1. You manufacture and sell Objects. Your total cost, TC, is a linear function of quantity, q. If you make 100 Objects, your cost is \$1400. If you make 250 Objects, your cost is \$1820.

   You sell Objects for \$35.

   (a) Find a formula for total cost, TC, as a function of q.

   (b) At what quantity, q, do you break even?

   (c) What is your marginal cost?

   (d) At what quantity is average cost, AC, equal to \$11?

2. You make and sell Things. For a quantity of q Things, your total cost is

$$TC(q) = 0.1q^2 + 2q + 8000.$$

You sell Things for \$170.

(a) At what quantities do you break even?

(b) At what quantity do you have the greatest profit?

(c) What is your maximum possible profit?

(d) Give a formula for marginal cost, MC.

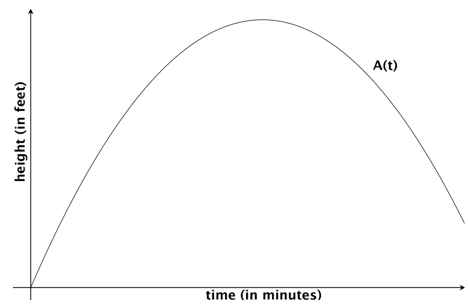
(e) At what quantity q is marginal cost equal to \$40?

3. (13 points)

Balloon A is released from ground level at $t = 0$. The height (in feet) of Balloon A after t minutes is given by

$$A(t) = -0.15t^2 + 9t.$$

The graph of $A(t)$ is given at right.



(a) Give a formula for $\dfrac{A(5+h) - A(5)}{h}$ and simplify as much as possible.

ANSWER: $\dfrac{A(5+h) - A(5)}{h} =$ ____________________

(b) Balloon B has a linear height vs. time graph. Balloon B is at the same height as Balloon A at $t = 0$ and $t = 55$. Find the formula for the height (in feet) of Balloon B after t minutes.

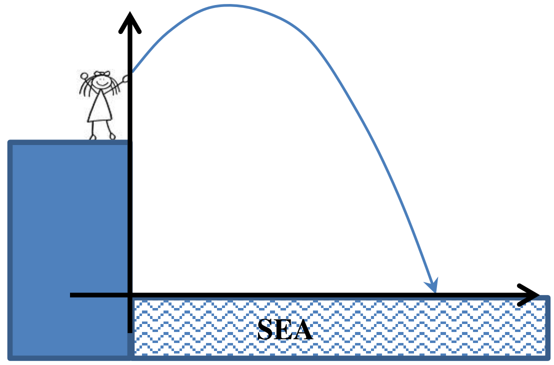
ANSWER: $B(t) =$ ____________________

(c) The average rate of ascent of Balloon A is $\frac{A(t)}{t}$ and the average rate of ascent of Balloon B is $\frac{B(t)}{t}$. Find the time at which the average rate of ascent of Balloon B is exactly 4 feet per minute faster than the average rate of ascent of Balloon A.

ANSWER: $t =$ ________________ minutes

4. (13 pts) Mary stands on a cliff and throws a rock toward the sea. At t seconds from the moment the rock was thrown, its height $H(t)$ above the sea level is given by the formula:

$$H(t) = -16t^2 + 24t + 15 \text{ (in feet)}$$



a) (3 pts) How high above the sea level is the rock at the moment when Mary throws it?

ANSWER:__________________ feet

b) (5 pts) What is the greatest height of the rock above the sea level?

ANSWER: ________________ feet

c) (5 pts) Compute the longest time interval during which the rock is at a height of at least 15 feet above the sea level.

ANSWER: from ____ to _____ seconds

5. (12 pts) You produce and sell Trinkets. Your total revenue, in dollars, from selling q Trinkets is

$$TR(q) = -2q^2 + 18q.$$

a) Recall that $MR(q) = TR(q+1) - TR(q)$. Compute the formula for $MR(q)$ in terms of q. Simplify it as much as possible.

ANSWER: $MR(q) =$ ______________________________ \$/Item

b) Suppose your variable cost, in dollars, is given by

$$VC(q) = q^3 - 10q^2 + 32q$$

Compute the Shutdown price.

