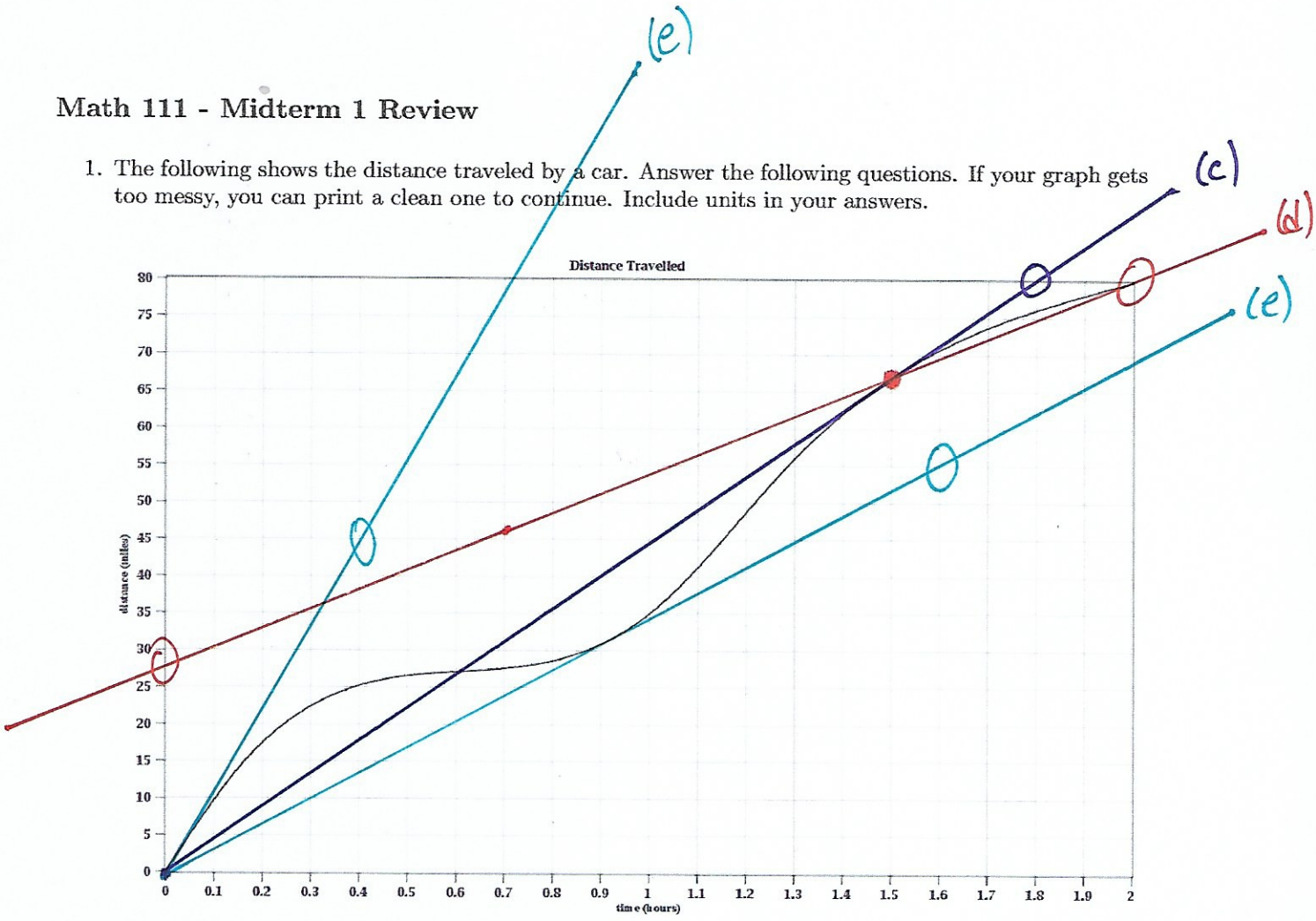
Math 111 - Midterm 1 Review

1. The following shows the distance traveled by a car. Answer the following questions. If your graph gets too messy, you can print a clean one to continue. Include units in your answers.



(a) When has the car travelled 50 miles?

$t \approx 1.22$ hours

(b) How far is the car from its starting point at the end of the first hour?

35 miles

(c) What is the Average Trip Speed of the car at 90 minutes?

90 min = 1.5 hrs

slope $\approx \frac{80}{1.8} \approx 44.4$ miles per hour

(d) What is the Average Speed of the car in the last 30 minutes of this journey?

