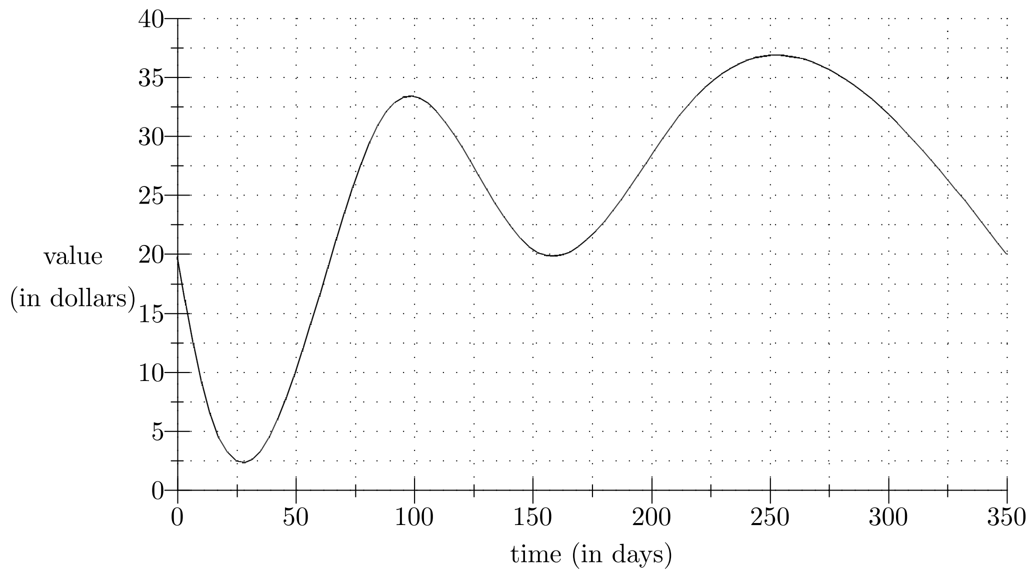
Name: ____________________ Section: __________

Math 111

Group Activity: Change and Average Change

1. Below is a graph of the value vs. time for one share of stock for a company named Banana Patch.



For the following, approximate as well as you can and include units with your answers.

(a) What is the value of one share of stock on day 300?

(b) How much does the value of one share of stock change during the first 300 days?

(c) How much, in dollars per day, does the value of one share of stock change *on average* in the first 300 days?

(d) Can you describe your answer to part (c) in graphical terms as the slope of a line on this graph? Be specific.

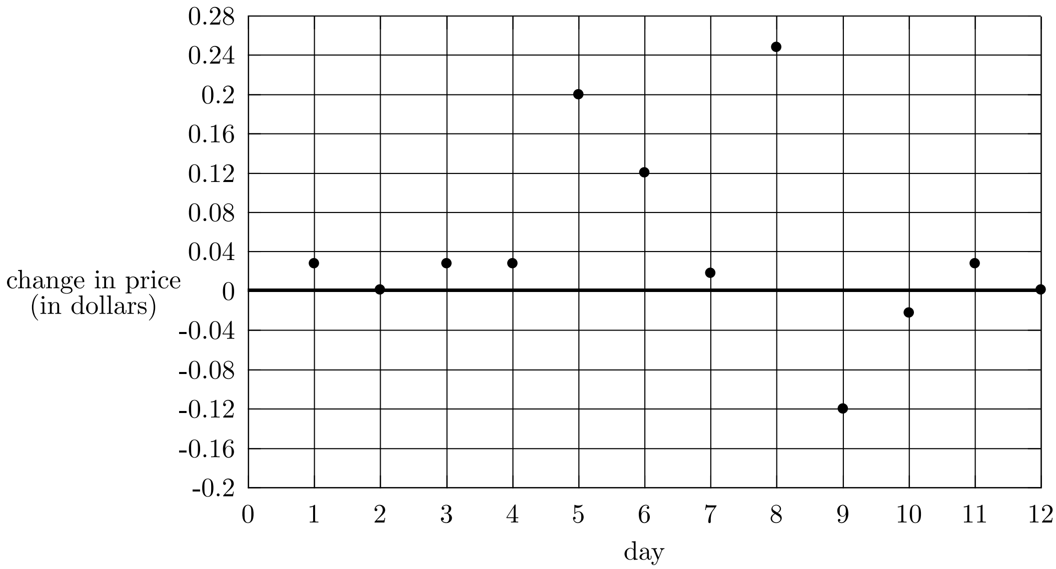
(e) How much, in dollars per day, does the value of one share of stock change *on average* from the 50^{th} day to the 100^{th} day?

(f) Can you describe your answer to part (e) in graphical terms as the slope of a line? Be specific.

(g) You have \$1000 that you would like to invest in Banana Patch. On day 0, you buy as many shares as you can afford. What is your investment worth on day 125?

(h) How much, in dollars per day, does the value of your investment from part (g) change *on average* from day 0 to day 125?

2. The *opening price* of a stock is the value of the stock when the stock market opens. The opening price of one share of stock for the Mambo Dog Company is monitored over several days. The graph below gives the **change** in the opening price since the previous day. For example, the change in price at $t = 1$ is \$0.03, which means that the opening price of the stock on day 1 is \$0.03 higher than the opening price on day 0.



Again, approximate as accurately as you can and include units.

(a) Suppose the opening price of Mambo Dog is \$5.35 on day 0. What is the opening price on day 4?

(b) How much higher is the opening price on day 5 than on day 3?

(c) Name all days on which the opening price of Mambo Dog was the same as the day before.

(d) i. Is the opening price higher on day 5 or day 6? Explain.

ii. Is the opening price higher on day 9 or day 10? Explain.

