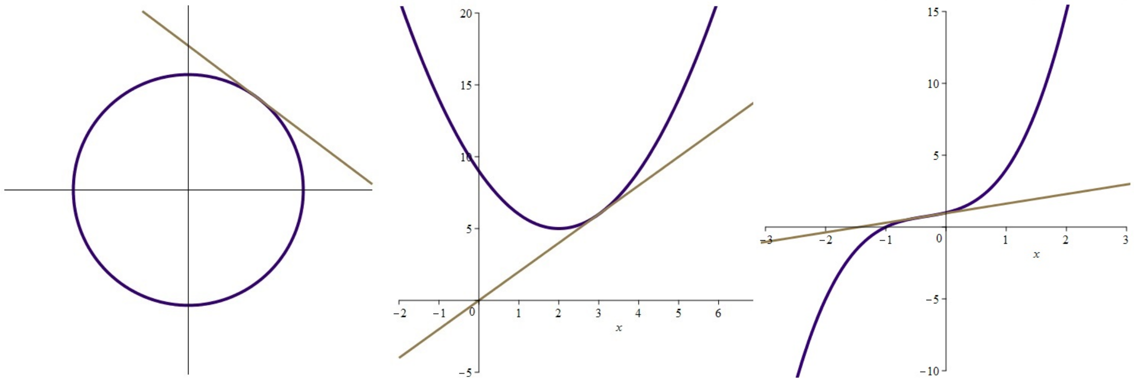
# Section 2.1 - Velocity and Tangent Problems

The aim of this section is state the problem of the **slope of the tangent line** or **instanteneous rate of change**. We'll see that while trying to compute these quantities - which will turn out to be the same problem - we will need a **new idea** . We'll develop that idea for the rest of this chapter making it the basis for the definition of the derivative.
*Hint: It is called the limit!*

## The Tangent Problem




The tangent line at a point is the line you see when you zoom in at that point. Given the equation of the curve and the point of tangency, we want to compute the **slope of the tangent line** . So given

$$y = f(x)$$

and a point

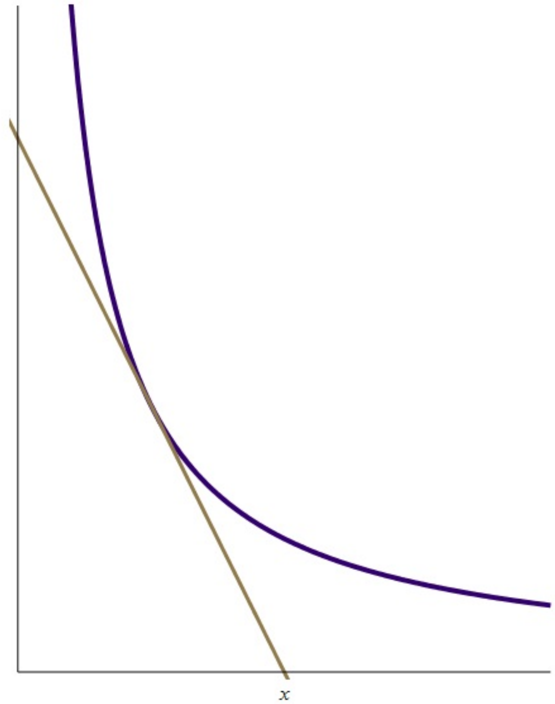
$$(a, f(a))$$

on the curve, can we compute the slope of the tangent line?

**Example 1:** Find the slope of the tangent line to the graph of

$$y = \frac{2}{x}$$

at the point $(1, 2)$.




**Problem:** In order to compute the slope of a line, we need **two** points. But, we only have one!

What do we do?

**BIG IDEA:** Choose a second point on the line. However, when we have a second point on the line, the line is no longer the tangent line. But if **we chose the second point very close to the first point**, then we get a good estimate for the slope of the tangent line.

Let's fill the table below and sketch the lines above:

| $x$-coordinate of second point | $y$-coordinate of second point | slope of line through $(1, 2)$ and the second point |
|---|---|---|
| 2 | $\frac{2}{1} = 1$ | $\frac{1-2}{2-1} = -1$ |
| 1.5 | $\frac{2}{1.5} \approx 1.3333$ | |
| 1.1 | | |
| 1.01 | | |
| 1.001 | | |
| 1.0001 | | |
| 1.00001 | | |

Now looking at the last column, can you guess the slope of the tangent line?

And write down the equation of the tangent line?

Could we have done this algebraically without a table?

