Math 124, Winter 2022 Midterm I

February 1, 2022

Name_____

Student Number_____

Instructions.

- **These exams will be scanned. Please write your name and student number clearly for easy recognition.**
- There are 4 questions. The exam is out of 50 points.
- You are allowed to use one page of notes written only on one side of the sheet in your own handwriting.
- You can only use a Ti-30x IIS calculator. Unless otherwise stated, you have to give exact answers to questions. ($\frac{2\ln 3}{\pi}$ and $1/3$ are exact, 0.699 and 0.333 are approximations for the those numbers.)
- **Show your work**. If I cannot read or follow your work, I cannot grade it. **You may not get full credit for a right answer if your answer is not justified by your work.** If you continue a question on the last page, make a note for me.

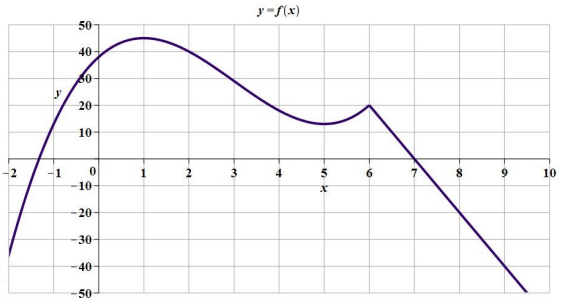
1. (15 points) Answer the following.

(a) Differentiate $f(x) = 3x^2 - \dfrac{1}{\sqrt{x}} + 5e^x$.

(b) Differentiate $g(t) = \dfrac{3t^2 - 1}{t\cos t + \sin t}$. *You do not need to simplify your answer, but be careful with your use of parentheses.*

(c) Use the *limit definition of the derivative* to compute $\dfrac{d}{dx}\sqrt{x+3}$.

(d) A graph of $y = f(x)$ with domain $(-2, 9.5)$ is given on the right.



(i) Use the graph arrange the following in *increasing order* from smallest to largest:

$$f'(-1), f'(0), f'(1), f'(3), f'(8)$$

(ii) At what value(s) of x is the derivative $f'(x)$ not defined?

2. (12 points) Answer the following. Show your (algebra) work or explain your reasoning.

(a) $\lim_{x \to 0} x^2 \sin\left(\frac{1}{x}\right)$

(b) $\lim_{t \to \infty} \frac{3 + t^2}{5t^2 + \sqrt{1 + 9t^4}}$

(c) $\lim_{h \to 0} \left(\frac{1}{h} - \frac{1}{h^2 + h}\right)$

(d) Is there a value of a so that $\lim_{x \to 2} \frac{x^2 + ax - 10}{x^2 - 3x + 2} = 7$? If yes, find it.

3. (14 points) The height of an object on Jupiter is given by

$$h(t) = -12.5t^2 + 80t + 12$$

where $t \geq 0$ is in seconds and the height is given in meters.

(a) Find the average velocity of the object during the first 2 seconds?

(b) What is the velocity of the object at $t = 2$? Is it going up or down?

(c) At what time, other than $t = 2$, does the object have the same speed as in part (b)?

(d) What is the acceleration due to gravity on Jupiter?

(e) What is the velocity of the object when it reaches its maximum height?

(f) What is the maximum height the object reaches?

4. (9 points) Find the two equations of the two tangent lines to the graph of

$$y = \frac{5x+1}{2x-1}$$

perpendicular to the line $9x - 7y = 19$.

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