Example (chemistry): You have two solutions, one with 5% alcohol and the other with 8% alcohol. You need to prepare 200 ml of 7% alcohol solution. How much do you mix of each solution?

Variables: 
- \( x \): ml from 5% sol. 
- \( y \): ml from 8% sol.

Equation for amount: 
\[ x + y = 200 \]

Equation for amount of alcohol: 
\[ 5\% \text{ of } x + 8\% \text{ of } y = .7 \% \text{ of } 200 \]
\[ \left( \frac{5}{100} x + \frac{8}{100} y = \frac{7}{100} \cdot 200 \right) \cdot 100 \]

\[ x+y = 200 \]
\[ 5x+8y = 1400 \]

Exercise!
Check: 
- \( x = \frac{200}{3} \text{ ml} \)
- \( y = \frac{400}{3} \text{ ml} \)
In your equations, both sides must have the same units.

Example: Lakeview PTSA needs $4000 for the new playground equipment. They are raising this money by selling 500 tickets to the Halloween Party. The party will cost $800 to set up.

There are two types of tickets:
- $12 - regular price
- $6 - reduced price

How many reduced price tickets can they sell and still reach their target?

Variables:
- \( x \): # of reduced price tickets
- \( y \): # of regular price tickets
Equation for # of hcuts:
\[ x + y = 500 \]

Equation for money
\[ 6x + 12y = 4000 + 500 \]

Exercise
\begin{align*}
x &= 200 \\
y &= 300
\end{align*}

1.6 Applications of Linear Equations in Business & Economics

<table>
<thead>
<tr>
<th>Supplement</th>
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<tbody>
<tr>
<td>Revenue</td>
<td>( R(x) )</td>
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<tr>
<td>Cost</td>
<td>( C(x) )</td>
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<tr>
<td>Profit</td>
<td>( P(x) )</td>
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</table>

\[ \text{Profit} = \text{Revenue} - \text{Cost} \]

You break even when Revenue = Cost or Profit = 0.
**NEW!** Supply - Demand - Market Equilibrium

$q$: quantity (independent variable, $x$-axis)

$p$: price (dollars, dependent variable, $y$-axis)

Theory:

Law of Demand: When prices decrease (there's sale), demand will increase (quantity)

Law of Supply: Quantity supplied will increase when prices increase.
Example 1  Demand
Supply

Market equilibrium: common solution
(Simple) Substitution

\[ 3q + 2 = -5q + 42 \]
\[ 8q = 40 \]
\[ q = \frac{40}{8} = 5 \]

Thus, \( p = -5(5) + 42 = 17 \).

ME: 5 things at 17 dollars each.
example 2: Nordstrom will buy 500 pairs of shoes if the price is $80 per pair and 600 pairs if the price is $75.

The cobbler will make 300 pairs of shoes if the price is $70 and 400 pairs if the price is $90.

Write down the Demand + Supply functions and find the equilibrium price and quantity.

$P$ is like $y$, $Q$ is like $x$

slope = \frac{\text{change in price}}{\text{change in quantity}}$

**Demand:** $(500, 80)$ $(600, 75)$

slope = \frac{80 - 75}{500 - 600} = \frac{5}{-100} = -0.05$

point-slope: $P - 80 = -0.05(Q - 500)$
clean-up: \( p = -0.05q + 105 \)

**Supply:** \((300, 70), (400, 90)\)

Slope: \[ \frac{70 - 90}{300 - 400} = \frac{-20}{-100} = 0.2 \]

\[ p - 70 = 0.2(q - 300) \]

clean up: \( p = 0.2q + 10 \)

**Market Equilibrium:**

\[ 0.2q + 10 = -0.05q + 105 \]

\[ 0.25q = 95 \]

\[ q = \frac{95}{0.25} = 380 \]

Then, \( p = 0.2(380) + 10 = 86 \)

380 pairs of shoes at $86 a pair.
Government Interferes: \textit{TAX} (per item)

Supply: \[ p = f(q) \]

The supplier is taxed (he makes the money) a fixed amount \( t \) per item. They reflect this on their prices. After \textit{TAX}, new supply function is

\[ p = f(q) + t \]
If the King County imposes $5 tax per pair, then the cobbler will change his supply:

$$ p = 0.2q + 10 + \frac{5}{p} $$

$$ (p = 0.2q + 15 \text{ after tax supply.}) $$

Exercise: Find new equilibrium point.