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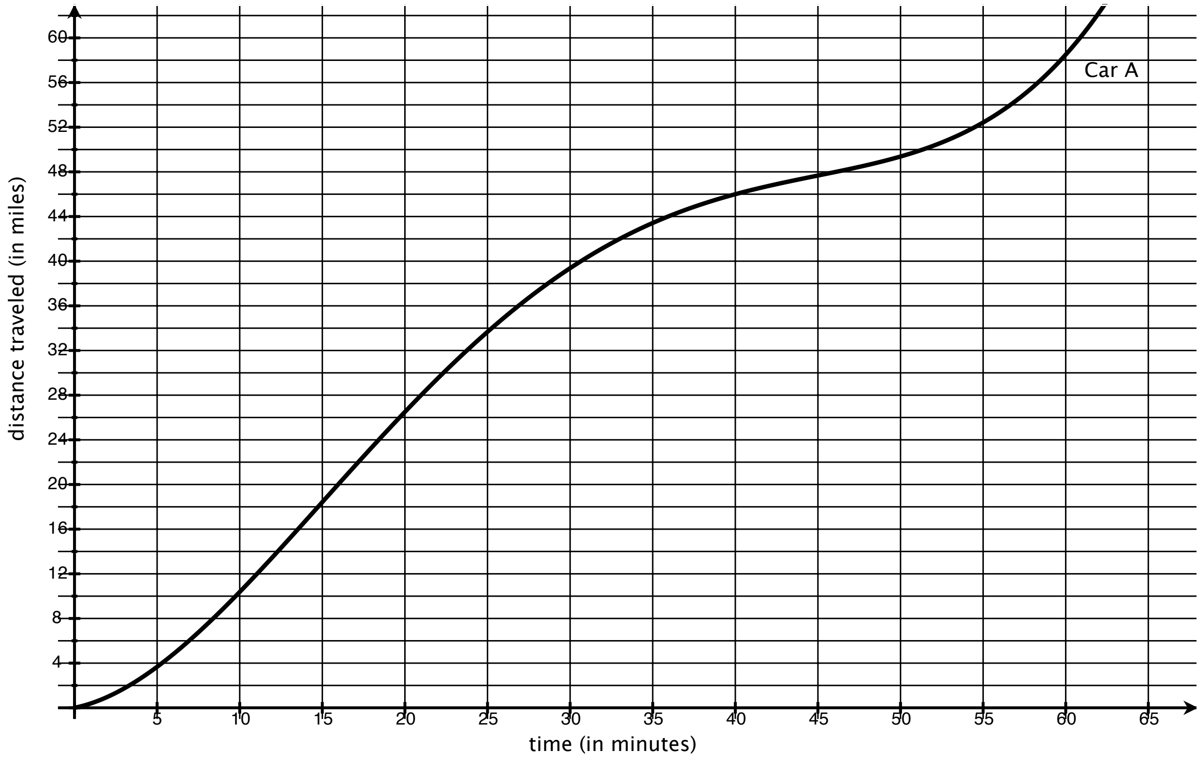
Math 111

Group Activity: More Rates of Change

1. The following is the graph of distance traveled versus time for Car A.



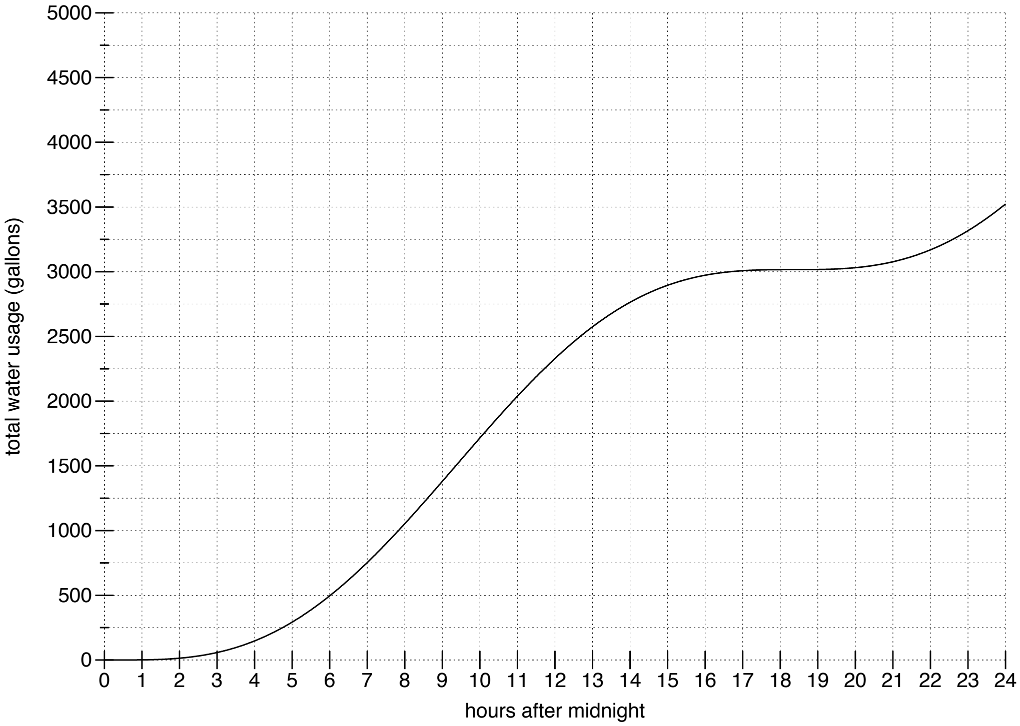
(a) Compute Car A's average speed during the 10-minute interval beginning at $t = 40$ minutes.

(b) Find the highest value of Car A's average trip speed. (HINT: Think about the graphical interpretation of average trip speed.)

(c) A second car, Car B, is next to Car A at $t = 0$ and travels 10 miles every 10 minutes. Give the longest time interval during which Car A is ahead of Car B. (HINT: Draw the graph of Car B's distance traveled on the axes above.)

(d) Give a 5-minute interval during which both cars have the same average speed and Car B is ahead of Car A.

2. A town is using water from a reservoir that is being refilled with a system of aqueducts. The graph below shows the total water drawn from the reservoir over the course of a day, starting at midnight.



