Solutions to Math 124 Au 18 MT II (Section A)

1. Compute $\frac{dy}{dx}$ for the following. You do not have to simplify your answers. be careful with your parentheses.

(a) $y = 3x\sqrt{5x + \sqrt{1+7x}}$

$$\frac{dy}{dx} = 3\sqrt{5x+\sqrt{1+7x}} + \frac{3x\left(5 + \frac{7}{2\sqrt{1+7x}}\right)}{2\sqrt{5x+\sqrt{1+7x}}}$$

(b) $y = \frac{xe^x}{(x^2+1)^2}$

$$\frac{dy}{dx} = \frac{(e^x + xe^x)(x^2+1)^2 - xe^x \cdot 2 \cdot (x^2+1)2x}{(x^2+1)^4}$$

(c) $x = \ln t$, $y = \arctan(t^2 + e^t)$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{\frac{2t+e^t}{(t^2+e^t)^2+1}}{(1/t)} = \frac{(2t+e^t)t}{(t^2+e^t)^2+1}$$

(d) $y = (x^2+1)^{(x^3+4)}$

$$\ln y = (x^3+4)\ln(x^2+1)$$

$$\frac{1}{y}\frac{dy}{dx} = 3x^2\ln(x^2+1) + \frac{(x^3+4)(2x)}{x^2+1}$$

$$\frac{dy}{dx} = \left(3x^2\ln(x^2+1) + \frac{2x(x^3+4)}{x^2+1}\right)(x^2+1)^{x^3+4}$$

2. Find the absolute minimum and absolute maximum of the function

$$f(x) = \frac{x-2}{x^2+5}$$

on the interval $[0, 6]$.

$$f'(x) = \frac{x^2+5-(x-2)2x}{(x^2+5)^2} = \frac{-x^2+4x+5}{(x^2+5)^2} = 0$$

when $x = \frac{-4 \pm \sqrt{16+20}}{-2} = \frac{-4 \pm 6}{-2} = 5$ or -1 not on $[0,6]$

$f(6) = \frac{4}{41} < \frac{4}{40} = 0.1$

$f(5) = \frac{3}{30} = 0.1$ abs. max.

$f(0) = -\frac{2}{5} = -0.4$ abs. min.

3. Use linear approximation near $(0, 1)$ to estimate the value of b if $(-0.2, b)$ is on the curve given by

$$e^{xy} + 3y^3 + y = 6.$$

Use the value of y'' to determine whether your approximation is an overestimate or underestimate of the actual value of b.

Differentiate

$$e^{xy}(y + xy') + 9y^2y' + y' = 0$$

differentiate again

when $x = 0$ $y = 1$

$$1 + 9y' + y' = 0 \rightarrow y' = \frac{-1}{10} = -0.1$$

Tangent Line

$$y - 1 = -0.1(x - 0)$$

$$y = -0.1x + 1$$

Linear App. $y \approx -0.1x + 1$ so $b \approx -0.1(-0.2) + 1$
$= 1.02$

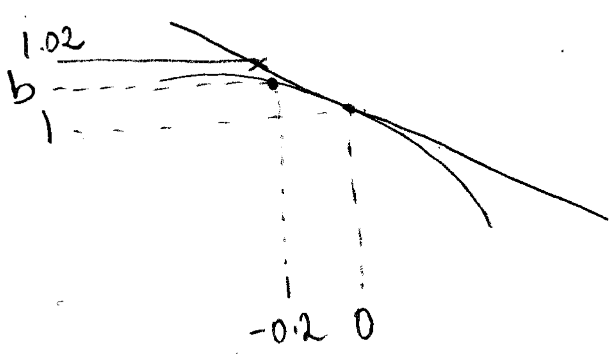
$$e^{xy}(y + xy')^2 + e^{xy}(y' + y' + xy'') + 18yy'y' + 9y^2y'' + y'' = 0$$

when $x = 0$ $y = 1$ $y' = -0.1$

$$1 + (-0.2) + 18(-0.1)^2 + 9y'' + y'' = 0$$

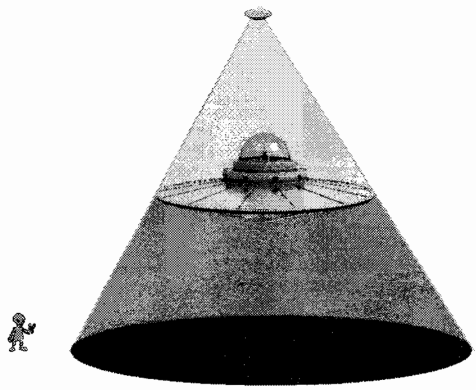
$$y'' = \frac{-1 + 0.2 - 0.18}{10} < 0$$

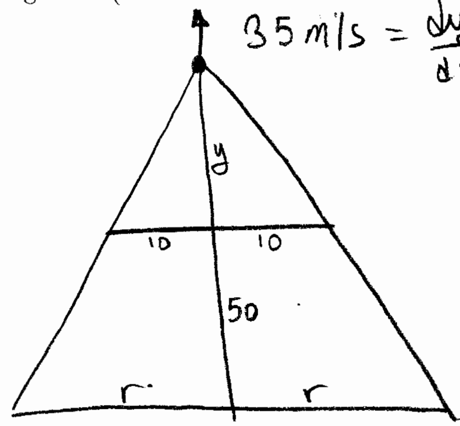
So $b < 1.02$, we have an overestimate.



4. A UFO (flying disc) of diameter 20 meters hovers at a fixed height of 50 meters above the ground. They send a small probe above which rises vertically at a rate of 35 meters per second. The small probe has a light under it so the UFO casts a circular shadow on the ground. How fast is the area of the shadow decreasing when the small probe is 150 meters above ground (so 100 meters above its mother ship)?

$35 \, m/s = \frac{dy}{dt}$





similar Δs

$A = \pi r^2$

$$\frac{y}{y+50} = \frac{10}{r} \rightarrow r = \frac{10(y+50)}{y}$$

$$\rightarrow A = \pi \left(\frac{10(y+50)}{y} \right)^2 = 100\pi \cdot \frac{(y+50)^2}{y^2}$$

$$\frac{dA}{dt} = 100\pi \, \frac{2(y+50)y^2 - (y+50)^2 2y}{y^4} \, \frac{dy}{dt}$$

$$= 200\pi(y+50) \, \frac{y-(y+50)}{y^3} \, \frac{dy}{dt} = \frac{-10{,}000\pi(y+50)}{y^3} \frac{dy}{dt}$$

when $y = 100$

$$\frac{dA}{dt} = \frac{-10000\pi(150)}{(100)^3} (35) = -52.5\pi \; m^2/s.$$