Math 124 Spring 2022 Midterm II

May 17, 2022

Name_____

Student Number_____

Instructions.

- These exams will be scanned. **Please write your name and student number clearly for easy recognition.**

- There are four questions. The exam is out of 50 points.

- You are allowed to use one page of notes written only on one side of the sheet in your own handwriting.

- You can only use a Ti-30x IIS calculator. Unless otherwise stated, you have to give exact answers to questions. ($\frac{2\ln 3}{\pi}$ and $1/3$ are exact, 0.699 and 0.333 are approximations for the those numbers.)

- **Show your work**. If we cannot read or follow your work, we cannot grade it. **You may not get full credit for a right answer if your answer is not justified by your work.** If you continue a question on the last page, make a note for us.

1. (18 points) Find $\dfrac{dy}{dx}$ for the following equations.

(a) $y = \sin\left(4^x + \sqrt{1 - e^{2x}}\right)$

(b) $y = \left(x^2 + 5\right)^{\ln x}$

(c) $\left(3x^2 + 4y^2\right)^5 = 6x^2 - 7y^2$

2. (11 points) The curve on the right is traced by the parametric equations

$$x = 5t^2 - 5t + 1 \qquad y = 2t^3 - 4t^2 + t + 2.$$

The line ℓ_1 is tangent to the curve at $P(1, 2)$ and intersects the curve again at the point Q. The line ℓ_2 is tangent to the curve at the point Q. Find the equations of both tangent lines ℓ_1 and ℓ_2. *You have to use algebra and calculus to find the answers, but you can use the graph to check if your answers are reasonable.*
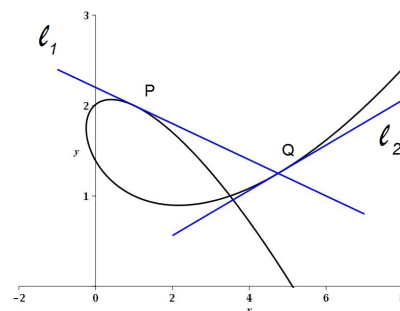
3. (9 points) Use *linear approximation* to estimate the value of $\sqrt{15}$.

4. (12 points) A tank is in the shape of a triangular prism, with the front and back isosceles triangles of base 1.2 meters and height 2.7 meters and length 5 meters as shown on the right. Water is being pumped into the tank at a rate of 4.5 cubic meters per minute. How fast is the depth of the water measured at its deepest rising when the total volume of water in the tank is 2.5 cubic meters?

1.2m

2.7m

5.0m
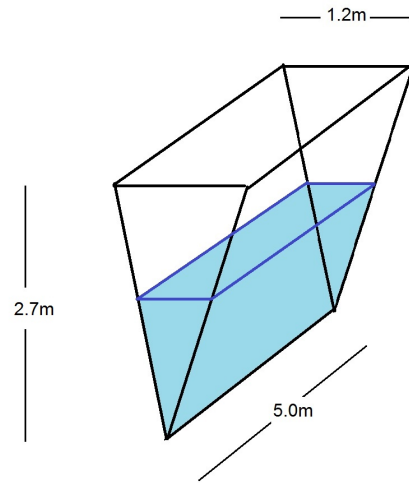
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