We will actually do this later in this chapter, you can give it a try now if you want:

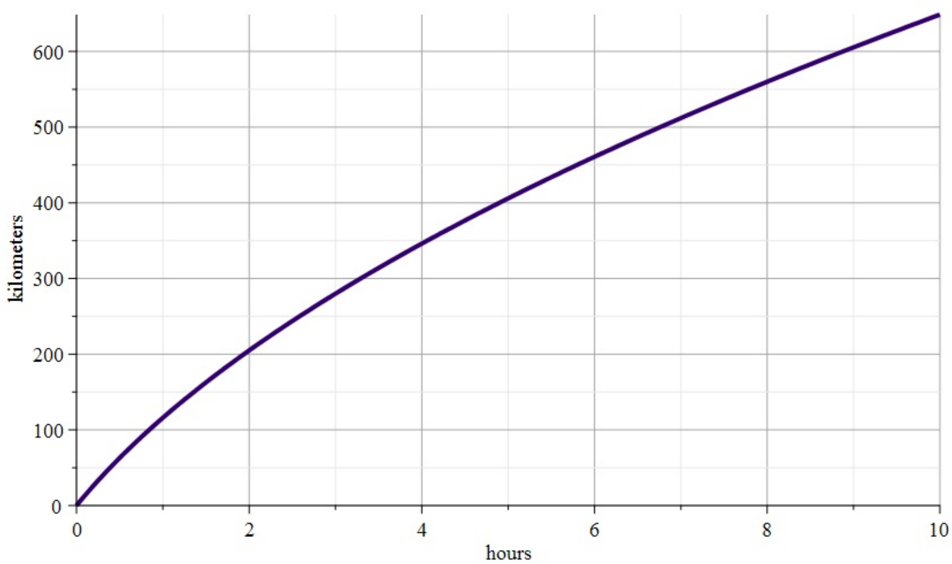
**Optional Exercise:** The first point was $(1, 2)$. The second point $(a, f(a)) = \left(a, \dfrac{2}{a}\right)$. First, write down the slope of the line through the points $(1, 2)$ and $\left(a, \frac{2}{a}\right)$ as rise over run. Then, simplify the quotient as much as you can (until you have a single fraction line). When it is in its simplest form, can you see what its value should be if $a$ is very very very close to 1?

## The Velocity Problem

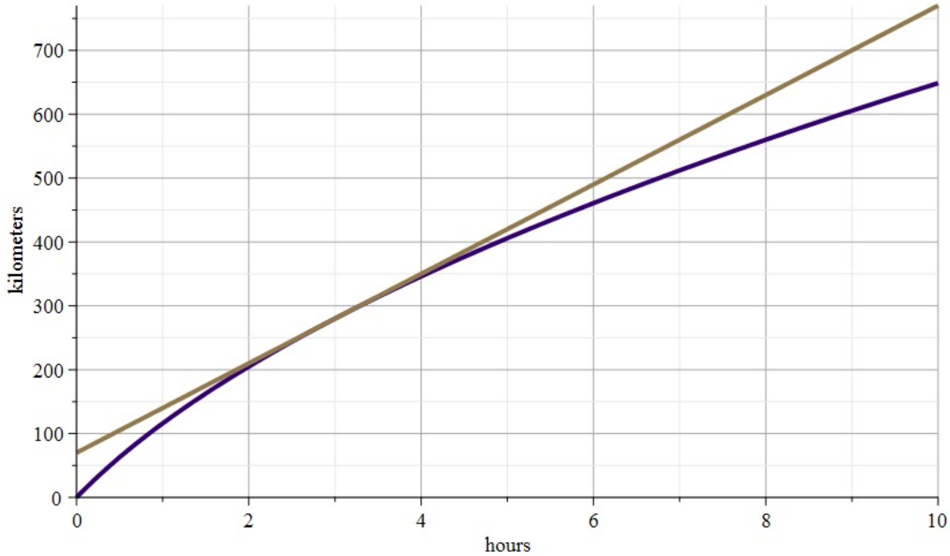
Given distance as a function of time $s(t)$, as a graph, table of values or formula, can we compute the instantaneous velocity at a given time? Here is the issue: To compute velocity, we divide the change in distance by the change in time. However, if we want the **velocity at an instant**, then we have neither a time interval, nor any distance is travelled!

**Example 2:** Given the following distance function, how fast is the car moving at exactly 3 hours after it starts its journey?

On the graph, average velocities are **secant slopes** and the instantaneous velocity is **the slope of the tangent line**! It is difficult to get precise values from the graph.



But, if we draw the tangent line, we can compute its slope using any two points!




If we had a table of values, then we could do estimations. However, the information on a table of values is limited.

| time (hours) | distance traveled (km) |
|---|---|
| 2.5 | 243.83 |
| 2.6 | 251.26 |
| 2.7 | 258.59 |
| 2.8 | 265.82 |
| 2.9 | 272.96 |
| 3 | 280 |
| 3.1 | 286.96 |
| 3.2 | 293.83 |
| 3.3 | 300.62 |
| 3.4 | 307.33 |
| 3.5 | 313.97 |

If we had the formula, we could get the distance values for $t = 3.01$, $3.001$, $3.0001$ hours or $t = 2.99$, $2.999$, $2.9999$ hours giving us much better estimates for the velocity at 3, which is also the slope of the tangent line at 3.

**Coming up:** We would like to formalize this process of **taking values closer and closer to a given number** to compute a quantity and eventually stop making tables of values. Once we have a good method that works, we will come back to slopes of tangents.