ANSWER: $SDP =$ ________ \$/Item

6. (13 points) Here are the formulas for two quadratic functions:

$$f(x) = -x^2 + 12x + 4 \text{ and } g(x) = 4x^2 - 12x + 10.$$

(a) Write out the formula for

$$\frac{f(x+5) - f(x)}{5}$$

and simplify as much as possible.

(b) Give the longest interval on which $g(x)$ and $f(x) - g(x)$ are both increasing.

ANSWER: from $x =$____________ to $x =$____________

(c) Which value of x in the interval from $x = 7.25$ to $x = 7.99$ makes $f(x)$ the largest?

ANSWER: $x =$____________

7. (20 points) You sell Things. The formula for total cost is

$$TC(q) = 0.1q^3 - 3q^2 + 35q + 15,$$

where q is in **hundreds of Things** and TC is in **hundreds of dollars.**

(a) Compute the **average cost** to produce 450 Things. Include units with your answer.

ANSWER: ____________________UNITS: ____________________

(b) Give formulas for **variable cost** and **average variable cost** for selling q hundred Things.

ANSWER: $VC(q) =$ ____

$AVC(q) =$ ____

(c) Find all values of q at which **average variable cost** is 18 dollars per Thing.

ANSWER: (list all) $q =$ ____________________hundred Things

(d) Compute the shutdown price.

ANSWER: ____________________dollars per Thing

(e) The graph of total revenue is a straight line and **profit** is 0 when $q = 20$ hundred Things. Find the formula for $TR(q)$.

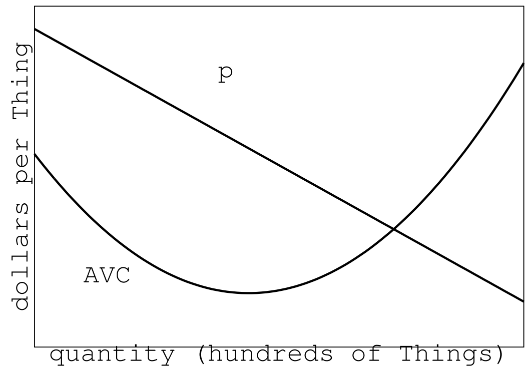
ANSWER: $TR(q) =$ ____________________

8. (16 points)

You sell Things. The price, p, in dollars per Thing and the average variable cost, AVC, in dollars per Thing on an order of q **hundred** Things are

$$p(q) = 28 - 3q \quad \text{and} \quad AVC(q) = q^2 - 9q + 22.$$

The **fixed cost** is $1,800. ($FC = 18$ hundred dollars). The graphs of price, p, and average variable cost, AVC, are given.



(a) (4 pts) Write out the formulas for the total revenue, variable cost, total cost, and average cost for selling q hundred Things.

$TR(q) =$ ________________

$VC(q) =$ ________________

$TC(q) =$ ________________

$AC(q) =$ ________________

(b) (4 pts) Find the profit for selling 460 Things.
(Give your final answer in dollars rounded to the nearest cent.)

________________ **dollars**

(c) (4 pts) Find the largest interval of quantities over which the **average variable cost** is less than or **equal to** 6 dollars per Thing. (Your values of q should be accurate to the nearest Thing).

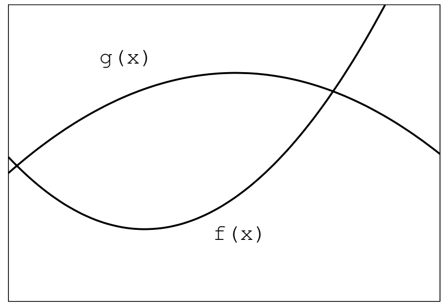
$q =$ ________________ **hundred Things** to $q =$ ________________ **hundred Things**

(d) (4 pts) Find the Shutdown Price (SDP).

________________ dollars per Thing

9. (19 points)

The graphs to the right are parabolas with formulas:



$$f(x) = x^2 - 6x + 18, \quad \text{and}$$

$$g(x) = -\tfrac{1}{2}x^2 + 5x + 16.$$

Give all final answer accurate to two digits after the decimal.

(a) (4 pts) Find all values of x at which the two graphs cross.

