Math 124 A Fall 2018 Solutions to Quiz 5

$$\frac{2}{x+y} + 2x^2 - y = 1$$

1. Differentiate to get
$$\frac{-2(1+y')}{(x+y)^2} + 4x - y' = 0$$
Clean denominator
$$-2(1+y') + 4x(x+y)^2 - y'(x+y)^2 = 0$$
Solve for y'
$$y' = \frac{-2 + 4x(x+y)^2}{(x+y)^2 + 2}$$

2. One way to go is from the y' formula:
When $x = 0$ and $y = 1$
$$y' = \frac{-2+0}{(0+1)^2+2} = -\frac{2}{3}$$
then we differentiate y' using the quotient rule
$$y'' = \frac{\left(4(x+y)^2 + 4x \cdot 2(x+y)(1+y')\right)\left((x+y)^2+2\right) - \left(-2 + 4x(x+y)^2\right)\left(2(x+y)(1+y')\right)}{\left((x+y)^2+2\right)^2}$$
When $x = 0$, $y = 1$ and $y' = -\frac{2}{3}$
$$y'' = \frac{(4)\,(1+2) - (-2)\left(2(1-\frac{2}{3})\right)}{(1+2)^2} = \frac{40}{27}$$

OR
to avoid fractions and quotient rule, we start with
$$-2(1+y') + 4x(x+y)^2 - y'(x+y)^2 = 0$$
When $x = 0$ and $y = 1$
$$-2(1+y') - y' = 0$$
so $y' = -2/3$ and differentiating
$$-2(y'') + 4(x+y)^2 + 4x \cdot 2(x+y)(1+y') - y''(x+y)^2 - y' \cdot 2(x+y)(1+y') = 0$$
When $x = 0$, $y = 1$ and $y' = -\frac{2}{3}$
$$-2(y'') + 4 - y'' - \frac{2}{3} \cdot 2(1 + \frac{2}{3}) = 0$$
so $y'' = 40/27$.