from $t = 1.5$ to $t = 2$ hours

slope $\approx \frac{80 - 27.5}{2 - 0} \approx 26.3$ miles per hour

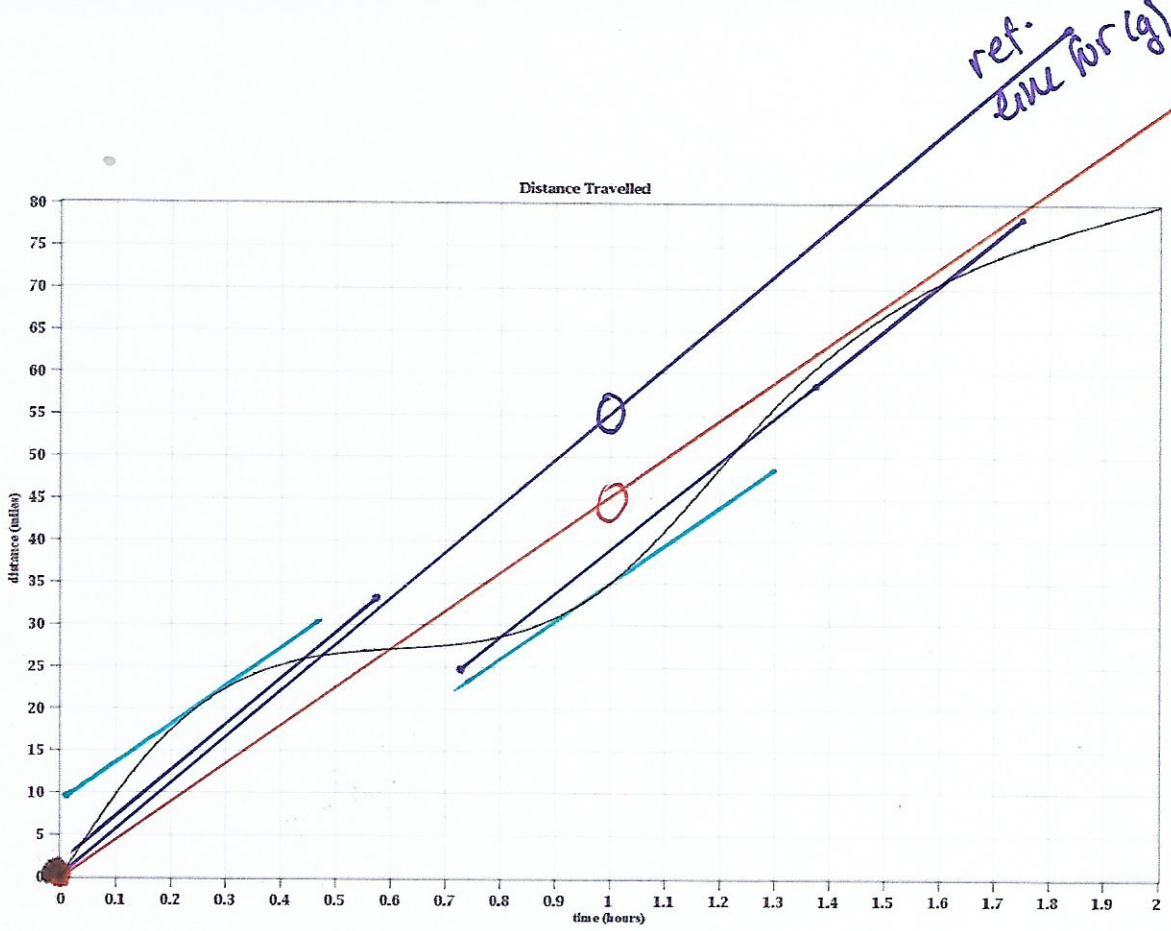
(e) What are the lowest and highest values of the car's Average Trip Speed?

Lowest at about $t \approx 0.93$ hrs, slope $\approx \frac{55}{1.6} \approx 34.4$ miles per hour

Higest ~~as~~ at the very beginning $t \approx 0.01$ hrs

slope $\approx \frac{45}{0.4} \approx 112.5$ miles per hour

(when this data starts at $t=0$, the car is already in motion and going very fast)



(f) Find the time(s) when the Average Trip Speed of the car is 45 mph.

at 0.6 hours

(g) Find a 24 minute time interval when the Average Speed is 55 mph.

24 min = 0.4 hrs

$t = 0.05$ to $t = 0.45$ hrs

$t = 0.6$ to 1.2 hrs

$t = 1.2$ to 1.6 hrs.

(h) A second car starts at the same place and travels in the same direction at a constant speed of 45 mph. What is the Average Trip Speed of the second car at 30 minutes? At one hour? At 90 minutes? At 1.23456789 hours?

all 45 mph

(i) List the intervals when the second car is ahead of the first car.

when $t > 0.6$ the second car is ahead

(j) Find the maximum distance between the two cars in this 2 hour period. Which car is ahead?

note how I did the parallel line trick to find max. vertical distance like we do for Revenue, Cost + Profit

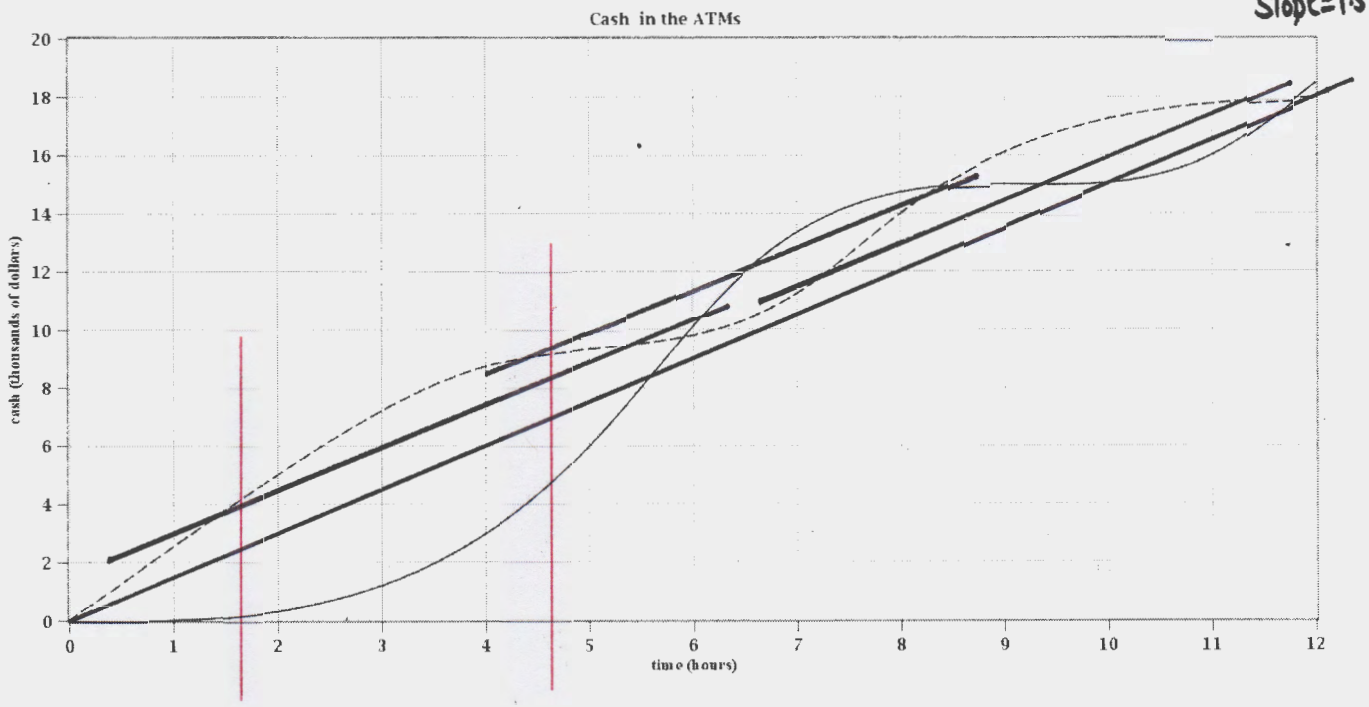
At $t \approx 0.25$, the first car is ahead by about $21 - 0.25(45) = 9.75$ miles.