$x =$ ____________________

(b) (4 pts) Find the longest interval when $f(x)$ and $g(x)$ are both increasing.

$x =$ ______________ to $x =$ ______________

(c) (4 pts) Find the size of the biggest vertical gap when $g(x)$ is above $f(x)$.
(That is, find the largest value of $g(x) - f(x)$.)

____

(d) Let $h(x)$ be a new parabola given by the relationship $h(x) = f(x - 4)$.

i. (4 pts) Write out the formula for $h(x) = f(x-4)$ and simplify into the expanded quadratic form, $h(x) = (\quad)x^2 + (\quad)x + (\quad)$.

$h(x) =$ ________________________

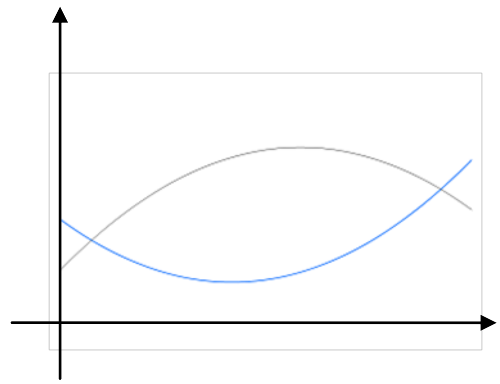
ii. (3 pts) Find the x and y values of the vertex for $h(x)$.
(Hint: There is a way to answer using the previous part and a way to answer directly from $f(x)$. Either way is fine, just show and explain your work.)

$x =$ ______________

$y =$ ______________

10. (15 points) The graphs to the right are parabolas, with the following formulas:

$$f(x) = x^2 - 5x + 15$$
$$g(x) = -x^2 + 6x + 10$$



a) Find the values of x at which the two graphs cross.

ANSWER: at $x =$______ and at $x =$________

b) Find the longest interval over which both functions are increasing. Fully justify your answer.

ANSWER: From $x =$ __________ to $x =$______________

c) Find the value of x at which $\frac{f(x)-f(0)}{x} = 7$

ANSWER: at $x =$_____________

11. (20 pts) You produce and sell Cans of Cat Food, in order sizes of 1 to 150 Cans.

Each Can costs \$1.50 to produce. Your fixed costs are \$30.

To encourage larger orders, you offer a volume discount as follows: the price for an order of **one** Can is \$2.99, and you **decrease** the price per Can **by 1 cent** for each additional Can ordered. For instance, if a customer buys 5 Cans, the selling price is \$2.95 per Can.

a) (3 pts) Write down a linear formula, in terms of quantity q ordered, for the selling price per Can.

$p(q) =$ ______________________________

b) (4 pts) Write down formulas in terms of q for the Total Revenue and the Total Cost for an order of q Cans.

$TR(q) =$ ______________________________

$TC(q) =$ ______________________________

Note: To answer the following questions you need the TR function from part (b). If you could not answer part (b), circle and use the formula $TR(q) = 5q - 0.02q^2$ instead (this is not the correct answer in part b!)

c) (5 pts) Compute the formula in terms of q for the Marginal Revenue. Show all steps & simplify your answer.

ANSWER: $MR(q) =$ ____________________

d) (6 points) Compute the largest profit possible.

ANSWER: Max profit is ____________________ dollars

12. (15 pts) The marginal revenue and marginal cost at q Items are given by the following linear functions:

$$MR(q) = -0.5q + b \qquad \text{dollars}$$

$$MC(q) = 0.4q + 7 \qquad \text{dollars}$$

In addition, the average cost (in dollars per Item) is given by the function:

$$AC(q) = 0.2q + 6.8 + \frac{38}{q}$$

a) What is the change in total cost if q increases from 4 to 5 Items?

ANSWER: _______________ dollars

b) Recall that $MR(q) = -0.5q + b$. Compute a value of $\boldsymbol{b}$ that results in the profit being maximal at $q = 10$ Items.

