Math 124 Solutions to Quiz 6

1. Using L'Hospital's Rule twice:

$$\lim_{x\to 0}\frac{x-\sin x}{x-\tan x} \overset{\text{LH}}{=} \lim_{x\to 0}\frac{1-\cos x}{1-\sec^2 x} \overset{\text{LH}}{=} \lim_{x\to 0}\frac{\sin x}{-2\sec x\sec x\tan x} = \lim_{x\to 0}\frac{\cos^3 x}{-2} = -\frac{1}{2}$$

or using L'Hospital's Rule once and then using algebra:

$$\lim_{x\to 0}\frac{x-\sin x}{x-\tan x} \overset{\text{LH}}{=} \lim_{x\to 0}\frac{1-\cos x}{1-\sec^2 x} = \lim_{x\to 0}\frac{(1-\cos x)\cos^2 x}{\cos^2 x-1} = \lim_{x\to 0}\frac{-\cos^2 x}{\cos x+1} = -\frac{1}{2}$$

2. $\lim_{x\to 0}(1-2x)^{3/x}$

$$y = (1-2x)^{3/x}$$

$$\ln y = \ln\left((1-2x)^{3/x}\right) = \frac{3\ln(1-2x)}{x}$$

$$\lim_{x\to 0}\ln y = \lim_{x\to 0}\frac{3\ln(1-2x)}{x} = 3\lim_{x\to 0}\frac{\frac{-2}{1-2x}}{1} = 3\lim_{x\to 0}\frac{-2}{1-2x} = -6$$

So,

$$\lim_{x\to 0} y = \lim_{x\to 0} e^{\ln y} = e^{-6}.$$