At $t \approx 0.97$, the second car is ahead by about $0.97(45) - 33 =$ 10.65

So the max. distance is 10.65 miles and the 2nd car is ahead. (Very close, though. It's ok if you got it the other way.)

2. The following shows the total money deposited (dashed) to and the total money withdrawn (solid) from the ATM machines of a bank after noon. At noon, there is \$3450 in the machines. Include units in your answers.

Slope=1.5



(a) How much money is in the ATMs at 7 PM?

$3450 - 13200 + 11200 = 1450$ dollars OR 1.45 thousand dollars

(b) At what time(s) is there \$3450 in the ATMs?

When withdrawn = deposited : $t = 0, 5.8, 8.3, 11.8$

(c) What is the overall average rate of change out of the ATMs at 3 PM?

$\frac{1100}{3} \approx 367$ dollars per hour

(d) What is the overall average rate of change in to the ATMs at 3 PM?

$\frac{7200}{3} \approx 2400$ dollars per hour

(e) What is the overall average rate of change of money in the ATMs at 3 PM?

$\frac{7200-1100}{3} \approx 2033$ dollars per hour

(f) Find a 4 hour period when \$6000 is deposited to the ATMs.

rate $= \frac{6000}{4} = 1500 =$ slope

$1.4 \le t \le 5.4$ $\quad 4.3 \le t \le 8.3$

$7.2 \le t \le 11.2$

(g) Find the time ~~interval(s) when there is more than \$7450 in the machines.~~

When deposit - withdrawn $> 7450 - 3450 = 4000$

about $1.7 \le t \le 4.6$

3. Let $B(t)$ be the number of books in a small public library (in the building, not checked out) at time t in days from January 1, 2013. Translate the following sentences into math:

Example: There were 3657 books on February 12, 2013:

$$B(43) = 3657.$$

(a) The overall rate of change in the number of books for the first 100 days of the year was 125 books per day.

$$\frac{B(100)-B(0)}{100-0} = 125$$

(b) The average rate of change in the number of books from March 1 to June 5 was 132 books per day.

$$\frac{B(156)-B(60)}{156-60} = 132$$

(c) The number of books in the library decreased by 267 during the month of August (From July 31 to August 31).

$$B(243)-B(212) = -267$$

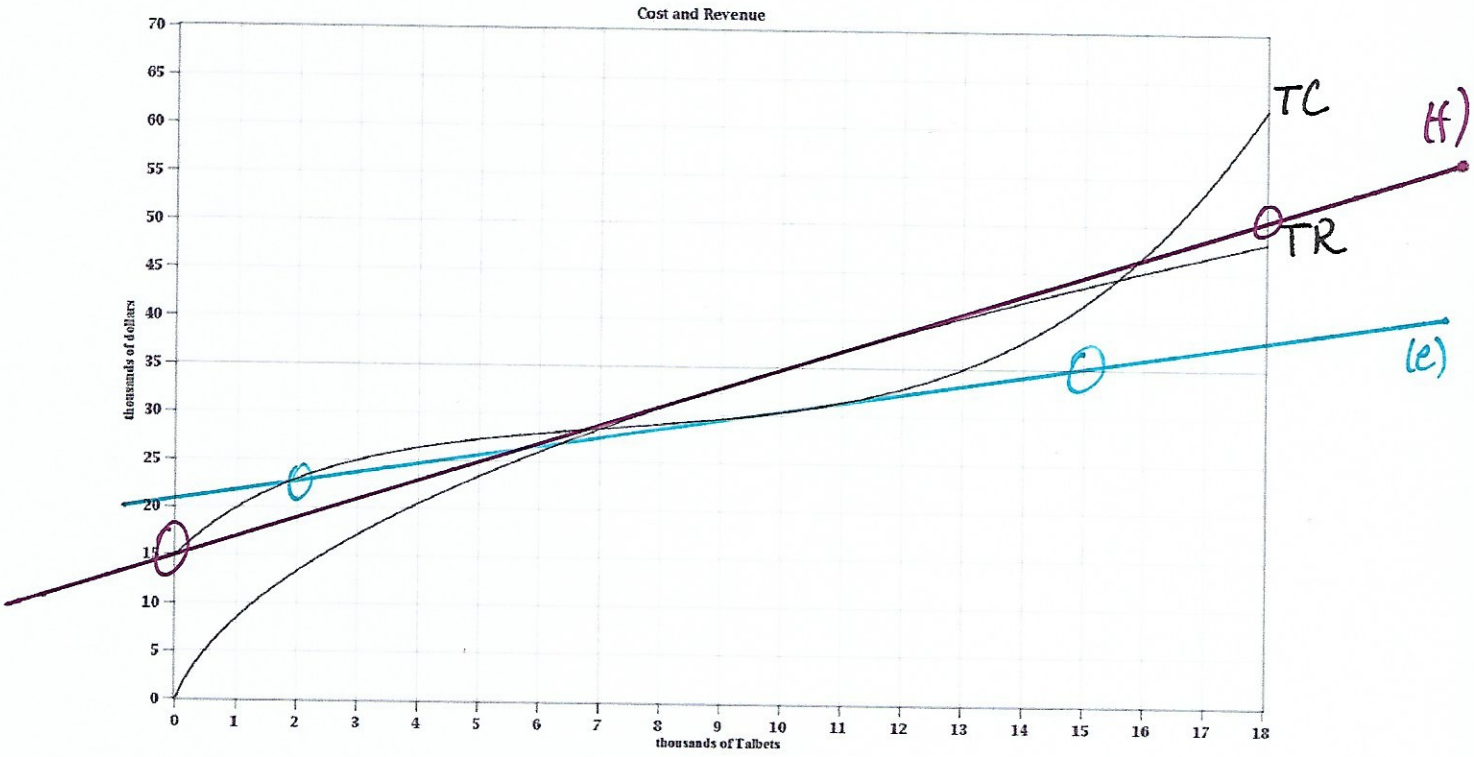
(d) The average rate of change in the number of books in the h days after January 6 was 121 books per day.

