Math 112, Spring 2019, Solutions to Midterm II

1. (a) $\displaystyle\int \frac{5}{x^2} - 3e^{0.2x} + \frac{5}{e^{4x}} + 2\ dx = \int 5x^{-2} - 3e^{0.2x} + 5e^{-4x} + 2\ dx$

   $\displaystyle = -5x^{-1} - \frac{3}{0.2}e^{0.2x} + \frac{5}{-4}e^{-4x} + 2x + C = -\frac{5}{x} - 15e^{0.2x} - \frac{5}{4}e^{-4x} + 2x + C$

   (b) $\displaystyle\int_1^5 \frac{7}{\sqrt{3x+1}}\,dx = \int_1^5 7(3x+1)^{-1/2}\,dx = 7\frac{(3x+1)^{1/2}}{(1/2)3}\Big|_1^5 = \frac{14}{3}\sqrt{3x+1}\Big|_1^5 = \frac{38}{3}$

   (c) $\displaystyle\int \frac{1+2e^{5x}}{e^{3x}}\,dx = \int \frac{1}{e^{3x}} + \frac{2e^{5x}}{e^{3x}}\,dx == \int e^{-3x} + 2e^{2x}\,dx = -\frac{e^{-3x}}{3} + e^{2x} + C$

2. (a) When $MC' = 0.8q - 4 = 0$ at $q = 5$, $MC(5) = 0.4(5^2) - 4(5) - 15 = 4$ dollars per bottle.

   (b) $MR(q) > 0$ when $-2q + 20 > 0$ or $q < 10$.

   $$TR(10) - TR(0) = \int_0^{10} -2q + 20\,dq = -q^2 + 20q0^{10} = 100$$

   Since $TR(0) = 0$, the maximum total revenue is $TR(10) = 100$ thousand dollars.

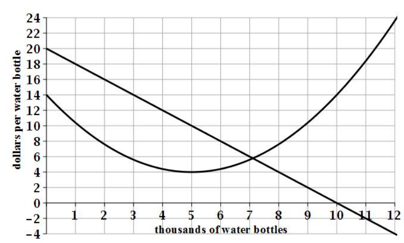
   (c) Profit is maximized when $MR = MC$:

   $$-2q + 20 = 0.4q^2 - 4q + 14$$
   $$0.4q^2 - 2q + 6 = 0$$
   $$q = \frac{2 \pm \sqrt{4 + 4(0.4)6}}{0.8} = 7.110$$

   $$P(7.110) - P(0) = \int_0^{7.11} -2q + 20 - (0.4q^2 - 4q + 14)\,dq = \int_0^{7.11} -0.4q^2 + 2q - 6\,dq = \frac{-0.4q^3}{3} + q^2 + 620^{7.11} = 45.293$$

   Since $P(0) = -FC = -12$, we get the maximum profit to be $P(7.11) = 45.293 - 12 = 33.293$ thousand dollars or \$33,293.

   (d) MC is a line with negative slope with y-intercept 20 and x-intercept worked out in part (b). MC is a parabola which opens up with y intercept 14 and lowest point worked out in part (a). Their intersection point is worked out in part (c).



3. (a)

$$f(3) - f(1) = \int_1^3 t^2 - 11t + 28\,dt = \frac{t^3}{3} - 11\frac{t^2}{2} + 28t\Big|_1^3 = \frac{62}{3}.$$

$$\text{Average Rate of Change} = \frac{f(3) - f(1)}{3-1} = \frac{\frac{62}{3}}{2} = \frac{31}{3}.$$

(b)

$$f'(t) = t^2 - 11t + 28 = (t-4)(t-7) = 0$$

when $t = 4$ or $t = 7$. To check which one is a local minimum and which one is a local maximum, you can do one of two things:
Check the second derivative: $f''(t) = 2t - 11$. $f''(4) = -3$ so $t = 4$ gives a local max. $f''(7) = 3$. So $t = 7$ gives a local min.
OR
Check the sign of the first derivative with, for example, $3 < 4 < 5 < 7 < 8$:

$$f'(3) = 9 - 33 + 28 = 4 > 0 \quad \text{so } f \text{ increasing}$$

$$f'(5) = 25 - 55 + 28 = -2 < 0 \quad \text{so } f \text{ decreasing}$$

So $t = 4$ gives a local max.

$$f'(8) = 64 - 88 + 28 = 4 > 0 \quad \text{so } f \text{ increasing}$$

So $t = 7$ gives a local min and

(c)

$$f''(t) = 2t - 11 > 0 \quad \text{when} \quad t > \frac{11}{2}$$

4. Equilibrium point:

$$\frac{2170}{x+10} = 1 + 0.02x$$

$$2170 = (1 + 0.02x)(x + 10) = 0.02x^2 + 1.2x + 10$$

$$0 = 0.02x^2 + 1.2x - 2160$$

$$x = \frac{-1.2 \pm \sqrt{1.44 + 0.08(2160}}{0.04} = 300, \text{the other } x \text{ is negative}$$

$$p = 1 + 0.02(300) = 7 \text{ dollars per pound}$$

So,

$$\text{Consumer's Surplus} = \int_0^{300} \frac{2170}{x+10}\,dx - 300 \cdot 7 = 2170 \ln(x+10)\Big|_0^{300} - 2100 = 2170 \ln 31 - 2100 \approx \$5,351.75$$