(a) Suppose the reservoir is empty at midnight and is filled by the aqueduct at a constant rate of 150 gallons per hour. Sketch a graph of the amount of water that has flowed into the reservoir on the above graph and use it to answer these questions: Will there be enough water in the reservoir to provide for the town during this 24-hour period? If not, when will there be a shortage? If the rate of flow must remain 150 gallons per hour, how much water must be in the reservoir at midnight to avoid a shortage? (HINT: How would you change the inflow graph so that there is never a shortage?)

(b) Suppose instead that the reservoir must be empty at midnight. Again, water flows into the reservoir at a constant rate. What is the smallest that rate could be to avoid a water shortage during this 24-hour period? (HINT: If water flows in at a constant rate, what does the graph of inflow look like? What feature of the inflow graph is represented by the rate of flow in? What must the inflow graph look like if there is never to be a shortage?)

(c) Suppose instead that there are 3000 gallons of water in the reservoir at midnight and that water flows in at a constant rate. What is the smallest that rate could be to avoid a water shortage during this 24-hour period?

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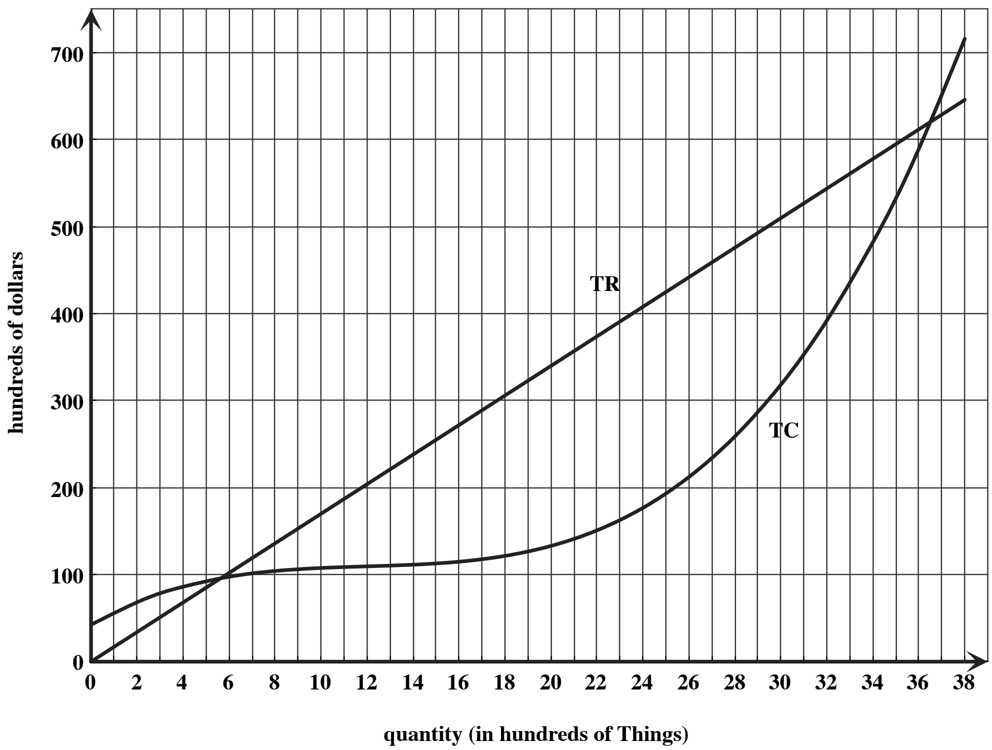
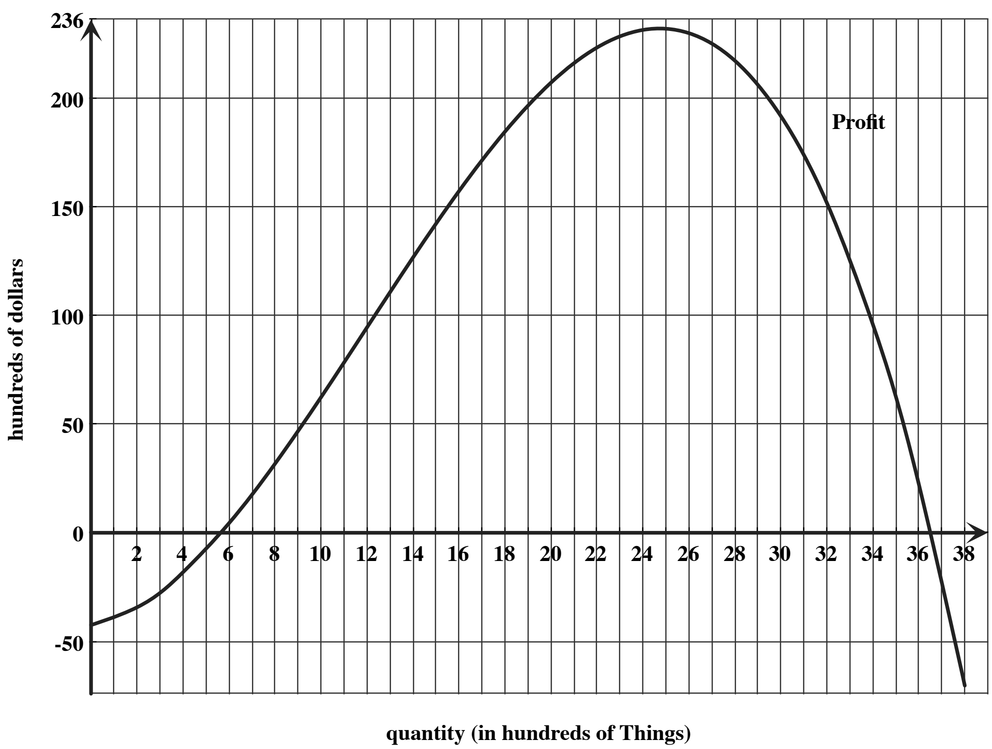
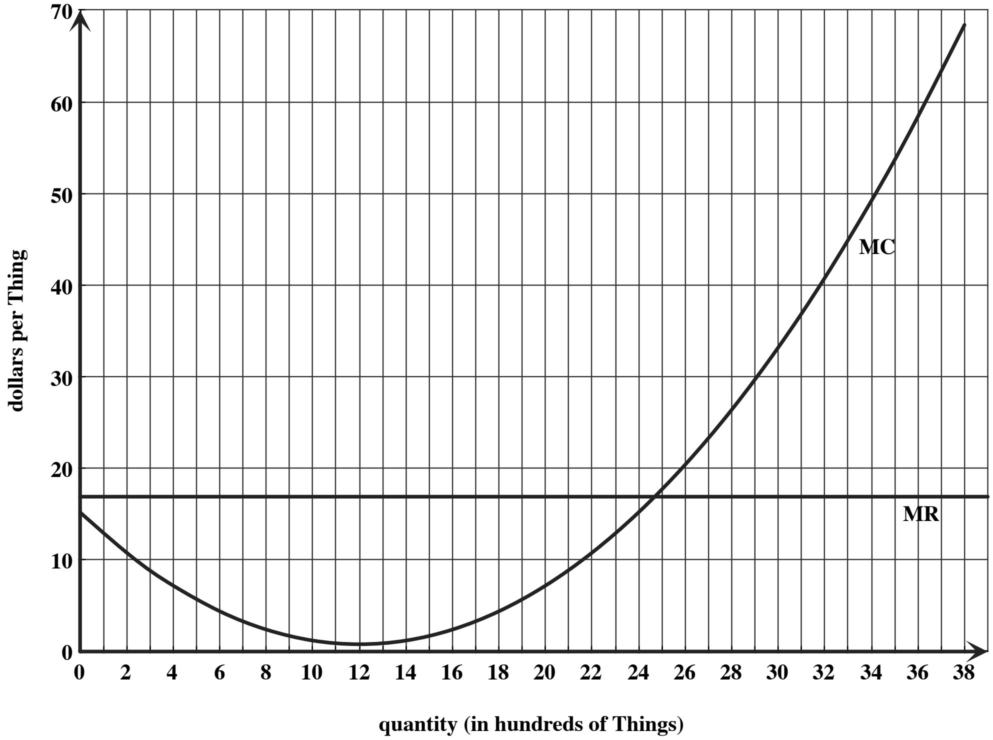
Math 111