$$\frac{B(6+h)-B(6)}{h} = 121$$

(e) The number of books on September 1 and October 15 were the same.

$$B(244) = B(288)$$

4. The following shows the Total Cost and Total Revenue functions for producing and selling Talbets. Include units in your answers.



(a) Which one is the Total Cost function? What is the Fixed Cost? Which one is the Total Revenue function. You can't make money if you don't sell anything.

FC = 15 thousand dollars

(b) Approximate the loss at 500 Talbets.

17.5 − 5 = 12.5 thousand dollars

(c) When do you break even?

At 7 thousand (and 15.5 thousand)

(d) Approximate the profit at 13500 Talbets.

$TR(13.5) - TC(13.5) \simeq 42 - 37 = 5$ thousand

(e) What is the Marginal Cost at 10000 Talbets?

Slope $\simeq \frac{35-23}{15-2} \simeq 0.92$ dollars per Talbet

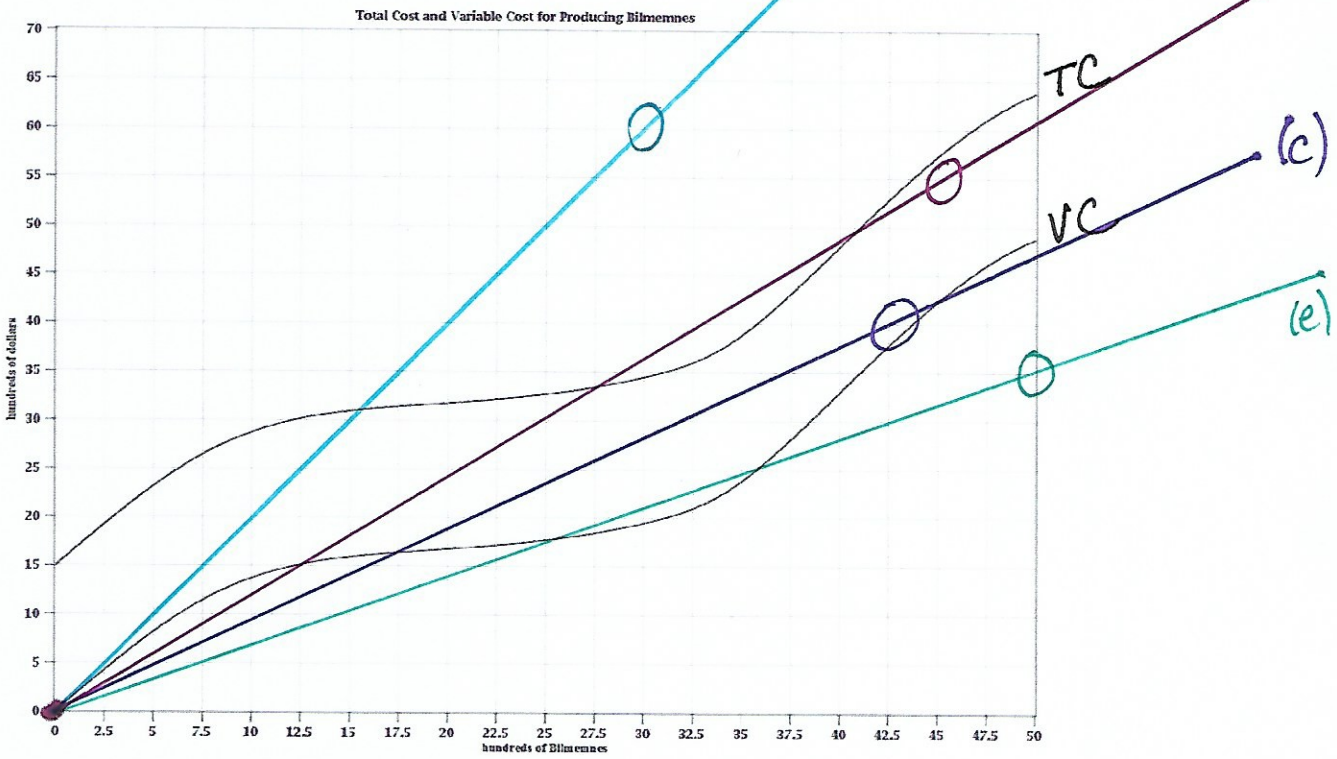
(f) What is the Marginal Revenue at 10000 Talbets?

Slope $\simeq \frac{50-15}{18-0} = 1.94$ dollars per Talbet

(g) At about 1000 Talbets, will your profits increase or decrease if you sell more Talbets?

MC < MR so your profits will increase

5. The following shows the Total Cost and Variable Cost functions for producing and selling Bilmemnes. Include units in your answers.



(a) Which one is the Total Cost function? Which one is the Variable Cost function? What is the Fixed Cost?

FC is 15 hundred dollars

(b) Compute the Average Cost at 2750 Bilmemnes.

slope $\approx \frac{55}{45} \approx 1.22$ dollar per B.

