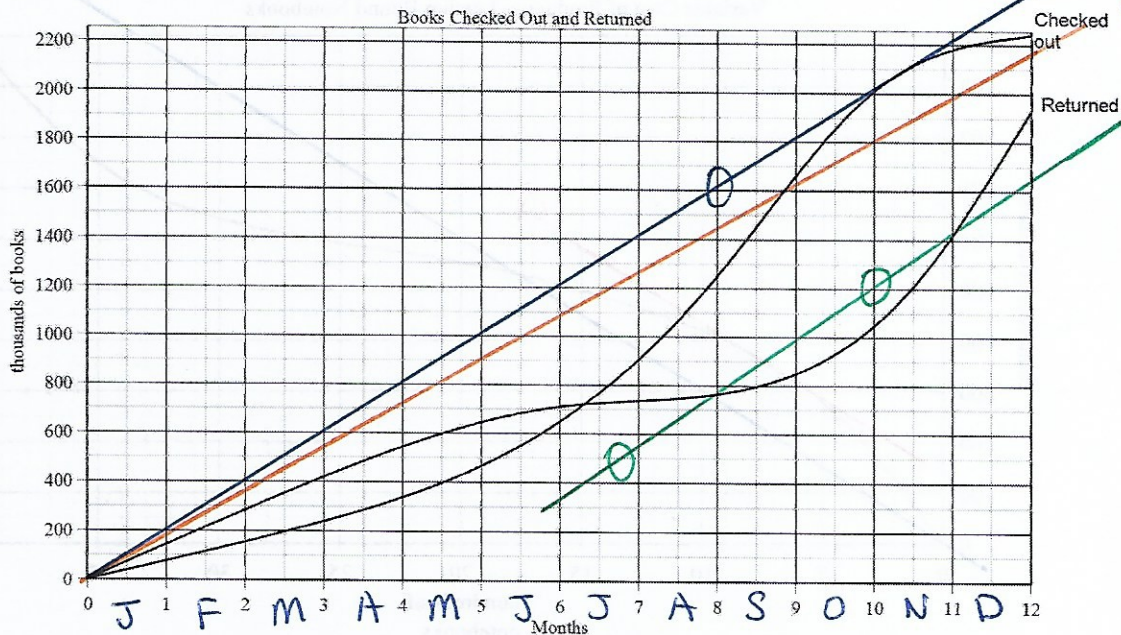


Solutions to Math III Fall 2019 Midterm 1 (Version 2)

1. The following shows graphs of the number of books checked out and returned to the Alexandria Library during the one year period starting in January 1, 2016. Answer the following questions based on these graphs. LABEL any lines you draw on the graph with the letter of the question so we can follow your work. Give UNITS with your answers.



- (a) Find the Average Rate of Change in number of books returned during the months of September, October and November. Round your answer to the nearest thousand book.

$$8 \leq t \leq 11$$

$$\frac{R(11) - R(8)}{11 - 8} \approx \frac{1420 - 760}{3} \approx 220 \text{ thousand}$$

$$\text{OR slope} \approx \frac{1300 - 500}{10 - 6.8} \approx 219 \text{ thousand}$$

- (b) Find a time when the Overall Rate of Change of materials checked out was 180,000 books per month.

about 8.8 months

- (c) If the library has nine million books, how many books were in the library at the beginning of March?

