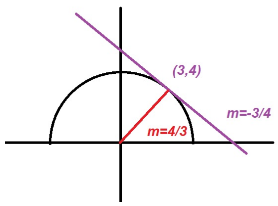
Math 124 Section A, Fall 2018 Solutions to Midterm I

1. (a)

$$\lim_{h\to 0}\frac{\sqrt{25-(3+h)^2}-4}{h}=\lim_{h\to 0}\frac{\sqrt{25-(3+h)^2}-4}{h}\cdot\frac{\sqrt{25-(3+h)^2}+4}{\sqrt{25-(3+h)^2}+4}$$

$$=\lim_{h\to 0}\frac{25-(3+h)^2-16}{h\left(\sqrt{25-(3+h)^2}+4\right)}=\lim_{h\to 0}\frac{-6h-h^2}{h\left(\sqrt{25-(3+h)^2}+4\right)}=\lim_{h\to 0}\frac{-6-h}{\left(\sqrt{25-(3+h)^2}+4\right)}=-\frac{3}{4}$$

(b) 

$$f(x)=\sqrt{25-x^2},\ \lim_{h\to 0}\frac{\sqrt{25-(3+h)^2}-4}{h}=f'(3)$$

2. (a) $\displaystyle\lim_{x\to 0}\frac{\sin(6x)}{\sin(5x)}=\lim_{x\to 0}\frac{\frac{6\sin(6x)}{6x}}{\frac{5\sin(5x)}{5x}}=\frac{6}{5}\lim_{x\to 0}\frac{\frac{\sin(6x)}{6x}}{\frac{\sin(5x)}{5x}}=\frac{6}{5}$

(b) $\displaystyle\lim_{x\to-\infty}\frac{\sqrt{3x^2-5}}{4x-16}=\lim_{x\to-\infty}\frac{\frac{\sqrt{3x^2-5}}{x}}{\frac{4x-16}{x}}=\lim_{x\to-\infty}\frac{\frac{\sqrt{3x^2-5}}{-\sqrt{x^2}}}{4-\frac{16}{x}}=-\lim_{x\to-\infty}\frac{\sqrt{\frac{3x^2-5}{x^2}}}{4-\frac{16}{x}}-\lim_{x\to-\infty}\frac{\sqrt{3-\frac{5}{x^2}}}{4-\frac{16}{x}}=-\frac{\sqrt{3}}{4}$

(c) $\displaystyle\lim_{x\to 2}\frac{x^2-5x+6}{x^2-4x+4}=\lim_{x\to 2}\frac{(x-2)(x-3)}{(x-2)(x-2)}=\lim_{x\to 2}\frac{x-3}{x-2}$ DNE because $\displaystyle\lim_{x\to 2^+}\frac{x-3}{x-2}=-\infty$ and $\displaystyle\lim_{x\to 2^-}\frac{x-3}{x-2}=\infty$.

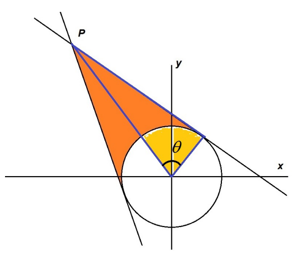
3. (a) Let (a,b) be a point of tangency. Then, the slope of the tangent line is give by

$$\frac{13-b}{-9-a}=-\frac{a}{b}$$

so $13b-9a=a^2+b^2=25$ or $b=\dfrac{25+9a}{13}$. Then,

$$25=a^2+b^2=a^2+\left(\frac{25+9a}{13}\right)^2$$

simplify to get $0=5a^2+9a-72$ so $a=3$ or $a=-4.8$.

(b) 

The area of the right triangle is

$$\frac{1}{2}\times 5\times\sqrt{(13-4)^2+(-9-3)^2}=\frac{75}{2}$$

The angle shown is $\theta=\tan^{-1}\left(\frac{15}{5}\right)$ so the area of the piece of the circle in yellow is $\frac{\tan^{-1}3}{2\pi}\times\pi 5^2$ so the shaded (orange) area is

$$2\left(\frac{75}{2}-\frac{25\tan^{-1}3}{2}\right)\approx 43.774.$$

4. (a) $f'(x)=\dfrac{x^2+4x-7}{(x+2)^2}=0$ when $x=-1\pm\sqrt{11}$.

(b) Increasing because $f'(3)=14/25>0$

(c) Slope is $f'(1)=-2/9$, $f(1)=-1/3$ so the tangent line equation is

$$y=-\frac{2}{9}x-\frac{1}{9}$$