(c) Compute the Average Variable Cost at 1750 Bilmemnes.

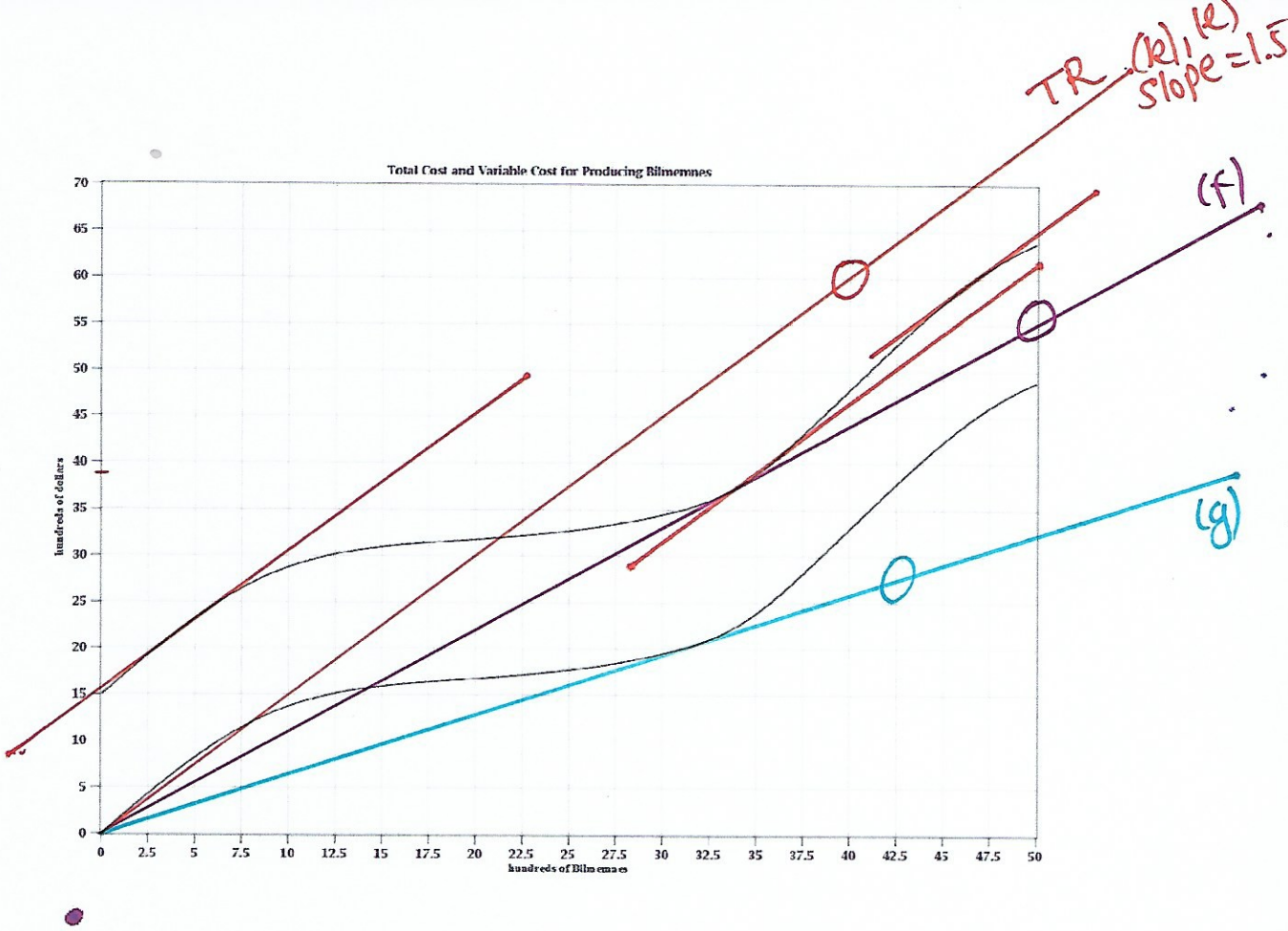
slope $\approx \frac{40}{42.5} \approx 0.94$ dollars per B.

(d) At what level of production is your Average Cost 200 cents per Bilmemne.

at about 15.5 hundred B.

(e) At what level of production is your Average Variable Cost 70 cents per Bilmemne.

at 25.5 and 35.5 hundred B.



(f) What is the smallest value of the Average Cost?

Slope $\simeq \frac{55}{50} \simeq$ 1.1 dollars per B.

(g) What is the smallest value of the Average Variable Cost?

Slope $\simeq \frac{27.5}{42.5} \simeq$ 0.65 dollars per B.

(h) What is the Breakeven Price?

1.1 dollars per B.

(i) What is the Shutdown Price?

0.65 dollars per B.

(j) What do you advise me to do if the market price is 50 cents per Bilmemne? Why?

Shutdown operations, sell everything & find another j

(k) If the I sell each Bilmemne for 150 cents, what is my maximum profit?

at about 35 hundred B.

