Math 124, Solutions to Spring 2022 Midterm I

1. (a) The numerator approaches 6 (which is positive), the denominator approaches 0 and when $x > 3$, it is positive, so the limit is ∞.

   (b) $$\lim_{x\to\infty}\left(3x-\sqrt{9x^2+4x-1}\right)\cdot\frac{3x+\sqrt{9x^2+4x-1}}{3x+\sqrt{9x^2+4x-1}}=\lim_{x\to\infty}\frac{9x^2-(9x^2+4x-1)}{3x+\sqrt{9x^2+4x-1}}=\lim_{x\to\infty}\frac{-4x+1}{3x+\sqrt{9x^2+4x-1}}$$
   $$=\lim_{x\to\infty}\frac{\frac{-4x+1}{x}}{\frac{3x+\sqrt{9x^2+4x-1}}{x}}=\lim_{x\to\infty}\frac{-4+\frac{1}{x}}{3+\sqrt{9+\frac{4}{x}-\frac{1}{x^2}}}=\frac{-4+0}{3+\sqrt{9+0-0}}=-\frac{2}{3}$$

   (c) $$f'(x)=\lim_{h\to 0}\frac{\frac{1}{x+h-2}-\frac{1}{x-2}}{h}\cdot\frac{(x+h-2)(x-2)}{(x+h-2)(x-2)}=\lim_{h\to 0}\frac{(x-2)-(x+h-2)}{h(x+h-2)(x-2)}$$
   $$=\lim_{h\to 0}\frac{(x-2)-(x+h-2)}{h(x+h-2)(x-2)}=\lim_{h\to 0}\frac{-h}{h(x+h-2)(x-2)}=-\lim_{h\to 0}\frac{1}{(x+h-2)(x-2)}=-\frac{1}{(x-2)^2}.$$

2. (a) $f'(x)=\dfrac{5e^x}{7}+\dfrac{3}{2\sqrt{x}}-143x^{12}+\pi x^{\pi-1}$

   (b) $g'(x)=\dfrac{(6x^2-5\sin x)(7x^{11}+13\sin x)-(2x^3+5\cos x)(77x^{10}+13\cos x)}{(7x^{11}+13\sin x)^2}$

   (c) $h'(x)=e^x\tan(x)+xe^x\tan(x)+xe^x\sec^2(x)$

3. (i) DNE, (ii) -1, (iii) $-\infty$, (iv) ∞, (v) -5, (vi)-1, (vii) $x=3,5$,
   (viii) $f'(2.5)<f'(0.5)<f'(0)<f'(8.7)<f'(5.1)$, (ix) $(-2,3)$ and $(5,\infty)$, (x) $x-2,3,5$

4. Let a be the x-coordinate of a point of tangency:

   $$\frac{4a^2+7-(-11)}{a-\left(-\frac{1}{2}\right)}=\text{SLOPE}=f'(a)=8a$$

   so

   $$4a^2+7+1=8a^2+4a$$

   which gives two solutions $a=-2$ and $a=1$.
   At the point $(-2,23)$ the tangent line is $y=-16x-9$.
   At the point $(1,11)$ the tangent line is $y=8x+3$.