

Math 111 Fall '19 MT2 (vi) solutions

1. You produce and sell Blooms. The Total Cost for producing Blooms is given by

$$TC(x) = 0.1x^2 + 1.5x + 3.4$$

hundreds of dollars, where the quantity x is in hundreds of Blooms. Include units with your answers.

- (a) (4 points) At what quantities is the Average Cost 6 dollars per Bloom. Round your answer to the nearest Bloom.

$$\begin{aligned} AC(x) &= \frac{0.1x^2 + 1.5x + 3.4}{x} = 6 \rightarrow 0.1x^2 + 1.5x + 3.4 = 6x \\ 0.1x^2 - 4.5x + 3.4 &= 0 \quad x = \frac{4.5 \pm \sqrt{(4.5)^2 - 4(0.1)(3.4)}}{0.2} \approx 44.23 \text{ OR} \\ &\quad 0.77 \text{ hundred Blooms} \end{aligned}$$

- (b) (3 points) The price p of each Bloom is constant. You break even at 400 Blooms. Write the formula for the Total Revenue function $TR(x)$.

$$\begin{aligned} TR(x) &= px \quad \text{when } x=4 \quad TR(4) = TC(4) \\ 4p &= 0.1(16) + 1.5(4) + 3.4 = 1.6 + 6 + 3.4 = 11 \\ p &= \frac{11}{4} = 2.75 \\ \text{so } TR(x) &= 2.75x \end{aligned}$$

- (c) (5 points) What is the Maximum Profit? Round your answer to the nearest cent.

$$\begin{aligned} P(x) &= TR(x) - TC(x) = 2.75x - (0.1x^2 + 1.5x + 3.4) \\ &= -0.1x^2 + 1.25x - 3.4 \\ \text{max at } x &= \frac{-1.25}{2(-0.1)} = 6.25 \text{ hundred Blooms} \end{aligned}$$

$$\begin{aligned} \text{max profit} &= P(6.25) = -0.1(6.25)^2 + 1.25(6.25) - 3.4 \\ &= 0.5063 \text{ hundred dollars} \\ &\text{OR \$50.63} \end{aligned}$$

2. (8 points) The supply function for SuperCalculators is linear. The supplier will produce 55 calculators if the price is \$117 and 75 calculators if the price is \$121. The local government imposes a tax of \$22 per calculator sold. The demand function for SuperCalculators is predicted to be

$$p = -0.001q^2 - .12q + 394,$$

where p is the price per SuperCalculator and q is the quantity of calculators which will be purchased at that price. Find the after-tax supply function and the equilibrium point for this scenario.

Supply

$$(55, 117) \quad (75, 121) \quad \text{slope} = \frac{121-117}{75-55} = \frac{4}{20} = 0.2$$

$$p - 117 = 0.2(q - 55)$$

$$\begin{aligned} p &= 0.2q - 11 + 117 \\ p &= 0.2q + 106 \end{aligned}$$

before tax

$$\begin{aligned} p &= 0.2q + 128 \\ \text{after tax} \end{aligned}$$

equilibrium:

$$-0.001q^2 - 0.12q + 394 = 0.2q + 128$$

$$0 = 0.001q^2 + 0.32q - 266$$

$$q = \frac{-0.32 \pm \sqrt{(0.32)^2 - 4(0.001)(-266)}}{0.002}$$

$$= \frac{-0.32 \pm 1.08}{0.002} = \boxed{380 = q} \quad (\text{or negative number})$$

$$p = 0.2q + 128 = 380(0.2) + 128 = 204 \text{ dollars}$$

3. (10 points) You need to buy bookcases for the Bibliophile Library. You know that Bookcase Billy costs \$100 per unit, requires six square feet of floor space, and holds eight cubic feet of books. Bookcase Hemnes costs \$200 per unit, requires eight square feet of floor space, and holds twelve cubic feet of books. You have been given \$1400 for this purchase and the library has room for no more than 72 square feet of bookcases.