Profit = $35(1.5) - 39 = 13.5$ hundred dollars

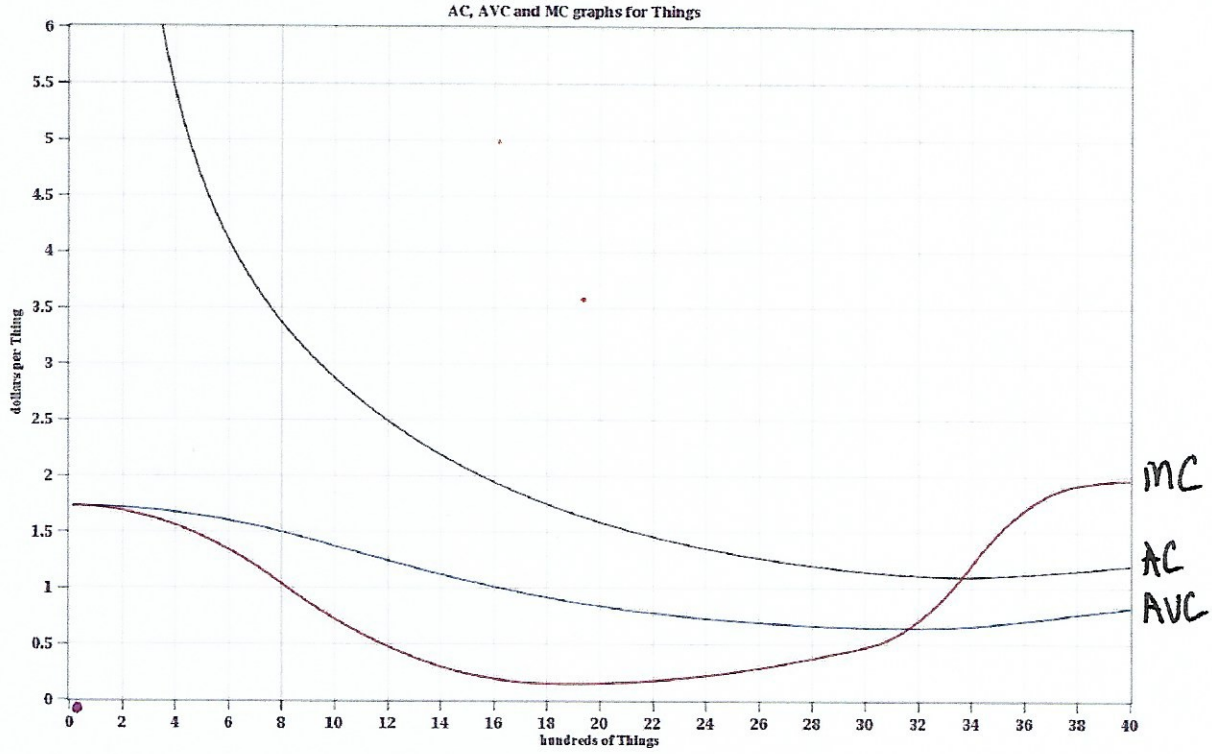
(l) Find the interval where the Marginal Cost is at most 150 cents per Bilmemne.

At about 5, 35, 46 hundred, MC = 1.5

when $5 \leq q \leq 35$ and $46 \leq q \leq 50$ the MC is

less than 1.5 dollars per B.

6. The following shows the AC, AVC and MC graphs for producing and selling Things. Include units in your answers.



(a) Compare the definitions of $MC(0)$ and $AVC(1)$. Which two graphs should be for MC and AVC?

$MC(0) = VC(1) - VC(0) = VC(1)$ (we can compute MC from TC or VC)

$AVC(1) = \frac{VC(1)}{1} = VC(1)$ so the graphs starting together are for MC + AVC

(b) Which one is higher: $AVC(1)$ or $AC(1)$? At this point, looking at where the three graphs start, you should know which one is the AC graph.

$AVC(1) = VC(1)$ $\quad AC(1) = \frac{TC(1)}{1} = TC(1) = VC(1) + FC$ so $AC(1) > AVC(1)$ (by FC)

(c) Can AC and AVC graphs ever intersect, i .e. can we have $AVC(q) = AC(q)$? What happens to the FC if you set the equa to each other? At this point you should have labelled all three. If not, look at the example from lecture to label all three. NO. $AC(q) = \frac{VC(q)}{q} + \frac{FC}{q} = AVC(q) + \frac{FC}{q}$

(d) Approximate the Total Cost of producing 1400 Things. Approximate the Variable Cost of producing 1400 Things. What is the Fixed Cost?

$TC(14) = AC(14) \times 14 \approx 2.2 \times 14 = 30.8$ hundred dollars

$VC(14) = AVC(14) \times 14 \approx 1.1 \times 14 = 15.4$ hundred dollars

$FC = TC(14) - VC(14) = 15.4$ hundred dollars

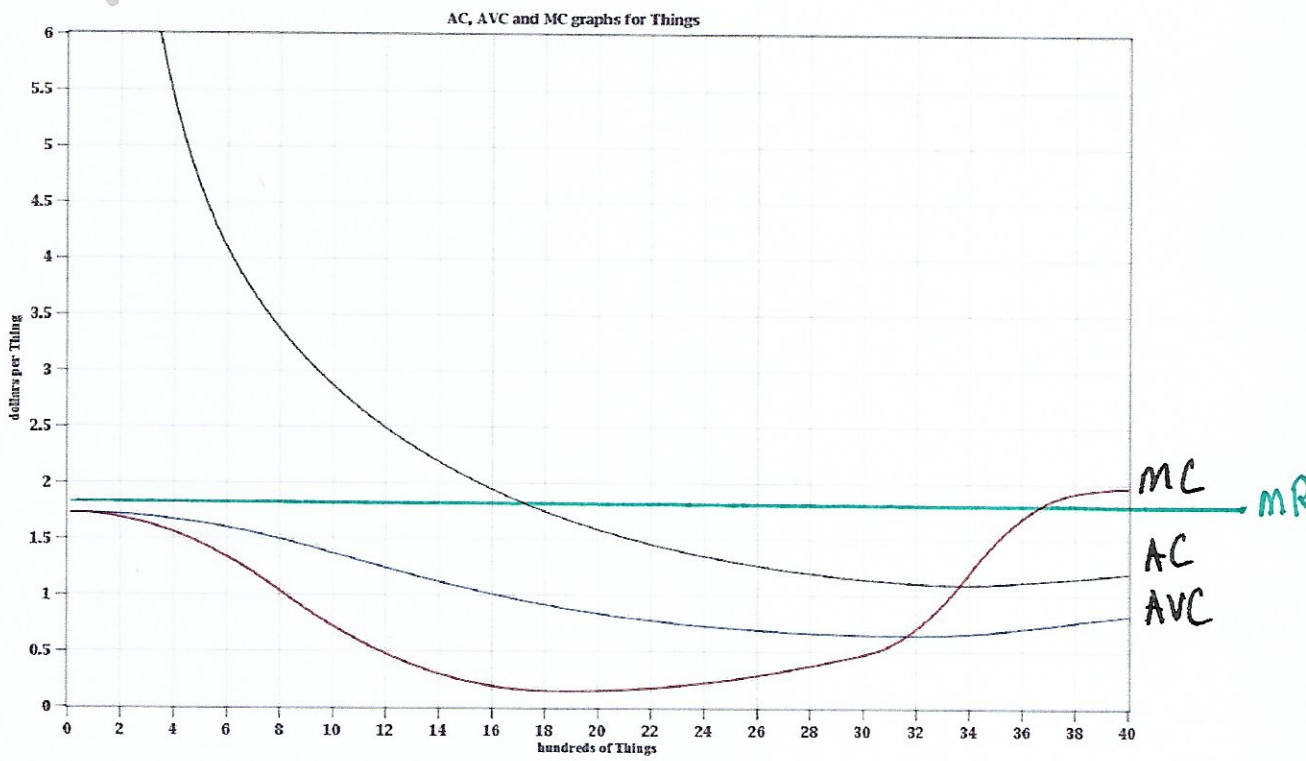
(e) Approximate the Total Cost of producing 3000 Things. Approximate the Variablel Cost of producing 3000 Things. What is the Fixed Cost? Compare your Fixed Cost with the one from the previous one. Hopefully, they are close, if not the same. Why?

