Math 111
Fall '19
MT2 (v2)
solutions

1. You produce and sell Blinks. The price p per Blink is given as a function of quantity sold by

$$p = 2.88 - 0.004q$$

dollars per Blink. The Total Cost of producing q Blinks is give by

$$TC(q) = 0.002q^2 - 0.84q + 128.2$$

dollars.

(a) (2 points) What is the Marginal Cost at 450 Blinks?

$$MC(450) = TC(451) - TC(450) = 0.002(451^2 - 450^2) - 0.84(451-450)$$
$$= 0.962 \text{ dollars}$$

(b) (1 point) What is the Total Revenue $TR(q)$?

$$TR(q) = (2.88 - 0.004q)q = -0.004q^2 + 2.88q$$

(c) (4 points) What is the maximum Profit?

$$P(q) = -0.004q^2 + 2.88q - (0.002q^2 - 0.84q + 128.2)$$
$$= -0.006q^2 + 3.72q - 128.2$$

max at $q = \dfrac{-3.72}{-0.012} = 310$ Blinks

max profit $P(310) = -0.006(310)^2 + 3.72(310) - 128.2$
$= 448.4$ dollars.

(d) (3 points) At which quantities do you break even? Round your answers to the nearest Blink.

when $0 = P(q) = -0.006q^2 + 3.72q - 128.2$

$$q = \frac{-3.72 \pm \sqrt{3.72^2 - 4(-0.006)(-128.2)}}{-0.012}$$

$\simeq 583$ OR 37 Blinks

2. The altitude of Balloon P is given by the function

$$p(t) = 1.25t^2 - 10t + 30$$

meters, where the time t is in minutes.

(a) (2 points) A second Balloon Q has a **linear** altitude function $q(t)$. It starts at 17 meters and reaches an altitude of 27 meters at 10 minutes. Write down the function $q(t)$ in meters of the altitude of Balloon Q as a function of time t in minutes.

$q(t) = mt + 17 \qquad 27 = 10m + 17 \rightarrow m = 1$

$q(t) = t + 17$

(b) (4 points) At what time is Balloon Q above Balloon P by the largest amount? How much is that amount?

maximize $f(t) = t + 17 - (1.25t^2 - 10t + 30)$

$= -1.25t^2 + 11t - 13$

at $t = \frac{-11}{-2.5} = 4.4$ minutes

max amount $f(4.4) = -1.25(4.4)^2 + 11(4.4) - 13$

$= 11.2$ meters

(c) (4 points) At what times are the balloons at the same altitude?