Name your variables, graph the feasible region with all its corners labeled, and determine how many of which model you should buy in order to maximize storage volume.

x : # of Billys y : # of Hemnes

budget

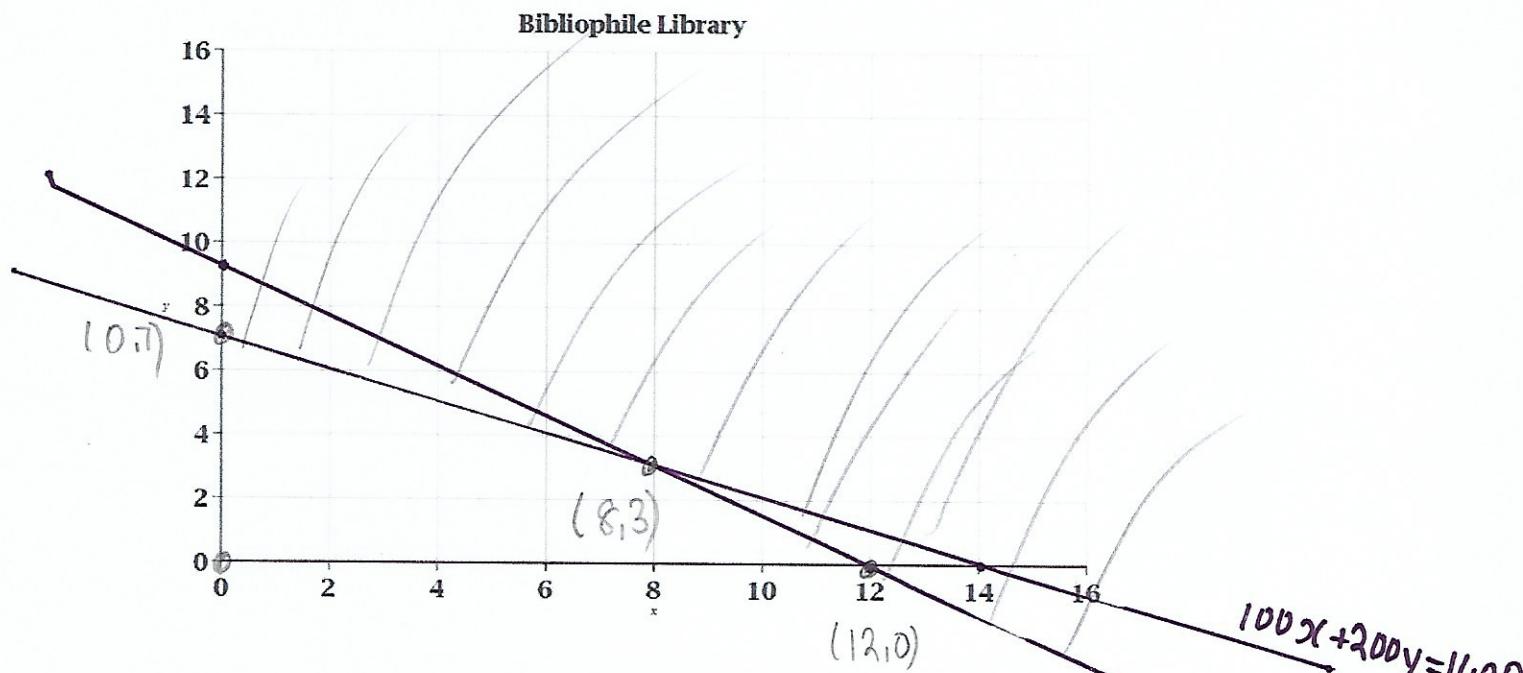
$$100x + 200y \leq 1400 \quad (0, 7) \quad (14, 0)$$

floor space

$$6x + 8y \leq 72 \quad (0, 9) \quad (12, 0)$$

maximize

$$\text{Volume } V = 8x + 12y$$



$$\begin{aligned} x + 2y &= 14 \rightarrow 6x + 12y = 84 \\ &\quad - 6x + 8y = 72 \\ &\quad \hline 4y = 12 \\ &\quad y = 3 \end{aligned}$$

$$x = 14 - 2y = 8$$

point	(0, 7)	(8, 3)	(12, 0)
$8x + 12y$	84	$64 + 36 = 100$	96

8 Billy
3 Hemnes

4. Solve the following equations for x . Round your answers to four digits after the decimal.

(a) (5 points) $7 = \frac{25}{1 + e^{2x}}$

$$7(1 + e^{2x}) = 25$$

$$7 + 7e^{2x} = 25$$

$$7e^{2x} = 18$$

$$e^{2x} = \frac{18}{7}$$

$$2x = \ln\left(\frac{18}{7}\right) \rightarrow x = \frac{\ln\left(\frac{18}{7}\right)}{2} \approx 0.4722$$

(b) (5 points) $1 - 3 \ln(2x - 6) = \frac{7}{9}$

$$-3 \ln(2x - 6) = \frac{7}{9} - 1 = -\frac{2}{9}$$

$$\ln(2x - 6) = \frac{2}{27}$$

$$2x - 7$$

$$2x - 7 = e$$

$$x = \frac{e^{\frac{2}{27}} + 7}{2} \approx 4.0384$$