Solutions to Common Review of Graphical and Algebraic Methods

The answers to pairs of questions are the same. The solutions below are algebraic unless it is not algebraically possible to solve the question as noted. In that case, an approximation from the graph is given.

Distance Question

The distance travelled by a Car is given by the function

$$D(t) = -0.021t^2 + 33.6t$$

where D is in miles and t is in hours.

1. What is the total distance travelled by the Car at 350 hours? Yes, it is a very long trip.

$$D(350) = -0.021(350)^2 + 33.6(350) = 9187.5 \text{ miles}$$

2. What is the Average Trip Speed at 500 hours?

$$ATS(500) = \frac{D(500)}{500} = 23.1 \text{ miles/hour}$$

3. What is the Average Speed of the car between 200 and 600 hours?

$$AS = \frac{D(600) - D(200)}{600 - 200} = 16.8 \text{ miles/hour}$$

4. At what time t is the Average Trip Speed equal to 25 miles per hour?
   Solve for t in
   $$25 = \frac{D(t)}{t}$$
   to get $t \approx 409.5$ hours.

5. Find a 250 hour time interval when the Average Speed of the car is 15.75 miles per hour?
   Solve for t in
   $$15.75 = \frac{D(t + 250) - D(250)}{250}$$
   to get $t = 300$. So from $t = 300$ to $t = 550$ hours.

6. If a Truck starts 4000 miles ahead of the first one and travels at a constant speed of 15 miles per hour, when will the Car catch up with the Truck?
   The distance function for the Truck is given by $R(t) = 4000 + 15t$. To see when the car catches up solve
   $$4000 + 15t = -0.021t^2 + 33.6t.$$
   The Car will catch up when $t \approx 367.71$. At the other t value, $t \approx 518$, the Truck will overtake the Car.

Total Cost, Variable Cost, Revenue, Profit Question

You produce and sell Things. The Total Cost for producing Things is given by

$$TC(q) = 1.75q^3 - 5.25q^2 + 5.5q + 1$$

where q is the number of thousands of Things and TC is in thousands of dollars.

1. What is the Total Cost of producing 300 Things?

   $$TC(0.3) = 2.22475 \text{ thousand dollars} = \$2224.75$$

2. What is the Variable Cost of Producing 300 Things?

   $$VC(0.3) = TC(0.3) - TC(0) = 1.22475 \text{ thousand dollars} = \$1224.75$$

3. What is the Fixed Cost?

   $$FC = TC(0) = 1 \text{ thousand dollars} = \$1000$$

4. What is the Average Cost at 200 Things?

   $$AC(0.2) = \frac{TC(0.2)}{0.2} = \$9.52 \text{ per Thing}$$

5. What is the Average Variable Cost at 800 Things?

   $$AVC(800) = \frac{VC(0.8)}{0.8} = \$2.42 \text{ per Thing}$$

6. At what quantity q is the Average Variable Cost equal to 1.75 dollars per Thing?
   Solve for q in

   $$1.75 = 1.75q^2 - 5.25q + 5.5$$

   to get $q \approx 1.82$ and $q \approx 1.17$ thousands of Things.

7. At what quantity is the Average Cost Equal to 3 dollars per Thing?
   The equation to solve is

   $$3 = 1.75q^2 - 5.25q + 5.5 + \frac{1}{q}$$

   which becomes $1.75q^3 - 5.25q^2 + 2.5q + 1 = 0$ when you simplify. We cannot solve the cubic so we get this from the graph to be about 1000 Things.

8. What is the Breakeven Price?
   You can do this from the graph: $BP \approx \$2.22$ per Thing.

9. What is the Marginal Cost at 400 Things? Remember q is in thousands of things and TC is in thousands of dollars.

   $$MC(400) = TC(0.44 + 0.001) - TC(0.4) = 0.002137 \text{ thousand dollars/Thing} = \$2.137 \text{ per Thing}$$

10. What is the Shutdown Price?
    The lowest value of

    $$AVC(q) = 1.75q^2 - 5.25q + 5.5$$

    is at $q = \frac{5.25}{3.5} = 1.5$ thousand Things and $SP = AVC(1.5) = \$1.5625$.

11. If you sell each Thing for 3 dollars, when do you break even?
    From the graph, you break even at about 1000 Things. You can check that $q = 1$ satisfies $TR = TC$ indeed.

12. If you sell each Thing for 3 dollars, what is the Maximum Profit?

    $$MC(q) = TC(q + 0.001) - TC(q) \approx 5.25q^2 - 10.5q + 5.5 \text{ dollars per Thing}$$

    Solving $3 = MC(q)$, we get $q \approx 1.72$ or $q \approx 0.28$. We check the Profit$= TR - TC$ in both: $P(1.72) \approx \$1320$ and $P(0.28) \approx -\$1330$ so the Maximum Profit is \$1330.

Marginal Cost, Average Variable Cost and Average Cost Question

You produce and sell Cosas. The Marginal Cost and The Average Cost for q hundred Cosas are

$$MC(q) = 0.525q^2 - 10.5q + 55 \qquad AC(q) = 0.175q^2 - 5.25q + 55 + \frac{100}{q}$$

in dollars per Cosa.

1. What is the Average Cost at 800 Cosas?

   $$AC(8) = \$36.7 \text{ per Cosa}$$

2. What is the Average Variable Cost at 2000 Cosas?

   $$AVC(20) = \$20 \text{ per Cosa}$$

3. What is the Marginal Cost at 1200 Cosas?

   $$MC(12) = \$4.6 \text{ per Cosa}$$

4. At what quantity q is the Average Variable Cost equal to 30 dollars per Cosa?
   Solving for q in $30 = 0.175q^2 - 5.25q + 55$ we get $q \approx 24.06$ or $q \approx 5.94$ hundred Cosas.

5. At what quantity q is the Average Cost equal to 25 dollars per Cosa?
   The equation to solve $25 = 0.175q^2 - 5.25q + 55 + \frac{100}{q}$ becomes $0.175q^3 - 5.25q^2 + 30q + 100 = 0$ which we cannot solve. From the graph we see that $q \approx 12.25$ and $q \approx 20$ hundred Cosas.
   (You can check that $q = 20$ satisfies the cubic equation. If you know synthetic division, knowing one of the roots $q = 20$ of the cubic equation, you can reduce it to a quadratic and find the others using the quadratic formula, one of which should be close to 12.25.)

6. At what quantity is the Marginal Cost equal to 5 dollars per Cosa?

   $$5 = 0.525q^2 - 10.5q + 55$$

   gives $q \approx 12.18$ or $q \approx 7.82$ hundred Cosas.

7. What is the Total Cost of producing 1000 Cosas?

   $$TC = 1000AC(10) = \$30,000$$

8. What is the Variable Cost of Producing 600 Cosas?

   $$VC = 600AVC(6) = \$17,880$$

9. What is the Fixed Cost?

   $$FC = 100 \text{ hundred dollars} = \$10,000$$

10. What is the Shutdown Price?
    Solve $MC = AVC$ and plug in the q value to one of the two OR find the lowest value of AVC using the vertex formula. $SP = \$15.625$

11. What is the Breakeven Price?
    Simplifying $MC = AC$ you get $0.35q^3 - 5.25q^2 - 100$ which we cannot solve. From the graph you see that $BP \approx \$22.5$.

12. If you sell each Cosa for 40 dollars, what is the maximum profit?
    Solving $40 = MC$ gives $q \approx 18.45$ or $q \approx 1.548$. Evaluate at the profit function: $P(18.45) = 311.29$ and $P(1.548) = -111.29$ hundred dollars. So the Maximum Profit is $\$31,129$.