at $t=2$

$$9000 - 155 + 285 = 9130 \text{ thousand books}$$

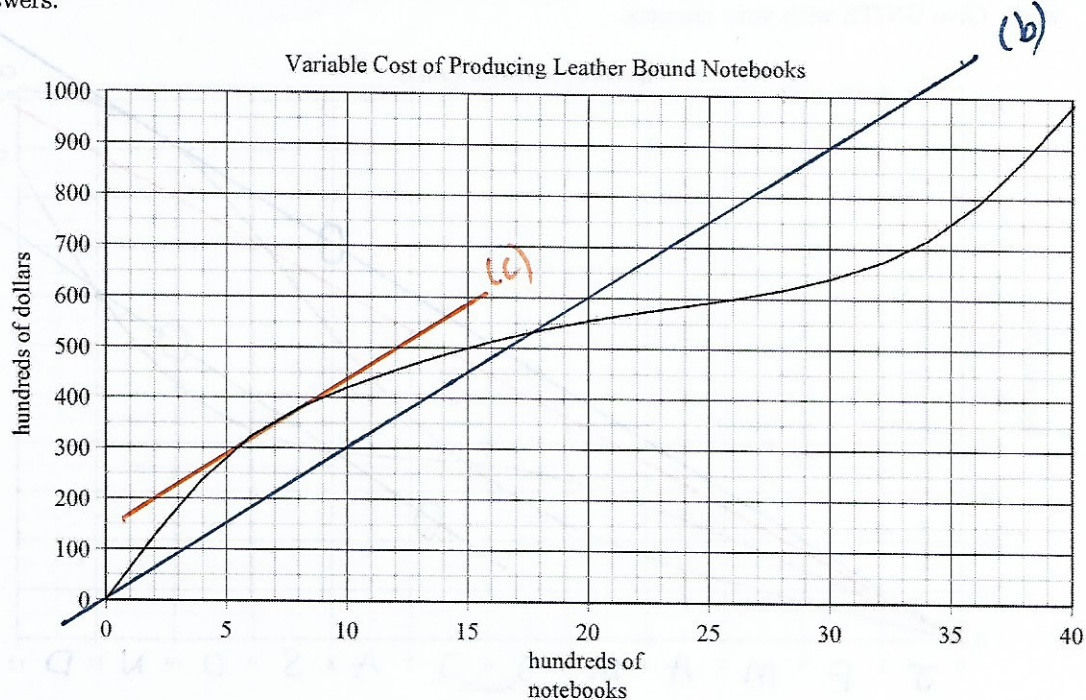
- (d) At what time was the Overall Rates of Change of books returned and books checked out the same?

$t \approx 6.3$ months

- (e) What is the maximum Overall Rate of Change of books checked out? Round your answer to the nearest thousand book.

$$\text{slope} = \frac{1610}{8} \approx 201 \text{ thousand books per month.}$$

2. The picture below shows the Variable Cost of producing handmade leather bound notebooks. The fixed costs are \$15,000. Answer the following questions based on the graph. LABEL any lines you draw on the graph with the letter of the question so we can follow your work. Give UNITS with your answers.



- (a) What is the Average Cost at 900 notebooks? Give your answer to the nearest cent.

$$AC(900) = \frac{TC(900)}{q} = \frac{VC(q) + FC}{q} \approx \frac{400 + 150}{9} \approx 61.11 \text{ dollars per notebook}$$

- (b) Assume you sell each notebook for 30 dollars. Graph Total Revenue above.