ANSWER: $b =$ _____________

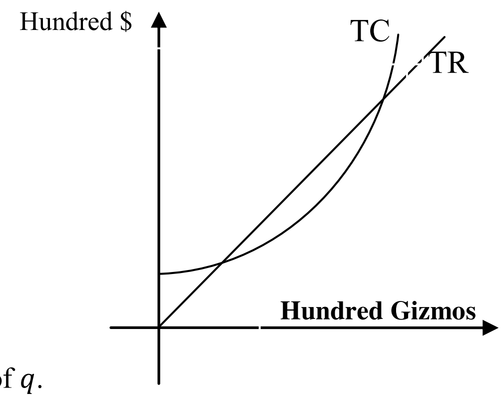
c) Compute the breakeven price.

ANSWER: BEP = ________________ dollars per item.

13. (20 pts) The graphs to the right are of the total cost TC and total revenue TR for producing and selling Gizmos.
The formula for the total cost is:

$$TC(q) = q^2 + 4.5q + 5$$

with q in **hundreds** of Gizmos, and TC in hundreds of dollars.
The TR graph is a line that goes through the origin and crosses the graph of TC at $q = 10$ hundred Gizmos.



a) (4 pts) Write down a formula for the total revenue as a function of q.

$TR(q) =$ ______________________________

b) (6 pts) What is the smallest quantity q at which the average cost is \$10 per Gizmo? (round your answer to 4 decimal digits)

ANSWER: at $q =$ ____________ hundred Gizmos.

c) (4 pts) What is the marginal cost at 3 hundred Gizmos? (caution: q is in **hundreds** of Gizmos!)

ANSWER: $MC(3) =$ ________ hundred dollars

d) (6 pts) Compute the largest profit possible.

ANSWER: Max profit is ____________ hundred dollars.

14. (18 points) A car drives for 4 hours on a straight road. Its distance, in miles, from its starting place at t hours is given by the formula:

$$D(t) = 100t - 25t^2$$

a) (3 pts) At what time is this car the farthest away from its starting place?

ANSWER: at______________ hours

b) (6 pts) Sketch the graph of $D(t)$ and compute the time interval when this car will be at a distance of at least 50 miles from its starting place.

ANSWER: from $t =$ __________ to $t =$ __________ hours

c) (5 pts) Write the following expression as a linear function of t:

$D(t + 0.5) - D(t) =$

ANSWER: $D(t + 0.5) - D(t) =$ _______________________

d) (4 pts) Find a time t such that the car traveled 25 miles during the half-hour time interval starting at t.

ANSWER: $t =$ ____________ hours

15. (16 pts) The distance, in miles, from some starting line for **Car** A and **Car** B at time t hours are respectively given by

$$D_A(t) = t^3 - 7t^2 + 20t \quad \text{and} \quad D_B(t) = 70t - 2t^2.$$

(a) (4 pts) Find and completely simplify the average trip speed formulas for **Car** A and **Car** B.

$ATS_A(t) =$ ______________________ mph

$ATS_B(t) =$ ______________________ mph

(b) (4 pts) How long does it take **Car** B to travel the first 50 miles? (Round your answer to two digits after the decimal point)

$t =$ ______________________ hours

(c) (4 pts) Give all times when the average trip speed for **Car** B is 25 mph.

$t =$ ______________________ hours

(d) (4 pts) For each car, give the smallest **value** of the Average trip speed between the times $t = 1$ and $t = 20$ hours. (Explain your answers)

smallest ATS value for **Car** $A =$ ______________________ mph

smallest ATS value for **Car** $B =$ ______________________ mph

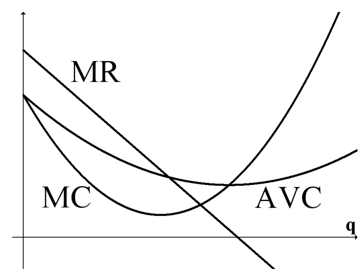
(e) (4 pts) Once again, the distance formula for **Car** B is given by: $D_B(t) = 70t - 2t^2$.
Write out and completely simplify the formula for the average speed for **Car** B over the two-hour interval starting at time, t. (Your answer will be in the form $at + b$)

$$\frac{D_B(t+2) - D_B(t)}{(t+2) - t} = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \text{ mph}$$

16. (14 pts)

You sell *Items*. Your marginal revenue, marginal cost and average variable cost are given by the formulas:

$MR(q) = 25 - 4q \quad MC(q) = q^2 - 10q + 19 \quad AVC(q) = \frac{1}{3}q^2 - 5q + 19,$



where q is measured in **hundreds of Items** and marginal revenue, marginal cost, and average variable cost are in **dollars per Item**.

