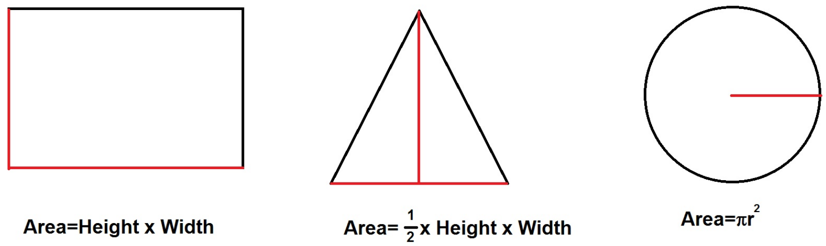
Geometry Review for Math 124

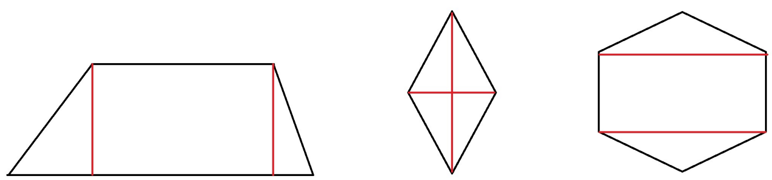
This is a review of basic geometry for use in story problems with pictures: Related Rates (and later in Applied Optimization). You are familiar with it all. Skim through and look at the suggested example related rate problems from my exam archive at https://sites.math.washington.edu/∼ebekyel/Archives/.

AREAS

You really need three area formulas:



For other shapes, you can add extra lines to divide up the region.

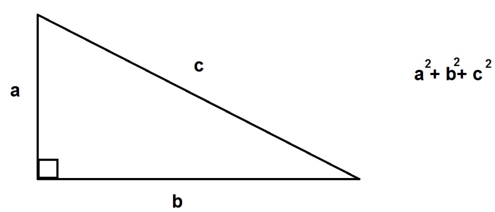


Example Problem: Winter 2013 MT2, #4

TRIANGLES

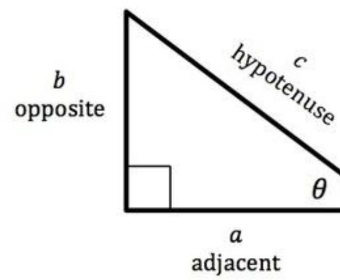
Triangles will come up in most problems. Here is all we use:

1. The Pythagorean Theorem



Example Problem: Autumn 2009 MT2, # 4

2. The trigonometric functions

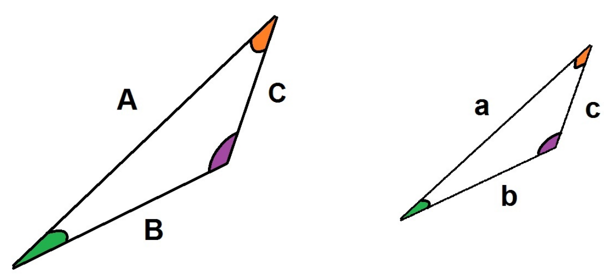


Function Name	Definition	Function Name	Definition
sine of θ	$\sin\theta = \frac{b}{c} = \frac{\text{opposite}}{\text{hypotenuse}}$	cosecant of θ	$\csc\theta = \frac{c}{b} = \frac{\text{hypotenuse}}{\text{opposite}}$
cosine of θ	$\cos\theta = \frac{a}{c} = \frac{\text{adjacent}}{\text{hypotenuse}}$	secant of θ	$\sec\theta = \frac{c}{a} = \frac{\text{hypotenuse}}{\text{adjacent}}$
tangent of θ	$\tan\theta = \frac{b}{a} = \frac{\text{opposite}}{\text{adjacent}}$	cotangent of θ	$\cot\theta = \frac{a}{b} = \frac{\text{adjacent}}{\text{opposite}}$

Example Problems: Winter 2017 MT2, # 4 and Spring 2013 MT2, #4

3. **Similar Triangles -** This is where most people get stuck. So, a good rule of thumb is, if the Pythagorean Theorem and trig functions did not solve your problem and you have a triangle in the picture consider similar triangles!

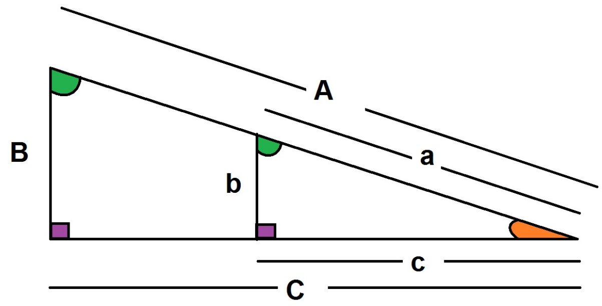
   Two triangles are similar if two (and therefore, all) of their angles are the same.



