Math 124, Winter 2022 Midterm II

February 22, 2022

Name_____

Student Number_____

Instructions.

- **These exams will be scanned. Please write your name and student number clearly for easy recognition.**
- There are 4 questions. The exam is out of 50 points.
- You are allowed to use one page of notes written only on one side of the sheet in your own handwriting.
- You can only use a Ti-30x IIS calculator. Unless otherwise stated, you have to give exact answers to questions. ($\frac{2\ln 3}{\pi}$ and $1/3$ are exact, 0.699 and 0.333 are approximations for the those numbers.)
- **Show your work**. If I cannot read or follow your work, I cannot grade it. **You may not get full credit for a right answer if your answer is not justified by your work.** If you continue a question on the last page, make a note for me.

1. (12 points)

(a) Differentiate $f(x) = 3e^{x^2} + 7\ln(1 + x^3) - \sec(6 + \sqrt{x})$.

(b) Differentiate $f(x) = \ln\left(\sin^3(4x) + 5\right)$.

(c) Differentiate $f(x) = (1 + x^2)^{\sqrt{x}}$.

2. (13 points) A curve is given implicitly by the equation

$$13x^2 - 8x^2y^2 + 9y^3 = 53.$$

(a) Compute y' in terms of x and y.

(b) Find the equation of the tangent line to this curve at the point $(1, 2)$.

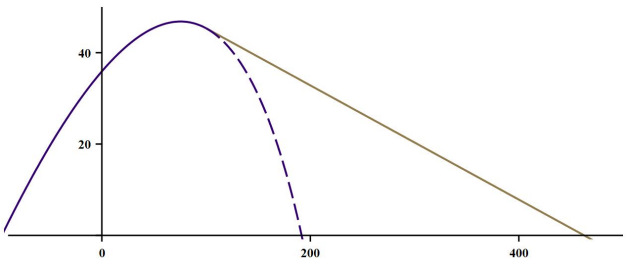
(c) Estimate the value of b if $(1.1, b)$ is on the curve.

(d) Find the value of y'' at the point $(1, 2)$.

3. (12 points) A magnetic field is set up so that a particle is going to travel on the parametric curve given by

$$x = -9t^2 + 116t - 96, \quad y = -\frac{t^3}{3} - 15t^2 + 54t, \quad t \geq 0.$$

However, the lab has not paid its electricity bill , so at $t = 2$, all power is cut off. Without any forces acting on it, instead of continuing on its curved path (dashed in picture) the particle keeps going on the tangent to its path as shown.



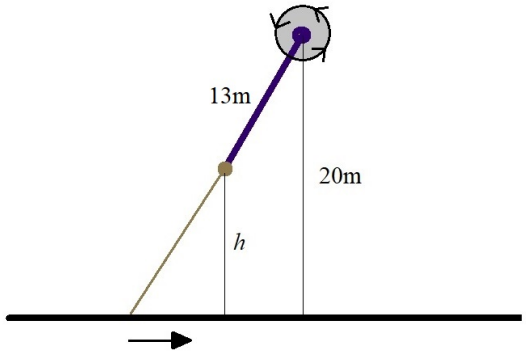
(a) Put arrows on the picture showing the direction of motion on the curve and the tangent line.

(b) If the power was not cut off, **when** would it have hit the x-axis. (The dashed path.)

(c) Since the power is cut off and it continues on the tangent line shown, **where** does it hit the x-axis?

(d) Given that its horizontal and vertical velocities will remain constant once the power is cut off, **when** will it hit the x-axis?

4. (13 points) One end of a 13 meter rod is fixed on a wall, at a spot 20 meters above the floor. The rod is rotating counterclockwise at an angular velocity of 5 radians per minute. At the end of the rod, there is a laser whose beam ends on the floor as shown. The picture is not to scale.



(a) How fast is the distance of the end of the rod from the floor, shown by h in the picture, decreasing when $h = 15$ meters?

(b) How fast is the beam on the floor moving at that time?

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