Group Activity: Revenue, Cost, and Profit

Objectives: Below are the graphs of total revenue and total cost, profit, and marginal revenue and marginal cost for producing and selling Things. This activity is designed to clarify how these concepts are related and to consolidate a lot of what you've already learned.






70
60
50
40
30
20
10
0
dollars per Thing
MC
MR
0 2 4 6 8 10 12 14 16 18 20 22 24 26 28 30 32 34 36 38
quantity (in hundreds of Things)

Instructions: The first column of this table gives values for you to find. Find each value and write it in the "Answer" column, including units. You should be able to find most of the values using more than one of the given graphs. In the remaining columns, describe where you see the value(s) on the indicated graph(s). For some of the cells in the table, you may write "unable to determine" if you cannot find the answer from the indicated graph. The first row is completed as an example.

Find...	**Answer (include units)**	**Using graphs of TR & TC**	**Using graph of Profit**	**Using graphs of MR & MC**
fixed cost	~40 hundred dollars or \$4000	the "y"-intercept of the TC graph	the absolute value of the "y"-intercept of the profit graph	unable to determine
marginal revenue at any quantity				
marginal cost at $q = 20$				
profit at $q = 10$				
quantities at which you break even				
quantities at which $TR > TC$				
quantities at which $MR > MC$				
quantities at which profit is increasing				
quantity at which profit is maximized				
quantity at which $MR = MC$				
maximum profit				

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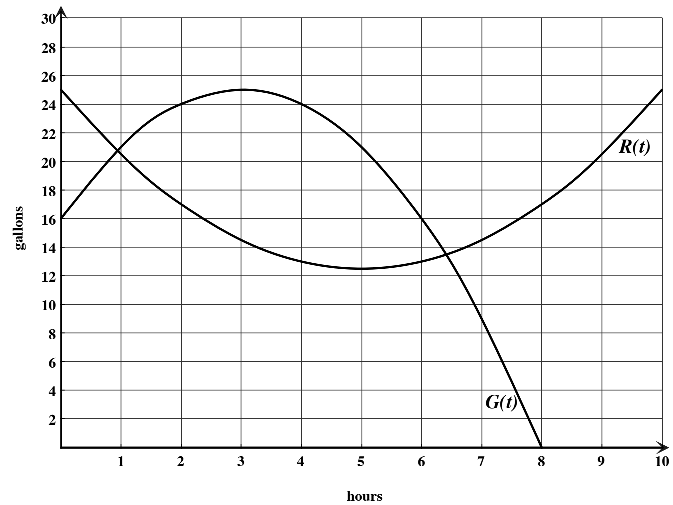
Math 111

Group Activity: Two Vats of Water

Water is flowing in and out of two vats, a red vat and a green vat. The graphs to the right show the amounts of water in the two vats over a several-hour period. We also have a gauge which measures the difference:

Amount in Green Vat
minus
Amount in Red Vat.

Note that the gauge starts out negative, is then positive for several hours, and again becomes negative.



1. Use the graphs to find approximate answers to the following.

   (a) Find the time at which the water in the red vat hits its lowest level.

   (b) Find the lowest value of the water level in the red vat.

   (c) Find the first time at which the two vats contain the same amount of water.

   (d) Find the time at which the difference gauge reaches its highest value.

   (e) Find the highest value the difference gauge reaches.

2. The formula for the amount of water in the red vat is

$$R(t) = 0.5t^2 - 5t + 25;$$

the formula for the amount of water in the green vat is

$$G(t) = -t^2 + 6t + 16.$$

We'll use algebra and these formulas to answer the same questions. For each, you should compare your answer to the responses you found using the graphs in #1.

(a) Find the time at which the water in the red vat hits its lowest level.

(b) Find the lowest value of the water level in the red vat.

(c) Find the first time at which the two vats contain the same amount of water.

(d) To answer part (d), we need a formula for the *difference*, $D(t)$. We said that our difference gauge measures the amount in the green vat minus the amount in the red vat. So,

$$D(t) = G(t) - R(t).$$

We have formulas for $G(t)$ and $R(t)$. Find a formula for $D(t)$ and use it to find the time at which the difference gauge reaches its highest value.

