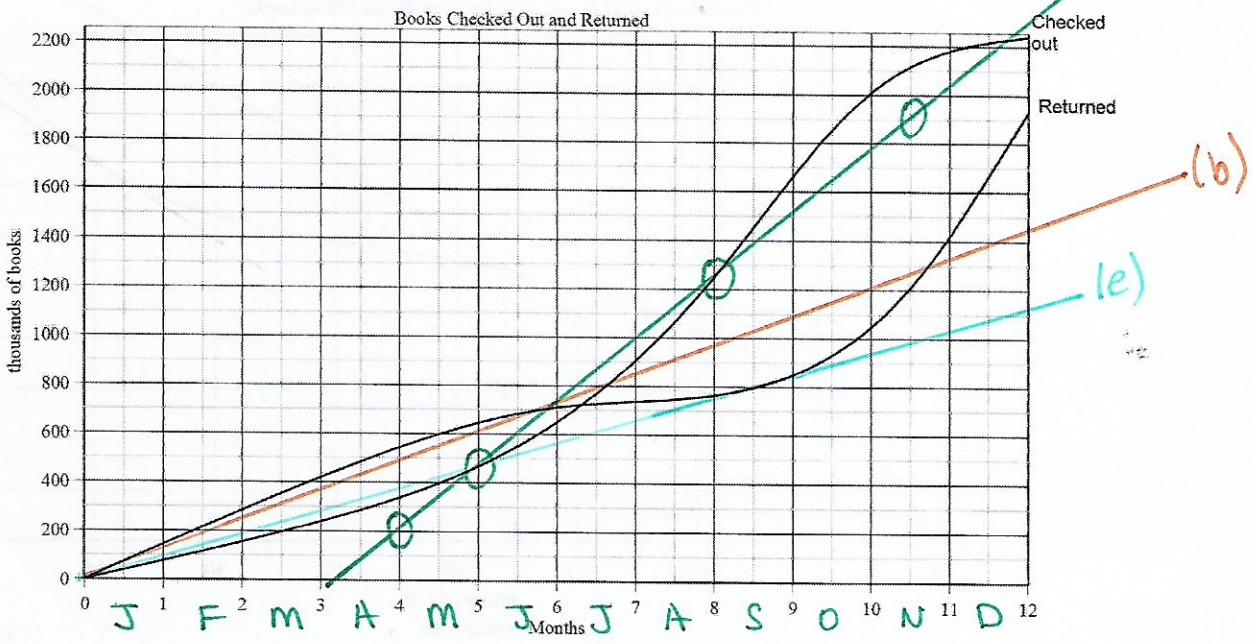
Solutions to Math 111 Fall 2019 midterm 1 (version 1)

1. The following shows graphs of the number of books checked out and returned to the Alexandria Library during the one year period starting in January 1, 2016. Answer the following questions based on these graphs. LABEL any lines you draw on the graph with the letter of the question so we can follow your work. Give UNITS with your answers.



(261) (a) Find the Average Rate of Change in number of books checked out during the months of June, July and August. Round your answer to the nearest thousand book.

$5 \le t \le 8$ $\frac{CO(8)-CO(5)}{8-5} \simeq \frac{1250-470}{3} \simeq 260$ thousand OR slope $\simeq \frac{1900-200}{10.5-4} \simeq 262$ thousand

(b) Find a time when the Overall Rate of Change of materials returned was 120,000 books per month.

$t \simeq 5.8$ or $t \simeq 10.7$

(c) If the library has eight million books, how many books were in the library at the beginning of October?