(a) (6 pts) In addition, the fixed costs (FC) are \$200.00. Find and completely simplify the formulas for variable cost, total cost and average cost.

$VC(q) =$ ____________________ hundreds of dollars

$TC(q) =$ ____________________ hundreds of dollars

$AC(q) =$ ____________________ dollars per item

(b) (4 pts) Find the quantity that maximizes profit. (Appropriately round to a whole number of Items)

____________________ Items

(c) (4 pts) Find the shutdown price.

____________________ **dollars per item**

17. Your company, "*RainCheck*", produces extra-large umbrellas. The selling price is \$19.99 per umbrella. Each umbrella costs you \$9.50 to produce. Your fixed costs are \$250.

a) (4 points) Write down formulas in terms of quantity q of umbrellas (and/or numbers), for each of the following:

$MR(q) =$ ______________________

$MC(q) =$ ______________________

$TR(q) =$ ______________________

$TC(q) =$ ______________________

b) (4 points) What is the smallest number of umbrellas which you need to produce and sell in order to make at least \$50 in profit?

ANSWER: $q =$ ______________ Umbrellas
(your answer should be a whole number of umbrellas)

c) (4 points) At what quantity is your average cost \$14.50 per umbrella?

ANSWER: At $q =$ ______________ Umbrellas

18. You produce Widgets. Your Total Revenue and Variable Cost are given by the following functions:

$$TR(q) = -0.5q^2 + 6q$$

$$VC(q) = 0.01q^3 - 0.2q^2 + 1.5q$$

with quantity q in **hundreds of Widgets**, and the total revenue and variable cost in **hundreds of dollars**.

a) (5 points) Sketch the Total Revenue graph and find the maximum Total Revenue.

ANSWER: Maximum TR is ____________ hundred dollars

b) (5 points) Find **<u>all</u>** the quantities q for which the Total Revenue is above $1200.
Pay careful attention to units!

ANSWER: From ______________ to ____________________ **Widgets**
(your answer should be a range of whole numbers of Widgets)

c) (5 points) Recall that the shutdown price can be computed as the lowest value of the average variable cost. Compute the shutdown price. Include correct units.

ANSWER: $SDP =$ _____________ Units:____________________________

19. (14 points) You produce and sell Trimbles. Your total revenue and total cost (both in dollars) for selling q Trimbles are:

$$TR(q) = -0.14q^2 + 14q \qquad TC(q) = 0.01q^3 - 0.75q^2 + 19.75q + 25.$$

(a) Use the fact that $MR(q) = TR(q+1) - TR(q)$ to write out a formula for $MR(q)$ in terms of q and simplify as much as possible.

ANSWER: $MR(q) =$ ____________________

(b) Set up the equation that you would solve in order to answer the question:

At what quantity does total revenue exceed total cost by 60 dollars?

Simplify your equation so that is in the form: $Aq^3 + Bq^2 + Cq + D = 0$.
DO NOT SOLVE THE EQUATION.

ANSWER: ____________________

(c) Use the fact that the shutdown price is the smallest value of average variable cost to find the shutdown price.

ANSWER: $____________________ per Trimble

20. (18 points) You sell Quipples on a sliding price scale. The price p per Quipple on an order of q **thousand** Quipples is

$$p = 20.10 - 0.1q \text{ dollars.}$$

(a) Write out a formula for your total revenue (in thousands of dollars) for selling q thousand Quipples.

ANSWER: $TR(q) =$ ____________________

(b) Name the longest interval over which TR is increasing.

ANSWER: from $q =$ ____________ to $q =$ ____________ thousand Quipples

(c) The Quipples cost \$2 each to produce and you have fixed costs of five thousand dollars. Write out a formula for the total cost (in thousands of dollars) to produce q thousand Quipples.

ANSWER: $TC(q) =$ ____________________

(d) On the interval from $q = 1$ to $q = 6$ thousand Quipples, what is the largest value of total cost?

ANSWER: ____________ thousand dollars

(e) What quantity maximizes profit?