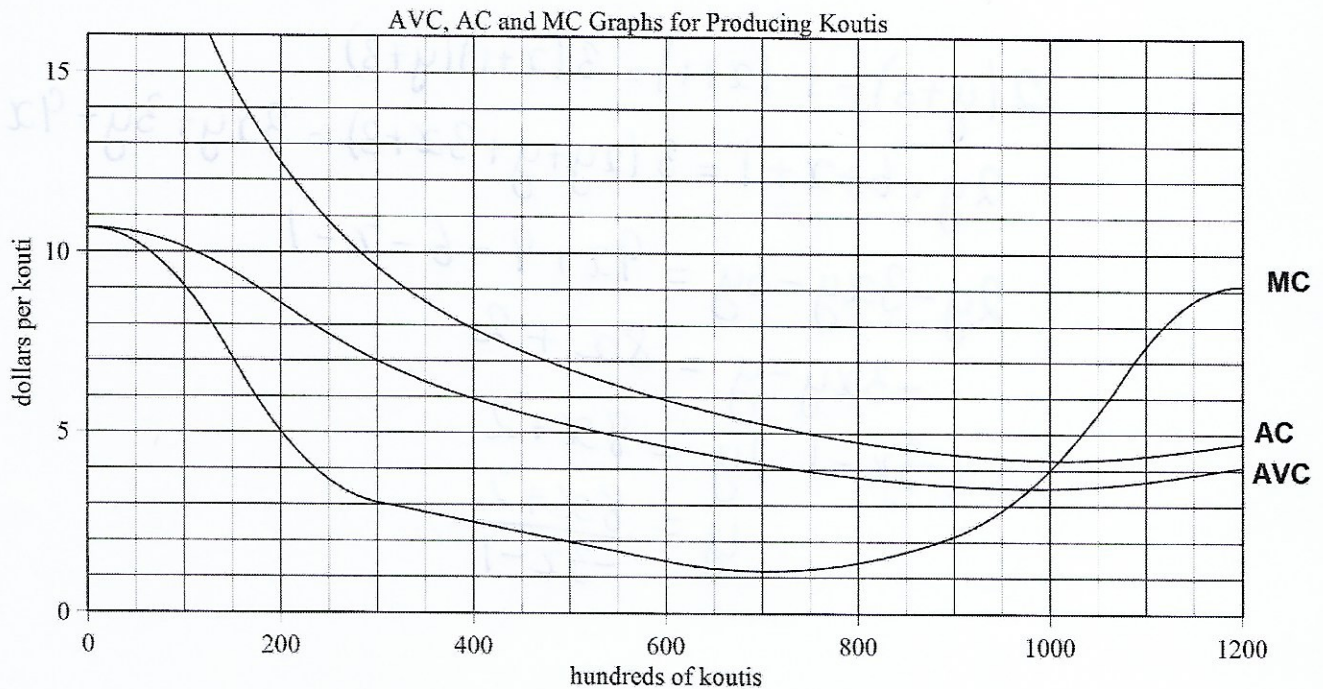
- (c) At what quantity do you have maximum loss? What is the maximum loss?

when $MC = MR = 30$ (+ $TR < TC$)
 at about 7 hundred notebooks
 $loss = TC - TR = VC(7) + FC - 7 \times 30$
 $\approx 350 + 150 - 210$
 $= 290 \text{ hundred dollars}$

- (d) At what quantity do you break even?

when $TR = TC = FC + VC$
 so when $TR - VC = FC = 150$
 at about 25 hundred notebooks

3. The following are graphs of Average Cost, Marginal Cost and Average Variable Cost for producing and selling Koutis. Answer the following questions based on this graph. Label any lines you draw on the graph with the letter of the question so we can follow your work. Include UNITS with your answers.



- (a) What is the Shutdown Price?

when $MC = AVC$ about 3.5 dollars per kouti

- (b) If you sell each Kouti for 7 dollars, at what quantity is the profit maximized?

when $MC = MR = 7$ about 1085 hundred koutis

- (c) What is the maximum profit?

$$\begin{aligned}
 P(1085) &= TR(1085) - TC(1085) \\
 &= 7 \times 1085 - 1085 \times AC(1085) \\
 &\approx 1085(7 - 4.3) \\
 &\approx 2930 \text{ hundred dollars}
 \end{aligned}$$

- (780) (d) Approximate the Fixed Cost.

using any q :

$$\begin{aligned}
 FC &= TC(q) - VC(q) \\
 &= q \times AC(q) - q \times AVC(q) \\
 &\approx 200 \times 12.5 - 200 \times 8.5 \\
 &= 800 \text{ hundred dollars}
 \end{aligned}$$

using $q = 200$

4. The two parts of this question are not related.

(a) Solve for y in terms of x :

$$\left(\frac{2}{x+1} + \frac{1}{y+3} = 3 \right) (x+1)(y+3)$$

$$2(y+3) + 1 \cdot (x+1) = 3(x+1)(y+3)$$

$$2y + 6 + x + 1 = 3(xy + y + 3x + 3) = 3xy + 3y + 9x + 9$$

$$2y - 3xy - 3y = 9x + 9 - 6 - x - 1$$

$$-3xy - y = 8x + 2$$

$$(-3x - 1)y = 8x + 2$$

$$y = \frac{8x + 2}{-3x - 1}$$

(b) Find the equation of the line through the points $(2, 3)$ and $(-1, 9)$. Give your answer in the form $y = mx + b$.

$$\text{slope} = \frac{9 - 3}{-1 - 2} = \frac{6}{-3} = -2$$

$$y - 3 = -2(x - 2)$$

$$y = -2x + 7$$