksheet on this.

5. (Section 14.4) Tangent planes and linear approximation. For explicit functions $z = f(x, y)$ and implicit functions. Once you get the tangent plane equation $z = Ax + By + C$, linear approximation is simply $f(x, y) \approx Ax + By + C$.

6. (Section 14.7) Optimization, including story problems where you set up the function $f(x, y)$. **You can be sure I will ask this question on the midterm.** Critical points are where both $f_x = 0$ and $f_y = 0$. There may be several and depending on the function it may involve some algebra. Once you find the critical points, you have two ways to go:

   (a) The domain is closed and bounded. You check the function on the boundary by reducing it to a one variable problem. The boundary equation may be of the form $y = g(x)$, $x = h(y)$, or you may choose to go parametric with $x = g(t)$ and $y = h(t)$. Think about the equation and decide accordingly. Finally, compare the function values at the critical points with those at the boundary.

   (b) The domain is not closed and bounded, for example all values (x, y). Then you use the second derivatives f_{xx}, f_{yy} and $f_{xy} = f_{yx}$ and the discriminant to decide if the critical point gives a local maximum, local minimum or a saddle point.

Polar Coordinates - Section 10.3

1. The polar coordinate system, what r and θ represent, plotting points. Different polar coordinates can give the same point on the xy-plane. Dealing with negative r and θ values.

2. Switching from Polar to Cartesian: $x = r\cos(\theta)$ and $y = r\sin(\theta)$, points and equations.

3. Switching for Cartesian to Polar: $r^2 = x^2 + y^2$ and $\tan(\theta) = y/x$. Here, you have to be careful using the inverse tangent function $\tan^{-1}(a) = \arctan(a)$.

4. Graphing basic polar curves, in particular the circles $r = a$, $r = 2a\cos(\theta)$, $r = 2a\sin(\theta)$ and the line $\theta =$ constant. Matching graphs for more complicated ones.

Double Integrals - Chapter 15

1. (Section 15.1) Definition of the double integral and approximating double integrals. The double integral as volume when $f(x, y) > 0$.

2. (Section 15.1) Double integrals over rectangles. **Do not forget** your *dxdy* or *dydx* when you set up an integral.

3. (Section 15.2) Double integrals over general regions D. Deciding to go $dxdy$ or $dydx$ depending on the region. Switching the order of integration if one way does not work. **Sketch your regions** D, especially when you are switching the order of integration. Splitting the integral if the region is neither Type I or Type II.

4. (Section 15.3) Double integrals with polar coordinates and regions. Remember the r in $rdrd\theta$. Switching from Cartesian to Polar as an integration technique. See 4. in Polar Coordinates above.

5. (Section 15.4) Applications: Area of D, volume under $z = f(x, y)$, average value and center of mass.

Look at questions from the book or the relevant assignment to review a topic in a particular section. Look at old midterm questions for general practice.