Rules of Integration

These are the main rules of integration we need:

$$\int x^m\,dx = \frac{x^{m+1}}{m+1} + C, m \neq -1 \qquad \int \frac{1}{x}\,dx = \ln|x| + C \qquad \int e^x\,dx = e^x + C$$

Exercises

1. $\int 2x^3 - 0.9x^2 + 7x + 1\,dx$

2. $\int 3e^x - 24e^{0.2x} + \frac{5}{e^{4x}} + 2\,dx$

3. $\int \frac{\sqrt{x}}{3} - \frac{4}{x} + \frac{0.1}{x^{11}}\,dx$

If, instead of x, we have $ax+b$, we can "reverse the Chain Rule" to get these integrals:

$$\int (ax+b)^m\,dx = \frac{(ax+b)^{m+1}}{a(m+1)} + C, m \neq -1 \qquad \int \frac{1}{ax+b}\,dx = \frac{\ln|ax+b|}{a} + C \qquad \int e^{ax}\,dx = \frac{e^{ax}}{a} + C$$

(Check by differentiating the right hand sides using the Chain Rule.)

Exercises

4. $\int \sqrt{x+1}\,dx$

5. $\int \frac{10}{5+x}\,dx$

6. $\int \frac{3}{7x+3}\,dx$

7. $\int \frac{3}{\sqrt{4x+2}}\,dx$

When we have any other type of formula to integrate, we may need to **do algebra before** we integrate. Each piece your integrate should be from one of the rules above. Integration does not have product or quotient rules!

Exercises

8. $\int \frac{3\sqrt{x} - x^2}{5x}\,dx$

9. $\int (3x+1)^2\,dx$ (You can do this one in two ways!)

10. $\int \frac{1 + 2e^{3x}}{e^{5x}}\,dx$

There are many more integration (and differentiation) problems in the Midterm 2 Archives.

The Indefinite Integral

The Indefinite Integral is a function. All the rules on the previous page were written as indefinite integrals. For example,

$$\int 2x + 5\, dx = x^2 + 5x + C$$

It answers the question: Which function has the given derivative? The C is there because there are many functions which have a particular derivative. For example $f(x) = x^2 + 5x + 1$ and $g(x) = x^2 + 5x - 17$ both have the derivative $f'(x) = g'(x) = 2x + 5$. To recover the value of C, we need more information.

Application: Recovering a function given its rate of change

1. Given the rate of change of altitude, we can find the altitude function **if** we know the altitude at $t = 0$ or at some other time.
2. Given Marginal Revenue $MR(q)$, we can find the (Total) Revenue function $TR(q)$ using the fact that $TR(0) = 0$.
3. Given Marginal Cost $MC(q)$, we can find the Variable Cost function $VC(q)$, using the fact that $VC(0) = 0$.
4. Given Marginal Cost $MC(q)$, we can find the Total Cost function, **if** we know the Fixed Cost $FC = TC(0)$ or we are given the Total Cost for some other quantity.

In general, you can recover a function from its derivative (rate of change) if you know its value at one place - usually, but not necessarily, at 0. For all these problems, **don't forget the** $+C$ as it is an important part of the problem. All of Section 12.4 assignment was on this. Review as necessary.

Solutions to the Integration Exercises

1. $\int 2x^3 - 0.9x^2 + 7x + 1\, dx = \dfrac{x^4}{2} - 0.3x^3 + \dfrac{7}{2}x^2 + x + C.$
2. $\int 3e^x - 24e^{0.2x} + \dfrac{5}{e^{4x}} + 2\, dx = 3e^x - 120e^{0.2x} - \dfrac{5}{4}e^{-4x} + 2x + C$
3. $\int \dfrac{\sqrt{x}}{3} - \dfrac{4}{x} + \dfrac{0.1}{x^{11}}\, dx = \dfrac{2}{9}x^{3/2} - 4\ln|x| - \dfrac{0.01}{x^{10}} + C$
4. $\int \sqrt{x+1}\, dx = \dfrac{2}{3}(x+1)^{3/2} + C.$
5. $\int \dfrac{10}{5+x}\, dx = 10\ln|5+x| + C$
6. $\int \dfrac{3}{7x+3}\, dx = \dfrac{3}{7}\ln|7x+3| + C$
7. $\int \dfrac{3}{\sqrt{4x+2}}\, dx = \dfrac{3}{2}\sqrt{4x+2} + C$
8. $\int \dfrac{3\sqrt{x} - x^2}{5x}\, dx = \dfrac{6}{5}\sqrt{x} - \dfrac{1}{10}x^2 + C.$
9. $\int (3x+1)^2\, dx = 3x^3 + 3x^2 + x + C$ OR $\dfrac{(3x+1)^3}{9} + C.$

   When you multiply out the second answer, you do not get exactly the first answer. Is that a problem?
10. $\int \dfrac{1 + 2e^{3x}}{e^{5x}}\, dx = -\dfrac{1}{5}e^{-5x} - e^{-2x} + C$