at $t=9$ $8000 - 1660 + 850 = 7190$ thousand books

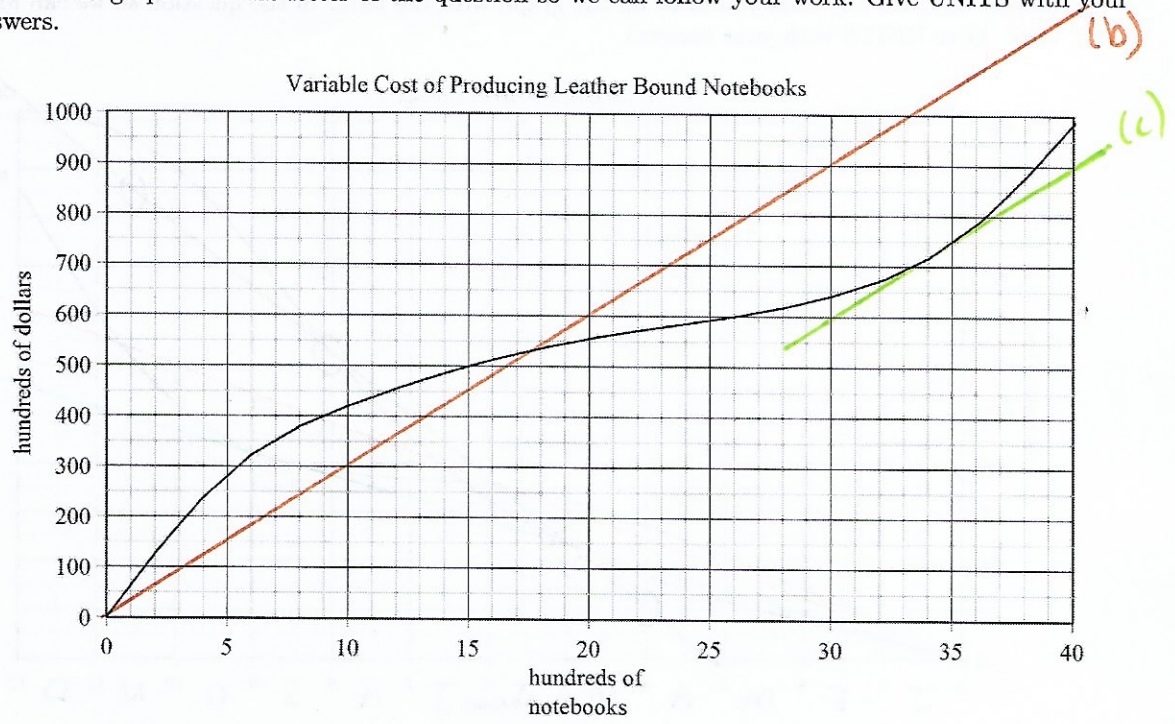
(d) At what time was the Overall Rates of Change of books returned and books checked out the same?

$t \simeq 6.3$ months

(e) What is the lowest Overall Rate of Change of books returned? Round your answer to the nearest thousand book.

using points $(1,100)$ & $(8.5,800)$ slope $= \frac{800-100}{8.5-1} \simeq 93$ thousand books

2. The picture below shows the Variable Cost of producing handmade leather bound notebooks. The fixed costs are \$15,000. Answer the following questions based on the graph. LABEL any lines you draw on the graph with the letter of the question so we can follow your work. Give UNITS with your answers.



(a) What is the Average Cost at 2600 notebooks? Give your answer to the nearest cent.

$AC(26) = \frac{TC(26)}{26} = \frac{VC(26)+FC}{26} \approx \frac{600+150}{26} \approx 28.85$ dollars per notebook

(b) Assume you sell each notebook for 30 dollars. Graph Total Revenue above.

(c) At what quantity do you have maximum profit? What is the maximum profit? (At what quantity do you have maximum loss? What is the maximum loss?)