(e) Find the highest value the difference gauge reaches.

3. Suppose there is a purple vat that always contains exactly 5 gallons more than the red vat.

   (a) Sketch a rough graph of the amount in the purple vat on the graph at the beginning of this Activity.

   (b) Give a formula for $P(t)$, the amount of water in the purple vat after t hours.

   (c) Answer each of the following as accurately and efficiently as possible. For several of these questions, you should be able to use your answers to #1 and/or 2 and the new graph of $P(t)$ to answer without doing any calculations. For some, however, you must do some calculations to get the exact answer.

      i. Find the time at which the water in the purple vat hits its lowest level.

      ii. Find the lowest value of the water level in the purple vat.

      iii. Find the first time at which the water levels in the **purple** and **green** vats are the same.

      iv. If there is a difference gauge that measures the amount in the green vat minus the amount in the purple vat, find the time at which this difference gauge reaches its highest value.

      v. Find the highest value this new difference gauge reaches.

4. Suppose there is a blue vat whose water level is described by the following: At time t, the blue vat contains the same amount that the red vat contained one hour earlier.

   (a) Sketch the amount in the blue vat on the graph at the beginning of the activity. Start at $t = 1$.

   (b) Let $B(t)$ be the amount in the blue vat at time t. At time t, the blue vat contains the amount that the red vat contained at $t - 1$. This means that $B(t) = R(t - 1)$. We obtain $R(t - 1)$ by replacing every t in the formula for $R(t)$ with a $t - 1$:

$$B(t) = R(t - 1) = 0.5(t - 1)^2 - 5(t - 1) + 25.$$

   Simplify this formula to get a quadratic expression for $B(t)$.

   (c) Answer each of the following as accurately and efficiently as possible. For several of these questions, you should be able to use your answers to #1 and/or 2 and the new graph of $B(t)$ to answer without doing any calculations. For some, however, you must do some calculations to get the exact answer.

      i. Find the time at which the water in the blue vat hits its lowest level.

      ii. Find the lowest value of the water level in the blue vat.

      iii. Find the first time at which the water levels in the **blue** and **green** vats are the same.

      iv. If there is a difference gauge that measures the amount in the green vat minus the amount in the blue vat, find the time at which this difference gauge reaches its highest value.

      v. Find the highest value this new difference gauge reaches.

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Math 111

Group Activity: Maximizing Profit Three Ways

You work for Tasty Tours, organizing tours of a local winery. In order to make your tour competitive with other companies, you offer price breaks for larger groups. The following table gives the price per person (in dollars), the total revenue (in dollars), and the marginal revenue (in dollars per person) for different values of q (in number of people).

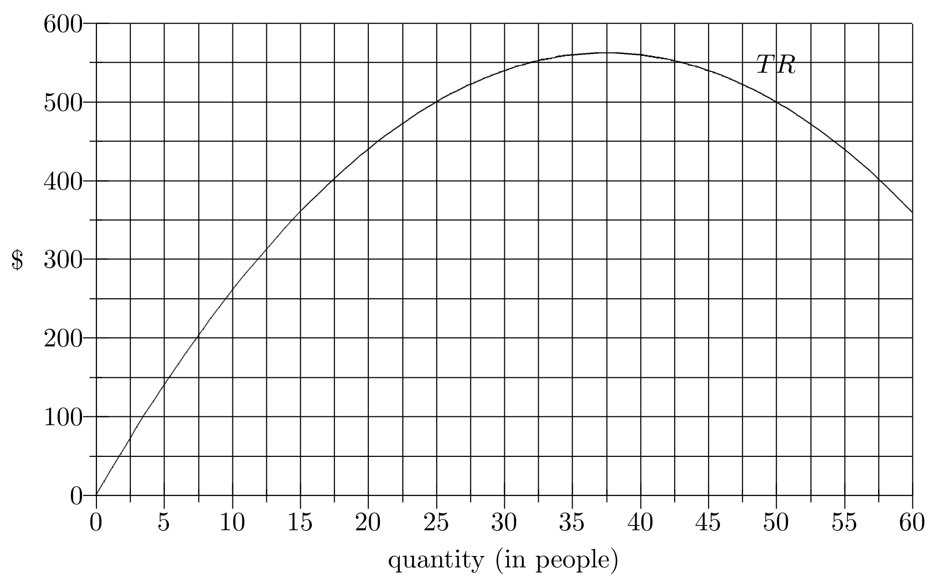
q	10	20	25	30	35	40	60
p	26	22	20	18	16	14	6
TR	260	440	500	540	560	560	360
MR	21.60	13.60	9.60	5.60	1.60	−2.40	−18.40

Your costs come from the winery and caterer, who charge you \$8 per person. You also have fixed costs in the amount of $\$FC$.

1. (a) The value of marginal cost is the same for all quantities. What is the marginal cost? Include units.

   (b) Recall how marginal revenue and marginal cost determine the quantity that maximizes profit and use the above table to estimate the number of people that yields maximum profit.

2. The following is the graph of total revenue.



   (a) Sketch the graph of variable cost on the axes above.

   (b) Use the graphs of TR and VC to approximate the number of people that yields maximum profit.

3. (a) The price per person p is a linear function of quantity q. Using the information given in the table, find this linear function.

(b) You now have a formula for the price of a tour per person for a group of q people. Use this to find the formula for your total revenue: $TR(q)$. What are the units associated with q in your formula? With $TR(q)$?

(c) Recall that $MR(q) = \dfrac{TR(q+1) - TR(q)}{1}$ and find a linear formula for $MR(q)$. What are the units associated with q in your formula? With MR(q)?

(d) Use the *formulas* for MR and MC to determine the number of people that yields maximum profit.

4. You used three different methods to find the quantity that maximizes profit: using a table of values of MR, graphs of TR and VC, and using formulas for MR and MC. The three methods should have yielded similar results. Discuss the advantages and disadvantages of using each method.