   Then, the corresponding sides satisfy

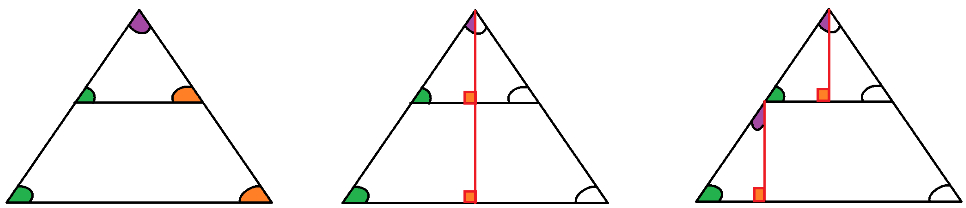
$$\frac{A}{a} = \frac{B}{b} = \frac{C}{c}$$

   Usually, just two of the sides (not all three) are used for the necessary equation. Frequently, the triangles will be right angles and possibly one will be inside the other.



Example Problem: Autumn 2016 MT2, # 4

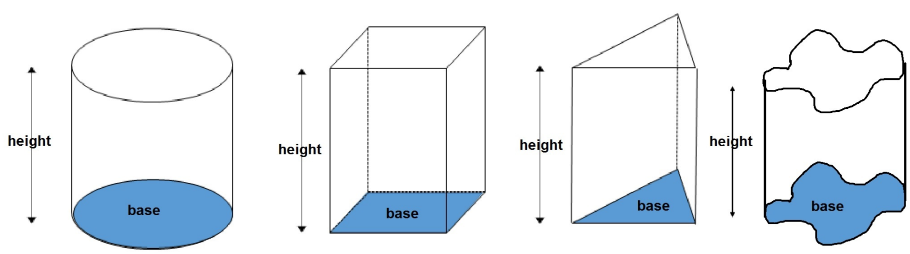
In the picture below, a line parallel to the base was drawn to make a smaller (similar to the original) triangle at the top. By adding vertical (in red) lines, you can make right triangles and create other similar triangles. Label the corresponding sides in the pictures below and write the ratios for all three.



VOLUMES

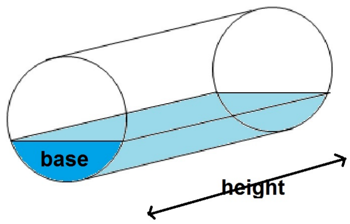
There are only three volume formulas you need to know.

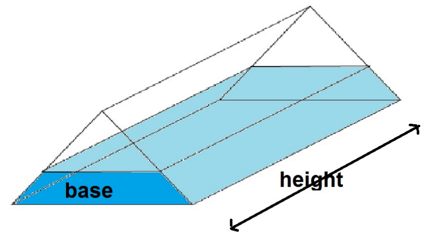
1. When the top and the bottom have the same shape the volume is always **base area times height**. The base could be any shape (whose area you know).



$$\textbf{Volume} = \textbf{ Base Area } \times \textbf{Height}$$

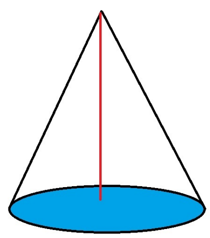
Here are two examples when the shape is lying on its side. You want to compute the volume of the shaded part (think of it as filled with water).

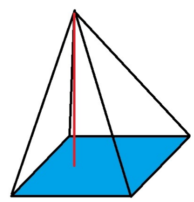


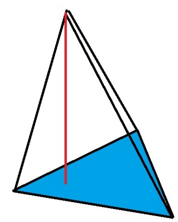


Example Problem: Spring 2011 MT2, # 4 (uses similar triangles)

2. When the top is **pointed** the volume is always **one third of base area times height**.





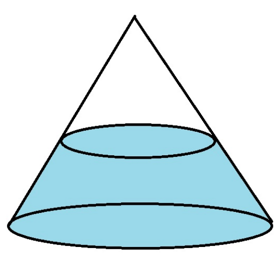


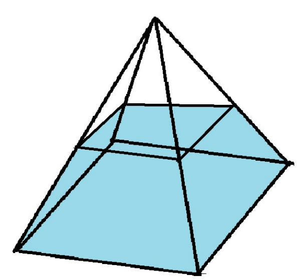
$$\textbf{Volume} = \frac{1}{3} \times \textbf{ Base Area } \times \textbf{Height}$$

where the height is the length of the perpendicular line from the tip to the base.

Example Problem: Winter 2007 MT2, # 4

How would you compute the shaded volumes below?





Example Problem: Autumn 2011 MT2, # 4 (uses similar triangles)

3. The volume of a **sphere** is

$$V = \frac{4}{3}\pi r^3$$

The surface area of a sphere is $S = 4\pi r^2$. Do you see how they are related?