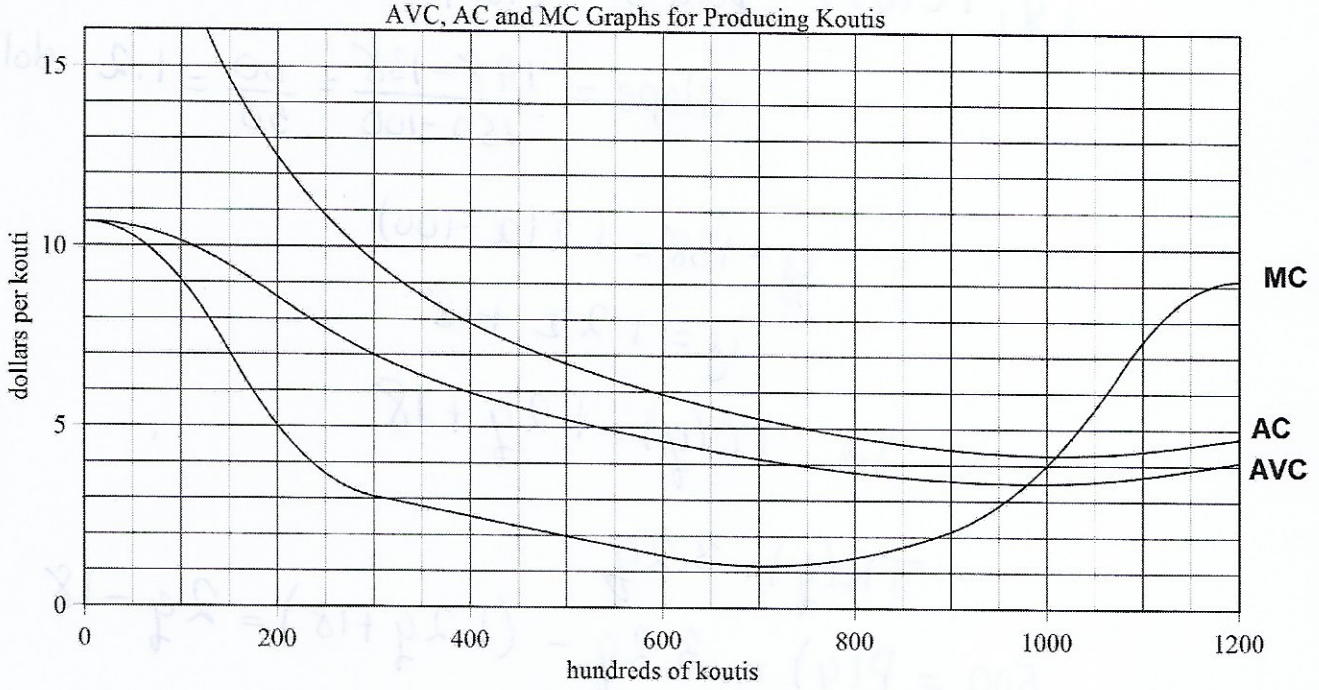
when $MR = MC$ (+ $TR > TC$)
at about 34.5 hundred notebooks

$P(34.5) = TR(34.5) - TC(34.5)$
$= 30 \times 34.5 - (VC(34.5) + 150)$
$\approx 1035 - 730 - 150 = 155$ hundred dollars

(d) At what quantity do you break even?

when $TR = TC = VC + FC$
so when $TR - VC = 150$
at about 25.1 hundred books.

3. The following are graphs of Average Cost, Marginal Cost and Average Variable Cost for producing and selling Koutis. Answer the following questions based on this graph. Label any lines you draw on the graph with the letter of the question so we can follow your work. Include UNITS with your answers.



(a) What is the Breakeven Price?

MC=AC about 4.2 dollars per Kouti

(b) If you sell each Kouti for 6 dollars, at what quantity is the profit maximized?

when MC=6 about 1065 hundred Koutis

(c) What is the maximum profit?

$P(1065) = TR(1065) - TC(1065)$
$= 1065 \times 6 - 1065 \times AC(1065)$
$\approx 1065(6-4.2) = 1917$ hundred dollars

(780) (d) Approximate the Fixed Cost.

using any q : $FC = TC(q) - VC(q)$
$= q \times AC(q) - q \times VC(q)$

using $q=200$ $\approx 200 \times 12.5 - 200 \times 8.5$
$= 800$ hundred dollars

4. The Total Cost of producing Things is given by a linear function. It costs 138 dollars to produce 100 things and 198 dollars to produce 150 Things. If you sell each Thing for \$3.20, at what quantity do you make a profit of 500 dollars?

$(q, TC(q))$ points $(100, 138)$ & $(150, 198)$

$$\text{slope} = \frac{198-138}{150-100} = \frac{60}{50} = 1.2 \text{ dollars per thing}$$

$$y - 138 = 1.2(x - 100)$$

$$y = 1.2x + 18$$

so $TC(q) = 1.2q + 18$

$TR(q) = 3.2q$

$$500 = P(q) = 3.2q - (1.2q + 18) = 2q - 18$$

$$518 = 2q$$

$259 \text{ things} = q$