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Math 111

Group Activity: The Method of Linear Programming

The *Grass is Greener* lawn care company produces two different lawn fertilizers: Regular and Deluxe. The profit on each bag of Regular is \$0.75, while the profit on each bag of Deluxe is \$1.20. Each bag of Regular contains 3 pounds of active ingredients and 7 pounds of inert ingredients. In contrast, each bag of Deluxe contains 4 pounds of active ingredients and 6 pounds of inert ingredients. Due to limited warehouse facilities, the company can stock up to 8,400 pounds of active ingredients and 14,100 pounds of inert ingredients.

Let x denote the number of bags of Regular fertilizer and y denote the number of bags of Deluxe fertilizer the company will produce.

In this activity, you'll use the method of linear programming to find the amount of each fertilizer the company should produce in order to maximize profit and investigate why this method gives the optimal solution.

1. Linear programming is a method for maximizing or minimizing an objective function subject to one or more constraints. In this problem, we wish to maximize profit. Find a formula $P(x, y)$ that gives the amount of profit the company will earn by selling x bags of Regular fertilizer and y bags of Deluxe fertilizer. This is the objective function.

2. A constraint in a linear programming problem can be expressed as an inequality (a statement involving the symbol "$\leq$" or "$\geq$".) In this problem, two of the constraints are simply

$$x \geq 0 \text{ and } y \geq 0.$$

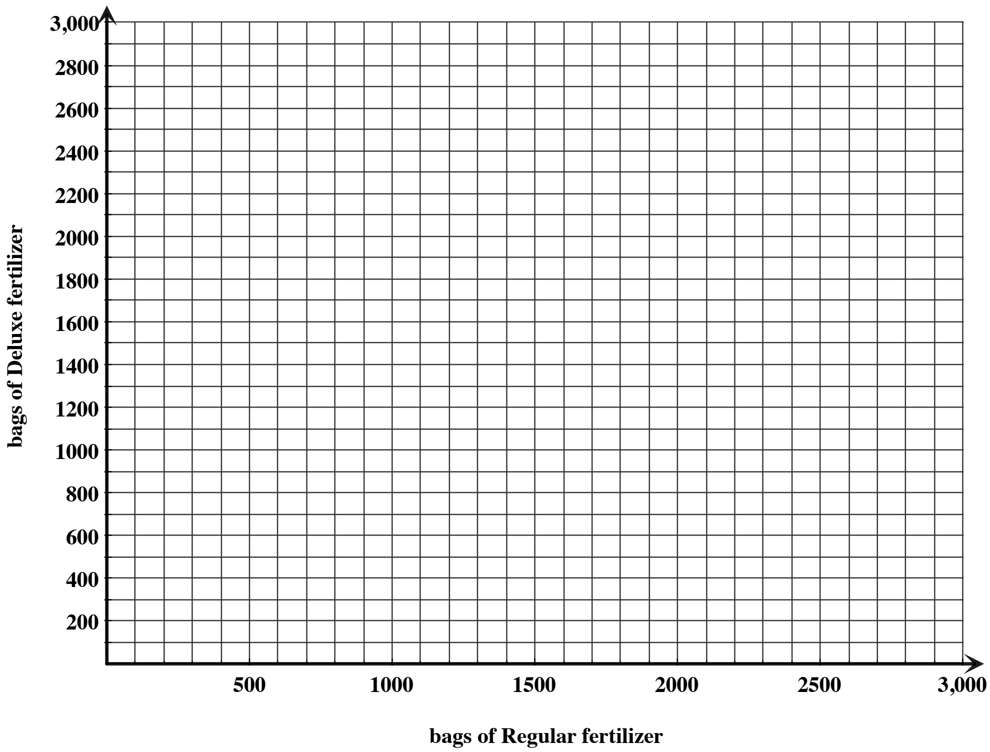
To get the remaining constraints, note that production is limited by the amounts of active ingredients and inert ingredients that are available.

- Determine the number of pounds of **active ingredients** needed to produce x bags of Regular fertilizer and y bags of Deluxe fertilizer and write an inequality that expresses the constraint due to the limitations on **active ingredients**.

- Determine the number of pounds of **inert ingredients** needed to produce x bags of Regular fertilizer and y bags of Deluxe fertilizer and write an inequality that expresses the constraint due to the limitations on **inert ingredients**.

You should now have four constraints.

3. Sketch and shade the system of inequalities representing your four constraints. This gives the feasible region for this problem: the set of points (x, y) that correspond to values of x and y that you can feasibly produce. Find the coordinates of all of the corners of the feasible region.



4. The maximum value of the objective function (profit, in this case) must occur at one of the corners of the feasible region. (We'll see why later in this activity.) This means that we can find the maximum value of the objective function by evaluating it at the corners and choosing the largest of those values. Evaluate the profit function from #1 at each of the corners you found in #3.

How many bags of each type of fertilizer should you make to maximize profit?