ANSWER: ____________ thousand Quipples

21. (16 points) You sell *Items*. Your marginal revenue and marginal cost are given by the formulas:

$$MR(q) = 2.539 - 0.4q \qquad MC(q) = 0.1q^2 - 0.8q + 1.85,$$

where q is measured in **hundreds** of Items and marginal revenue and marginal cost are in dollars per Item.

(a) Find the quantity that maximizes profit.

ANSWER: $q =$____________________hundred Items

(b) Find the quantity at which MR exceeds MC by exactly \$0.50 per Item.

ANSWER: $q =$____________________hundred Items

(c) At what quantity is MC lowest?

ANSWER: $q =$____________________hundred Items

(d) The formula for average variable cost is:

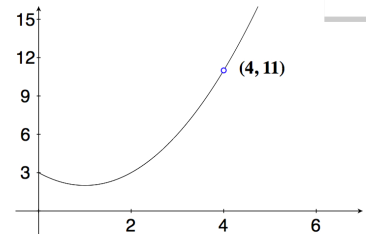
$$AVC(q) = \frac{1}{30}q^2 - \frac{2}{5}q + 1.85,$$

where q is in hundreds of Items and AVC is in dollars per Item. Find the shutdown price.

ANSWER: \$____________________per Item

22. (16 points) The parabola at right has a formula that looks like

$$f(x) = ax^2 - 2x + c.$$



(a) Use the fact that $f(0) = 3$ and $f(4) = 11$ to find the values of a and c.

ANSWER: $a =$____________________, $c =$____________________

Use your answers to part (a) to complete the formula for $f(x)$ and use this completed formula to answer the remaining questions:

$$f(x) = (\qquad)x^2 - 2x + (\qquad).$$

b) Find the slope of the diagonal line through $f(x)$ at $x = 2$.

ANSWER: ____________________

c) Write out a formula for $\dfrac{f(1+h) - f(1)}{h}$. Simplify your formula as much as possible.

ANSWER: $\dfrac{f(1+h) - f(1)}{h} =$ ____

d) Find a value of x such that $\dfrac{f(x) - f(0)}{x} = \dfrac{10}{3}$.

ANSWER: $x =$____________________

23. (10 points) You have two final exams coming up, one in Calculus and one in Spanish class. You decide to hire tutors to help you study.

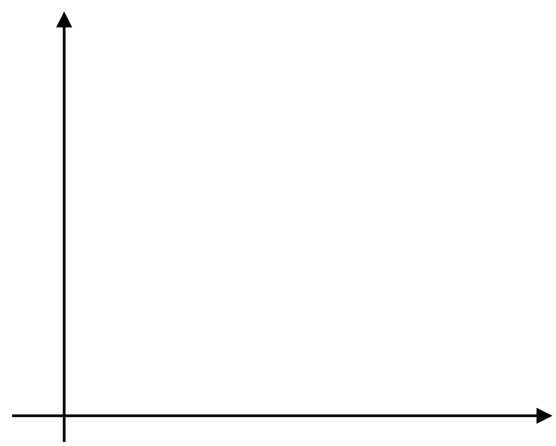
Your Calculus tutor charges \$30/hour, while your Spanish tutor charges \$15/hour.
For each hour you spend studying Calculus with your tutor, your GPA (grade point average) increases by 0.12.
For each hour you spend studying Spanish with your tutor, your GPA increases by 0.1.
You have a total of at most 16 hours you can dedicate to studying, and a budget of at most \$300.
Let x be the number of hours you study Calculus, and let y denote the number of hours spent studying Spanish.

a) (2 pts) You want to maximize your increase in GPA, subject to the given constraints. Write down the formula for your objective function $G(x, y)$. No need to show work.

$G(x, y) =$ ______________________________

b) (2 pts) Write the inequalities for your constraints. No need to show work.

c) (4 pts) Sketch the feasible region and list the coordinates of all the vertices.



Vertices (list all): $(x, y) =$ ______________________________

d) (2 pts) Calculate your maximum possible increase in your GPA. Show work.

