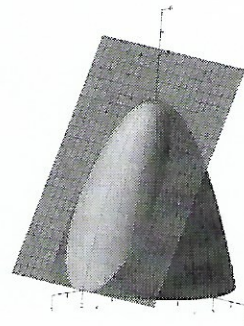1. (7 points) Set up an integral in **polar** coordinates to find the volume of the solid under the paraboloid

$$z = 4 - x^2 - y^2$$

and above the plane

$$z = 4 - 2x.$$

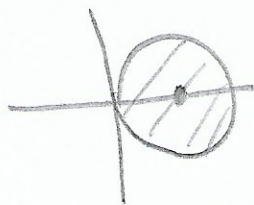A graph is given on the right to help you. Do **not** evaluate the integral.

D: intersect the surfaces

$$4 - x^2 - y^2 = 4 - 2x$$

$$0 = x^2 - 2x + y^2$$

$$1 = (x^2 - 2x + 1) + y^2$$

$$1 = (x-1)^2 + y^2$$

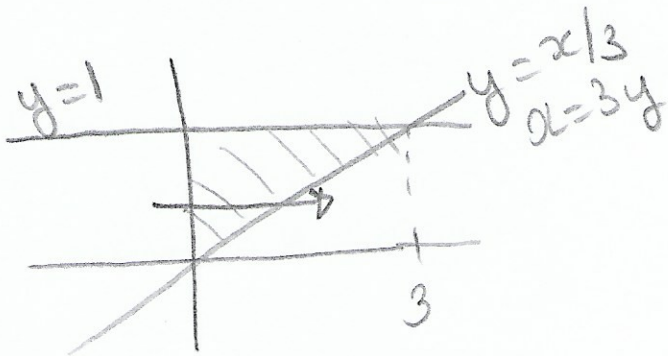polar: $r = 2\cos\theta$

$$\iint_D 4 - x^2 - y^2 - (4 - 2x)\, dA$$

$$= \int_0^\pi \int_0^{2\cos\theta} (2r\cos\theta - r^2)\, r\, dr\, d\theta$$

2. (10 points) Evaluate the integral

$$\int_0^3 \int_{x/3}^1 x^2 \sqrt{3+y^2}\, dy\, dx$$

by switching the order of integration.

region $\dfrac{x}{3} \le y \le 1$ $, 0 \le x \le 3$

$y=1$ $y=x/3$ $x=3y$

$$\int_0^1 \int_0^{3y} x^2 \sqrt{3+y^2}\, dx\, dy = \int_0^1 \frac{x^3}{3}\sqrt{3+y^2}\,\Big|_0^{3y}\, dy$$

$$= \int_0^1 9\, y^3 \sqrt{3+y^2}\, dy \qquad u = 3+y^2$$
$$\qquad\qquad du = 2y\, dy$$

$$= \frac{9}{2}\int_0^1 (u-3)\sqrt{u}\, du = \frac{9}{2}\int_3^4 u^{3/2} - 3u^{1/2}\, du$$

$$= \frac{9}{2}\left(\frac{2u^{5/2}}{5} - 2u^{3/2}\right)\Big|_3^4 = \frac{9}{2}\left[\left(\frac{2}{5}4^{5/2} - 2\cdot 4^{3/2}\right) - \left(\frac{2}{5}3^{5/2} - 2\cdot 3^{3/2}\right)\right]$$

$$= \frac{9}{2}\left[\left(\frac{64}{5} - 16\right) - \left(\frac{18}{5}\sqrt{3} - 6\sqrt{3}\right)\right] = \frac{9}{2}\left[\frac{-16 + 12\sqrt{3}}{5}\right]$$

$$= \underline{\frac{-72 + 54\sqrt{3}}{5}}$$

3. (13 points) Find the absolute maximum and absolute minimum values of the function

$$f(x,y) = x^3 - 3xy^2 + 2xy$$

on the closed and bounded domain $x \geq 0$, $y \geq 0$ and $x + y \leq 1$. You may round your answers to two digits after the decimal.

critical points:

$$f_x = 3x^2 - 3y^2 + 2y = 0$$

$$f_y = -6xy + 2x = 0$$

$$2x(1-3y) = 0$$

$x = 0$ OR $y = \frac{1}{3}$

$f_x = -3y^2 + 2y = 0$ $\qquad f_x = 3x^2 - \frac{1}{3} + \frac{2}{3} = 3x^2 + \frac{1}{3} = 0$

$y(-3y+2) = 0$ $\qquad\qquad$ no solution

$y = 0$ or $y = \frac{3}{2}$

$$\boxed{(0,0), (0, 3/2)}$$

Boundary

$g(y) = 0$

$x = 0$
$0 \leq y \leq 1$

$(0,0), \boxed{(1,0)}$

$y = 0$
$0 \leq x \leq 1$

$f(x) = x^3$
$f' = 3x^2 = 0$

$y = 1-x$
$0 \leq x \leq 1$

$f(x) = x^3 - 3x(1-x)^2 + 2x(1-x)$
$f' = 3x^2 - 3(1-x)^2 - 3x \cdot 2(1-x)(-1) + 2 - 4x$
$= 3x^2 - 3 - 3x^2 + 6x + 6x - 6x^2 + 2 - 4x$
$= -6x^2 + 8x - 1$

$x = \dfrac{-8 \pm \sqrt{64-24}}{-12} = \dfrac{-8 \pm 2\sqrt{10}}{-12}$

$= \dfrac{4 \pm \sqrt{10}}{6} \approx 1.2$ or 0.14

$\boxed{\left(\dfrac{4-\sqrt{10}}{6}, \dfrac{2+\sqrt{10}}{6}\right)}$ $\qquad \boxed{(0,1)} (1,0)$ $\qquad$ not in domain

$f(0,y) = 0$, $\quad f(1,0) = 1$ $\quad f\left(\dfrac{4-\sqrt{10}}{6}, \dfrac{2+\sqrt{10}}{6}\right) \approx f(0.14, 0.86)$

$\qquad\qquad$ max $\qquad\qquad\qquad\qquad\qquad \approx -0.07$

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ min.

4. Let
$$f(x, y) = \sin(xy - 1).$$

(a) (5 points) Compute all first and second partial derivatives for $f(x, y)$.

$f_x = y \cos(xy - 1)$

$f_{xx} = -y^2 \sin(xy - 1)$

$f_{xy} = \cos(xy - 1) - xy\sin(xy - 1)$

$f_y = x \cos(xy - 1)$

$f_{yy} = -x^2 \sin(xy - 1)$

$f_{yx} = \cos(xy - 1) - xy\sin(xy - 1)$

(b) (5 points) Use linear approximation at $(1, 1)$ to estimate $f(1.1, 1.2)$.

$f(1,1) = \sin 0 = 0$

$f_x(1,1) = 1 \cos 0 = 1$

$f_y(1,1) = 1 \cos 0 = 1$

$z - 0 = 1(x - 1) + 1(y - 1)$

$\boxed{z = x + y - 2}$

$f(1.1, 1.2) \approx 1.1 + 1.2 - 2 = 0.3$