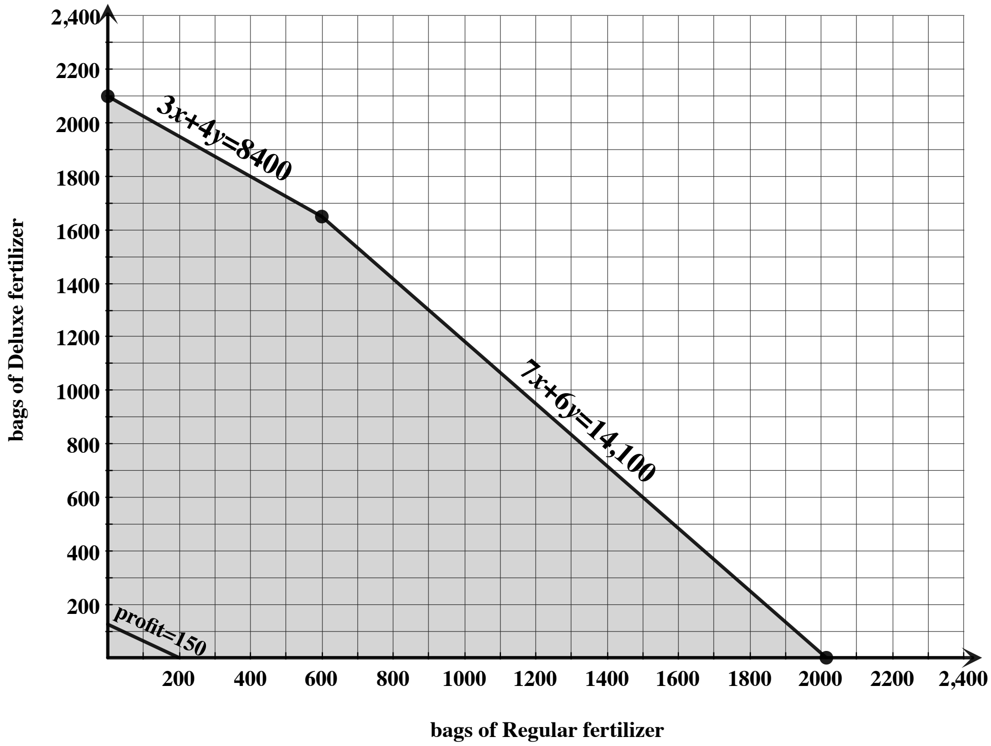
What is the maximum possible profit?

5. Now, we'll investigate why maximum profit occurs at one of the corners of the feasible region. The feasible region is graphed below. (Check that it's the same as the one you drew earlier.)

   (a) Recall that profit is given by the function $P(x, y) = 0.75x + 1.20y$. Let's look at the amounts of each type of fertilizer we'd need to make in order to have a profit of exactly \$150. Those would be the points (x, y) that satisfy the equation

$$0.75x + 1.20y = 150.$$

   This is a line. Find its x-intercept and y-intercept and verify that I've drawn this line on the graph of the feasible region below.



   (b) Repeat this process several times until you understand and can describe the behavior of these constant profit lines: draw the lines in the feasible region above that show the values of x and y that give a profit of \$300, \$600, \$900, and \$1200. What features do these lines share? What happens as you consider larger and larger profit?

   (c) Explain why the maximum profit must occur at one of the corners of the feasible region.

   (Hint: Look at how the constant profit lines change as profit increases and remember that only points in the feasible region matter.)

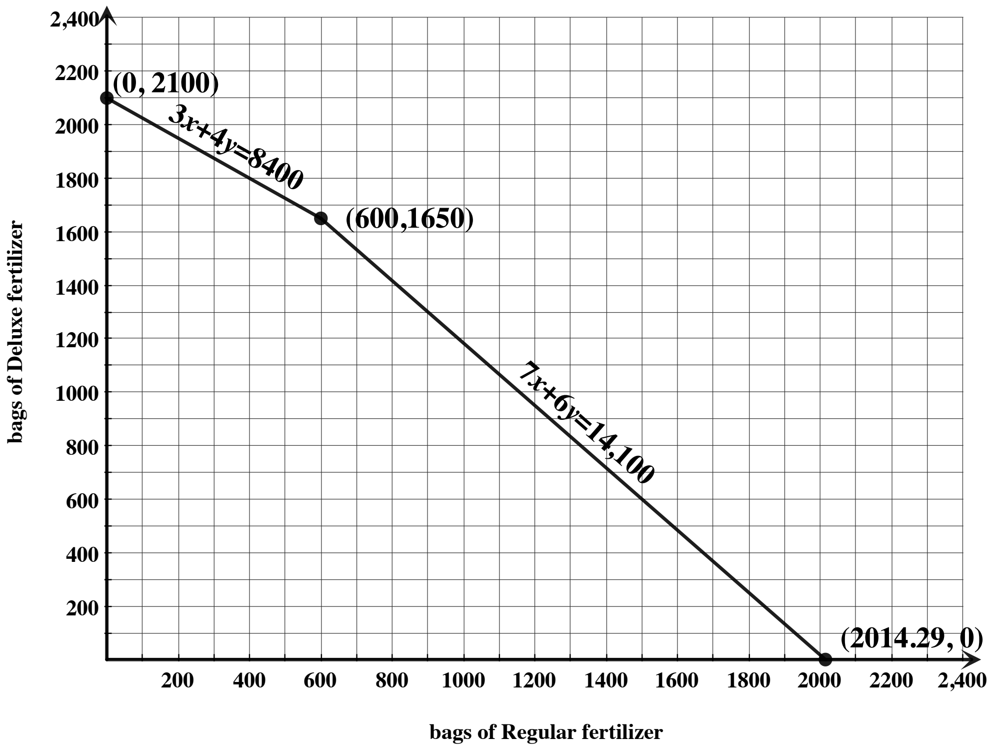
6. Finally, suppose you must make **at least** 400 bags of the Regular fertilizer and **at least** 600 bags of the Deluxe and you adjust your costs so that you earn a profit of \$1.00 for each bag of Regular. You continue to earn \$1.20 per bag of Deluxe. This means that your new objective function is

$$P(x, y) = 1.00x + 1.20y.$$

Your new constraints are:

$$3x + 4y \leq 8400,\ 7x + 6y \leq 14100,\ x \geq 400 \text{ and } y \geq 600.$$

Complete the feasible region on the graph below, find all the corners, and determine the maximum possible profit and the production levels that yield maximum profit.



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Math 111

Group Activity: Percent and Proportionate Change

Suppose you are awarded a 5% raise in your salary. We call 5% the **percentage change** (or just **percent change**) in your salary and that percentage expressed as a decimal, 0.05, the **proportionate change**. To compute your NEW salary, add your OLD salary to the dollar amount of the raise, which is 5% of your OLD salary:

$$\text{NEW} = \text{OLD} + 0.05 \cdot \text{OLD}.$$

1. If your current annual salary is \$38,000, what is your salary after a 5% raise?

2. You inherit a diamond ring appraised at \$5000. Diamonds are expected to appreciate by 8% per year. What would you expect the ring to be worth in one year?