when $f(t) = 0$ so

$t = \frac{-11 \pm \sqrt{121 - 4(-1.25)(-13)}}{2(-1.25)}$

$= \frac{11 \pm \sqrt{56}}{2.5} \simeq 7.39$ OR 1.41 minutes

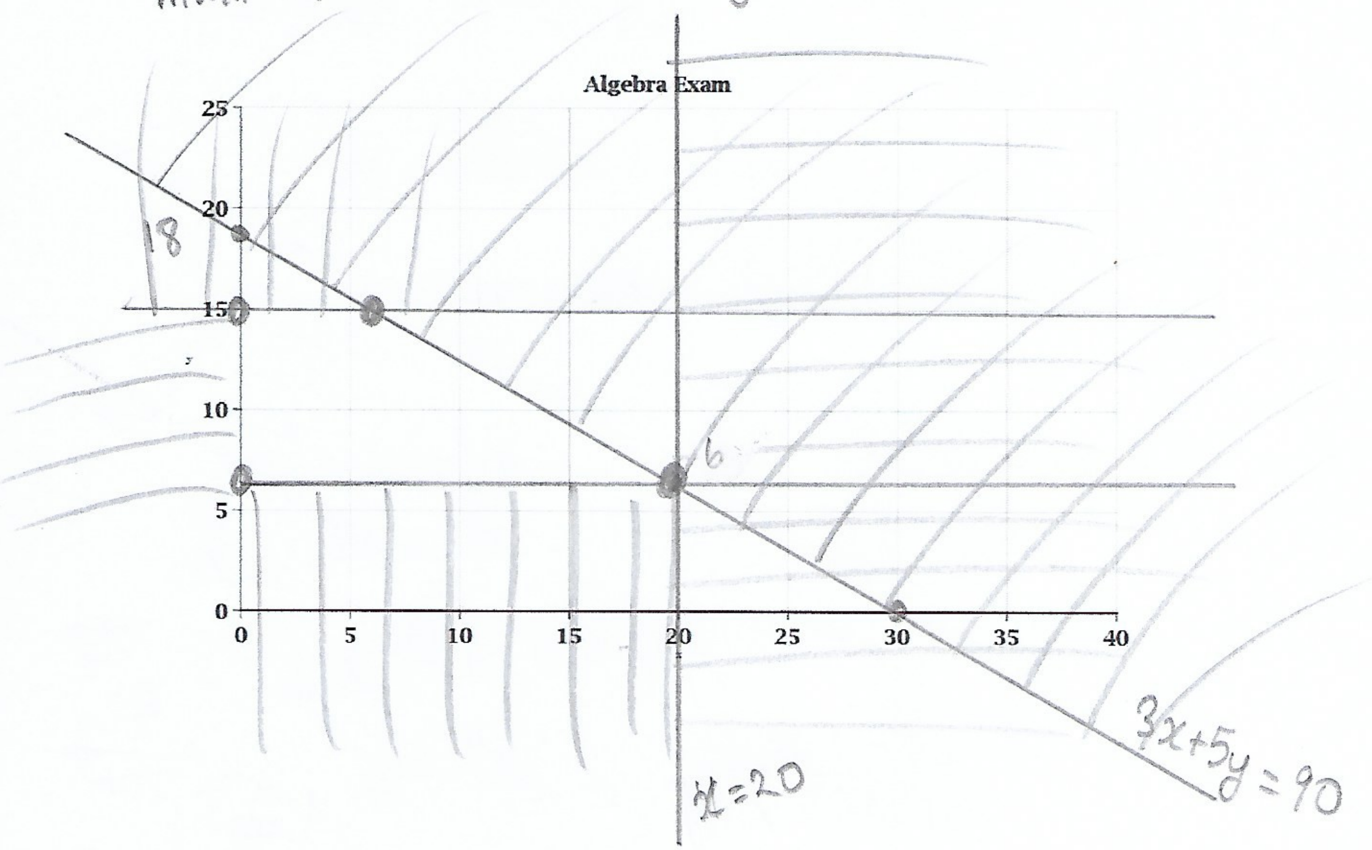
3. (12 points) Al-Khwarizmi is late for his algebra exam and he has 90 minutes left to complete it. The exam has 20 multiple choice questions and 15 short answer questions. It takes 3 minutes to answer a multiple choice questions and 5 minutes to answer a short answer question. He is required to answer at least 6 short answer questions. Each multiple choice question is worth 2 points and each short answer question is worth 5 points. How many of each should he answer (assuming he will get all of them right) to maximize his score on this exam?

x: multiple choice $\quad$ y: short answer

$0 \le x \le 20$ $\quad$ $6 \le y \le 15$

time $\quad 3x + 5y \le 90 \quad (0,18)\ (30,0)$

maximize $\quad P = 2x + 5y$



points

(x,y)	$(0,6)$	$(0,15)$	$\boxed{(5,15)}$	$(20,6)$
$2x+5y$	30	75	$\boxed{85}$	70

4. The population of Krokazia was 12.5 million in 1943 and 17 million in 1994. Assume that the population of Krokazia can be modelled by the exponential function

$$P(t) = P_0 e^{kt}$$

where P is in millions and t is in years.

(a) (6 points) Compute the constant k and write down the function modeling the population of Krokazia?

$P(t) = 12.5\,e^{kt}$

in 1994 $t = 1994 - 1943 = 51$

$17 = P(51) = 12.5\,e^{51k}$

$\frac{17}{12.5} = e^{51k}$

$\ln\left(\frac{17}{12.5}\right) = 51k \rightarrow k = \frac{1}{51}\ln\left(\frac{17}{12.5}\right) = \frac{\ln(1.36)}{51}$

$P(t) = 12.5\,e^{\frac{\ln(1.36)}{51}t}$

(b) (2 points) What is the population of Krokazia today? Round your answer to the nearest thousand people.

in 2019 $t = 2019 - 1943 = 76$

$P(76) = 12.5\,e^{\frac{\ln(1.36)}{51}(76)}$

≈ 19.76556215

$\simeq 19.766$ million