ANSWER: ___________

24. (13 points) You're ordering desserts for your company's annual shareholders' meeting. The Something Special Bakery has two specialty items: Café Latté Cupcakes and Key Lime Tartlets. Each box of Cupcakes costs \$30 and serves 12. Each box of Tartlets costs \$24 and serves 10. The bakery can provide no more than 22 boxes of Tartlets on the day of your event. You may spend up to \$1098 on the desserts.

Let x be the number of boxes of Cupcakes you order and y be the number of boxes of Tartlets. How many boxes of each should you order to maximize the number of servings and how many can you serve? (NOTE: The Bakery is willing to sell a fraction of a box. So, x and y need not be whole numbers.)

ANSWER: $x =$ ____________________ boxes, $y =$ ____________________ boxes,

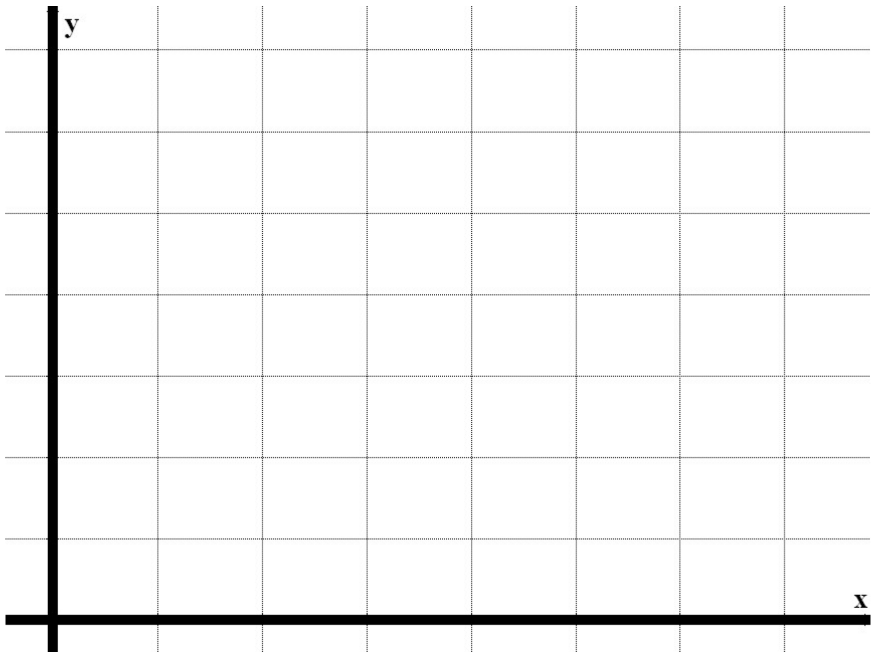
for a maximum of ____________________ servings

25. (10 pts) The constraints for a linear programming problem are

$$4x + 2y \leq 1200, \;\; y \leq 400, \;\; \text{and } x \leq 200.$$

and x and y both must be greater than or equal to zero.

(a) (7 pts) Sketch and shade the feasible region and **clearly label the exact coordinates of all vertices**.



(b) (3 pts) Subject to the given constraints, find the maximum and minimum values of the objective function:

$$f(x, y) = 2x + 3y + 200.$$

ANSWER: **minimum** value = ____________________

maximum value = ____________________

26. (9 points) You run a home-business, knitting and selling mittens and socks.

Each pair of mittens takes you 4 hours of work and 0.75 spools of wool to knit, and it sells for $20.

Each pair of socks takes you 2.5 hours and 1 spool of wool to knit, and it sells for $12.

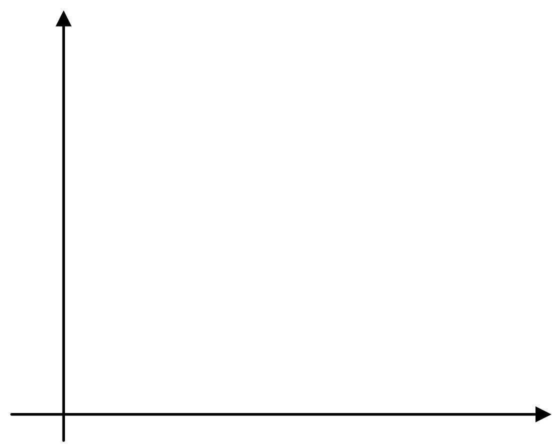
This week you have at most 40 hours to spend knitting and a supply of 15 spools of wool.