$TC(30) = AC(30) \times 30 \approx 1.2 \times 30 = 36$ hundred dollars

$VC(30) = AVC(30) \times 30 \approx 0.67 \times 30 = 20.1$ hundred dollars

$FC = TC(30) - VC(30) = 16.1$ hundred dollars.

They are close (15.4 and 16.1) but not the same because of ~~four~~ estimation errors.



(f) What is the smallest value of the Average Cost?

1.1 dollars per thing

(g) What is the smallest value of the Average Variable Cost?

0.67 dollars per thing

(h) What is the Breakeven Price?

1.1 dollars per thing

(i) What is the Shutdown Price?

0.67 dollars per thing

(j) If the I sell each Thing for \$1.80, what is my maximum profit?

when $MR = MC$ at about 36.7 hundred things

$\| \; 1.80$

$TR(36.7) = AC(36.7) \times 36.7 = 1.1 \times 36.7 \simeq 40.4$ hundred dollars

$P(36.7) = TR(36.7) - TC(36.7)$

$= (1.80)(36.7) - 40.4$

$= 25.7$ hundred dollars

7. Algebra.

(a) Solve for x in

$$9(x-12) = 30 - (x-2).$$

$9x - 108 = 30 - x + 2$

$10x = 108 + 30 + 2 = 140$

$x = \frac{140}{10} = 14$

Check: $9(14-12) \stackrel{?}{=} 30-(14$

$9(2) \stackrel{?}{=} 30-12$

(b) Solve for y in

$$\frac{y-1}{3} + 1 = \frac{2}{3}(y+2).$$

$\cdot 3$

$y - 1 + 3 = 2(y+2)$

$y + 2 = 2y + 4$

$2 - 4 = 2y - y$

$-2 = y$

Check: $\frac{-2-1}{3} + 1 \stackrel{?}{=} \frac{2}{3}(-$

$-1 + 1 \stackrel{?}{=} \frac{2}{3}$

(c) Solve the inequality and graph the solution;

$$3(2x+5) - 2 > 9x + 12.$$

$6x + 15 - 2 > 9x + 12$

$15 - 2 - 12 > 9x - 6x$

$1 > 3x$

$\frac{1}{3} > x$

(d) Solve the inequality and graph the solution:

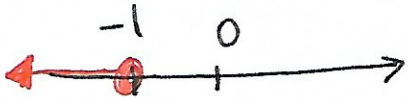
$$2(5x+3) \leq 6x + 2.$$

$10x + 6 \leq 6x + 2$

$10x - 6x \leq 2 - 6$

$4x \leq -4$

$x \leq -1$



(e) Solve for e in

$$\frac{e+R}{S} = Te + R.$$

$\cdot S$

$e + R = TSe + RS$

$R - RS = TSe - e$

$R - RS = (TS-1)e$

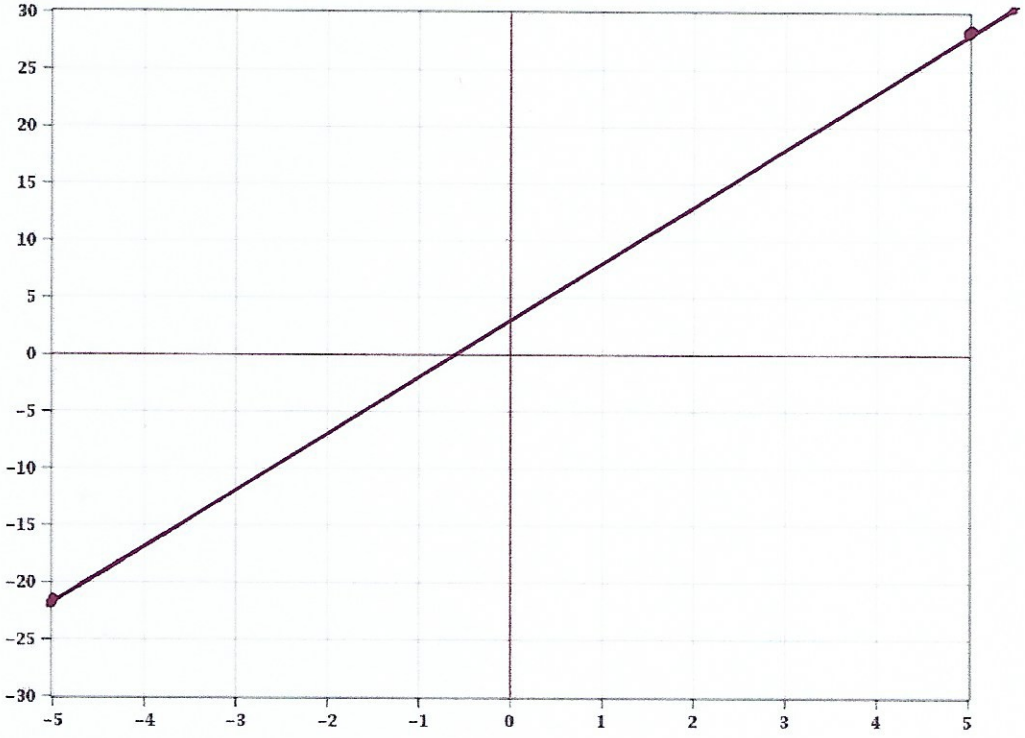
$\frac{R-RS}{TS-1} = e$

8. Graphing.

-3 | 5

(a) Graph $y = 5x + 3$. What is the x-intercept? What is the y intercept?

3



2)

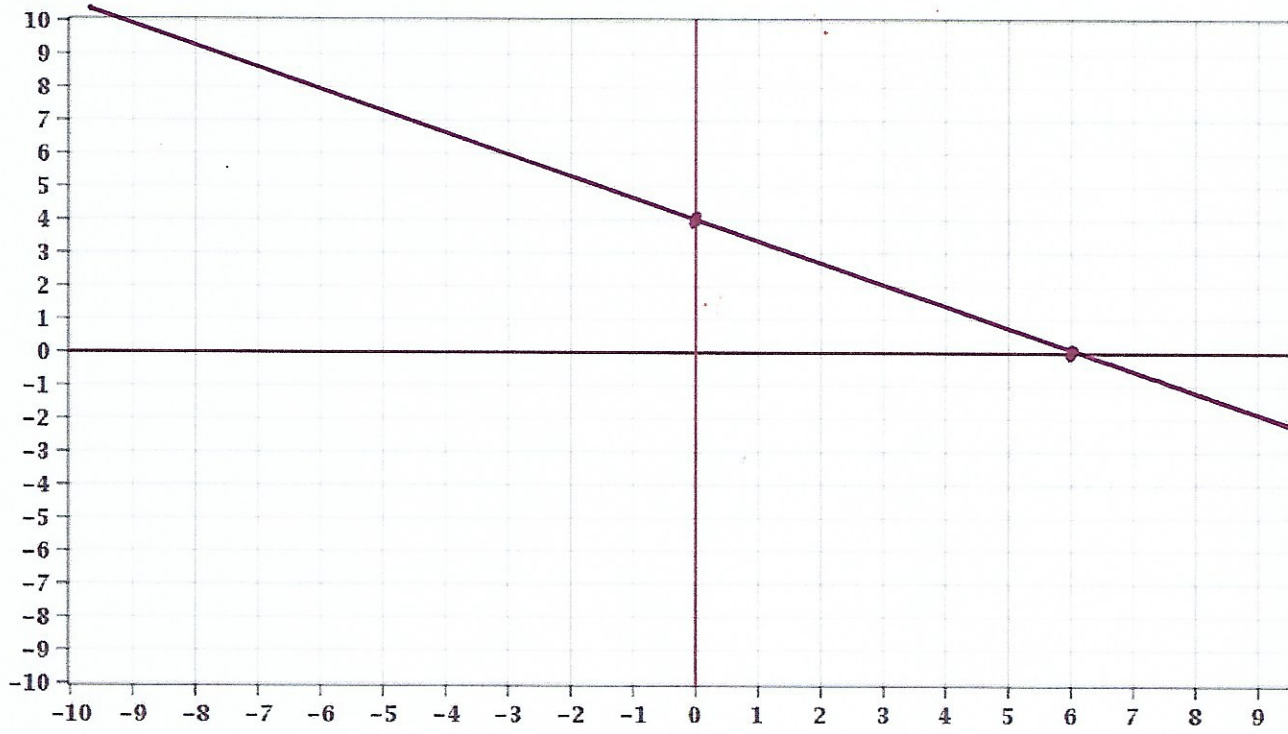
+2)

)

6

4

(b) Graph $2x + 3y = 12$. What is the x-intercept? What is the y intercept?



9. The water and sewage (what goes in must come out, right?) company determines the monthly bill for a residential customer by adding water usage of 3.54 cents per gallon to its base charge of \$32.29 per month. Write an equation for the monthly charge y in terms of x, gallons of water used. Let y be measured in dollars. What is my October bill if I use 450 gallons of water in my house? Be careful with units.

$y = 32.29 + 0.0354x$

when $x = 450$

$y = 32.29 + 0.0354(450)$

$= 48.22$ dollars

10. Warren has \$180,000 to invest.There is one relatively safe investment fund that has an annual yield of 7% and another, riskier fund that has a 12% annual yield. How much should he invest in each fund if he wants to earn \$17,500 per year from his investments?

x: amount he invests in the 7% fund

$180{,}000 - x$: amount he invests in the 12% fund

wants: $17{,}500 = 0.07x + 0.12(180{,}000 - x)$

$17{,}500 = 0.07x + 21{,}600 - 0.12x$

$0.12x - 0.07x = 21{,}600 - 17{,}500$

$0.05x = 4{,}100$

$x = 82{,}000$ in the 7% fund, 98,000 in the 12% fund

11. Saige is making and selling dresses for dolls. Her fixed costs are \$120. It costs \$6.50 to make each dress. She sells each dress for \$12. Write down her Total Cost $TC(x)$ and Total Revenue $TR(x)$ functions where x is the number of dresses she makes and sells. When will she make a profit?

$TC(x) = 120 + 6.5x$ $\quad TR(x) = 12x$

She profits when $TR(x) = TC(x)$

$12x = 120 + 6.5x$

$12x - 6.5x = 120$

$5.5x = 120$

$x = \frac{120}{5.5} \approx 21.8$

She will make a profit when she sells the 22nd dress