3. You plan to buy a TV that is regularly priced at \$450. The store is offering 25% off every product in the store. What is the sale price of the TV? (HINT: You can still use an equation similar to the salary equation. But, since this is a reduction in the cost of the TV, your proportionate change is *negative.*)

4. In general, if a quantity changes by $p \times 100\%$ from an OLD value to a NEW value,

   $$\text{NEW}=\text{OLD}+p\cdot\text{OLD}.$$

   Solve this equation for the proportionate change, p.

5. Use either the formula, NEW=OLD+p·OLD or the formula you found in #4 to answer the following questions.

   (a) The population of a town was 4000 people in 2010 and 4093 in 2012. What was the percentage change in the population?

   (b) A pair of shoes that regularly costs \$103 is on sale for \$60. What percent savings is this? (Again, your proportionate change will be *negative*.)

   (c) A business purchased for \$650,000 in 1994 sold for \$850,000 in 1997. What was the percent change in its value?

   (d) A collectible lunchbox increases in value by 1.4% per year. If it is worth \$507 one year from now, what is its value today?

   (e) You start a new job with a starting salary of \$40,000 and a 2% cost-of-living raise each year. Fill in the following table:

t (in years)	salary after t years	dollar amount of raise
0	\$40,000	\$40,000·0.02 = \$800
1	\$40,800	\$40,800·0.02 =
2		
3		
4		

   Is it true that a salary that increases by 2% per year increases by 8% in four years? Explain.

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Math 111

Group Activity: A Fair Deal

Situation: XYZ Corporation issues promissory notes in \$1000 denominations under the following terms: purchase a \$1000 note now and, two years from now, they will give the owner of the note \$2000. You purchased one of these notes a year ago. It will mature in another year but you need cash now to pay your rent. A friend has offered to buy the note from you. The purpose of this activity is to determine how much your friend should pay for the note.

Define the following values:

- The amount someone pays for the note is their **principal** and will be denoted P.
- The amount someone receives when they sell the note will be denoted A.
- The **return** on the investment will be denoted R. Note $R = A - P$.
- The **rate of return** will be the return R as a percentage of the principal P. The rate of return expressed as a decimal will be denoted p. Then, $p = \frac{R}{P}$ and the rate of return is $p \times 100\%$.

1. **Deal #1:** Your friend offers to pay you \$1000 for the note.

   (a) In this scenario, identify the values of P, A, and R for each person: you and your friend.

   (b) What is your rate of return in this scenario? Your friend's rate of return?

   (c) Does this deal seem fair? (There isn't necessarily one "correct" answer here. The point is to have a conversation with the members of your group. The numbers you computed in parts (a) and (b) should play a part in your decision.)

2. **Deal #2:** You say that, since the investment increases by a total of \$1000 during the two years, you should each get a return of \$500.

   (a) In this scenario, identify the values of P and A and the **rate** of return for each of you.

   (b) Does this deal seem fair?

3. **Deal #3:** You explain the situation to your Math 111 TA who says that the way to make the deal fair is for both of you to earn the same **rate** of return and suggests the following: let $\$X$ be the amount your friend will pay you for the note and let p be the rate of return that you will both earn, expressed as a decimal.

   (a) In this scenario, you will pay \$1000 and receive $\$X$ from your friend. The amount of your return should be $(p \times 100)\%$ of your original investment. That is,

   $$X - \$1000 = \$1000p \text{ OR } X = \$1000 + 1000p.$$

   Similarly, your friend will pay $\$X$ and receive \$2000. But the \$2000 should give your friend a return of $(p \times 100)\%$ of his/her investment. That is,

   $$\$2000 = X + Xp.$$

   You now have two equations in two variables. Solve the system for X and p. (Round p to five digits after the decimal and round X to the nearest cent.)

   HINT: You can simply use substitution and solve the equations as they are currently written. But it may make your computations easier if you factor as follows:

   $$X = 1000(1 + p) \text{ and } 2000 = X(1 + p).$$

   (b) In this scenario, what is the return for each of you?

   (c) Does this deal seem fair?

   (d) Which one of the three deals would you choose? Or is there another scheme that seems fairer to you than any of these. If so, identify the principal, return, and rate of return for each of you and explain why it appeals to you.

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Math 111

Group Activity: Population Growth and Percent Change

1. A bacteria colony doubles in size every hour. At noon, there are 1000 bacteria. Let $B(t)$ be the number of bacteria in the colony t hours after noon.

   (a) How many bacteria are in the colony at 1:00 pm? 2:00 pm? 3:00 pm? 6:00 pm?

   (b) Give a formula that gives $B(t)$ as a function of t.

   (c) If $B(t) = 2500$, what is $B(t+1)$? (That is, if you know that the population is 2500 at time t, what will the population be one hour later?)

   (d) If $B(t) = 10,000$, what is $B(t-1)$? (That is, if you know that the population is 10,000 at time t, what was the population one hour earlier?)

   (e) Use your formula from part (b) to answer the following.

      i. What is the population at 12:30 pm? at 1:45 pm? at 8:20 pm? (Round to the nearest bacterium.)

      ii. When will the colony contain 1,000,000 bacteria? (How many hours after noon?)

      iii. Recall that, if a quantity changes from an OLD value to a NEW value, then the percentage change in the quantity is given by

      $$\frac{\text{NEW} - \text{OLD}}{\text{OLD}} \times 100\%.$$

      What is the percentage change in the population from noon to 12:30 pm?

2. A second colony increases its population by 75% every 30 minutes. There are 5000 bacteria in this colony at noon.

   (a) How many bacteria are in this colony at 12:30 pm? 1:00 pm? 1:30 pm? 2:00 pm? 3:00 pm?

   (b) By what factor must you multiply the population at one specific time to get the population 30 minutes later?

   (c) By what factor must you multiply the population at one specific time to get the population one hour later?

   (d) Let $C(t)$ represent the population of this colony t hours after noon. Give a formula for $C(t)$ as a function of t. (Again, you'll need to relate $C(t)$ to the population at noon and use your answer to part (c) of this question.)

   (e) When will this population contain 1,000,000 bacteria?

   (f) What is the percent change in this population over any one-hour period? (Round to the nearest percent.)