Let x be the number of pairs of mittens you produce this week, and y be the number of pairs of socks.

a) (2 pts) Write down the formula for the function $R(x, y)$ that computes the total revenue you would earn from selling x pairs of mittens and y pairs of socks.

$R(x, y) =$ ______________________________

b) (5 pts) Draw your feasible region, label it "FR", and compute all its vertices.
(list both coordinates -- you may round them to 2 decimal digits)

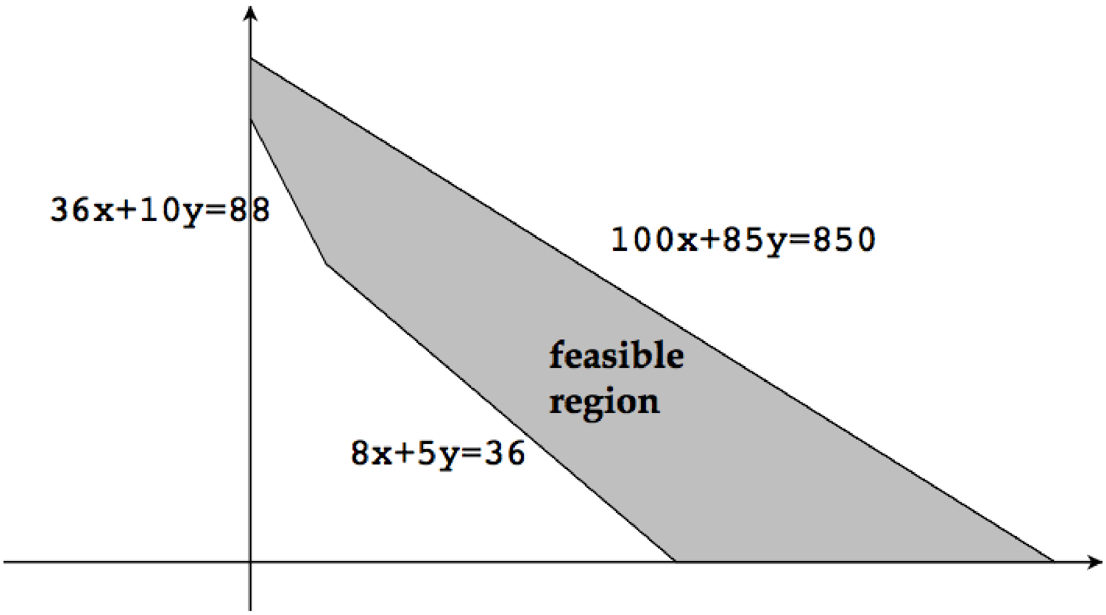


Vertices (list all): $(x, y) =$ ____

c) (2 pts) Find your maximum possible total revenue this week. Show work.

Max possible revenue is $____________.

27. (17 points) A linear programming problem has three constraints: $36x + 10y \geq 88$, $8x + 5y \geq 36$, and $100x + 85y \leq 850$. The feasible region is shown in the graph below:



Find the exact coordinates of the five vertices of the feasible region and use them to find the smallest and largest values of the objective function

$$P(x, y) = 5x + 4y$$

subject to these constraints.

ANSWER: smallest: ____________________

largest: ____________________

28. Find the market equilibrium point (q, p).

(a) supply: $1749 - p^2 + 3q = 0$ demand: $p^2 + 5q = 2485$

(b) supply: $p - q = 20$ demand: $q(2p - 30) = 408$

29. Solve for x. Round your FINAL ANSWER to 2 digits after the decimal.

(a) $\ln(6x + 7) = 3.2$

(b) $e^{4x-3} = 1.1$

(c) $\frac{1}{4}e^{-5x} = 0.2$

(d) $6 - e^{0.4x} = 1$

30. A function has the form $f(x) = Ae^{kx}$. You know that $f(0) = 4.2$ and $f(10) = 5.13$. Find the values of A and k and compute $f(100)$.

31. Let $f(x) = -2x^2 + 10x + 3$. Compute $\dfrac{f(x+h) - f(x)}{h}$ and simplify as much as possible.