Instructions.

- There are 4 questions. The exam is out of 40 points.

- If you finish early, check your work. You cannot leave until the end of class time. If you hand in your exam, you can read something else. You cannot use your phone or computer as long as you are in the classroom.

- You are allowed to use one page of notes written only on one side of the sheet in your own handwriting. Hand in your notes with your exam paper.

- You may use a calculator which does not graph and which is not programmable and which cannot take derivatives. Even if you have a calculator, give me exact answers. \( \frac{2 \ln 3}{\pi} \) is exact, 0.7 is an approximation for the same number.

- Show your work. If I cannot read or follow your work, I cannot grade it. You may not get full credit for a right answer if your answer is not justified by your work. If you continue at the back of a page, make a note for me. Please BOX your final answer.

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1. (10 points) Compute $\frac{dy}{dx}$ for the following.

(a) $y = \sqrt{1 + \sin^2(e^x)}$

$$y' = \frac{1}{2} \left(1 + \sin^2(e^x)\right)^{-\frac{1}{2}} \cdot 2\sin(e^x) \cdot \cos(e^x) \cdot e^x$$

(b) $\tan(x + y) - 4x^3 y^2 = e^{xy}$

$$\sec^2(x+y) \cdot (1 + y') - 12x^2 y^2 - 4x^3 2yy' = e^{xy}(y + xy')$$

$$\left(\sec^2(x+y) - 4x^3 2y - xe^{xy}\right)y' = -\sec^2(x+y) + 12x^2 y^2 + ye^{xy}$$

$$y' = \frac{-\sec^2(x+y) + 12x^2 y^2 + ye^{xy}}{\sec^2(x+y) - 8x^3 y - xe^{xy}}$$

(c) $y = (1 + x^2)^{5x}$

$$\ln y = \ln(1 + x^2)^{5x} = 5x \ln(1 + x^2)$$

$$\frac{y'}{y} = 5 \ln(1 + x^2) + 5x \cdot \frac{1}{1 + x^2} \cdot 2x$$

$$y' = \left[5 \ln(1 + x^2) + \frac{10x^2}{1 + x^2}\right] (1 + x^2)^{5x}$$
2. (10 points) The parametric curve

\[ x = 2t^2 - 40 \]
\[ y = t^3 - 12t \]

is graphed below together with a tangent line.

Find the coordinates of the point \( Q \) if the point \( P \) is at \((-38, -11)\). The line is tangent to the curve at the point \( P \).

\[ \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{3t^2-12}{4t} \]

When \( x = -38 = 2t^2 - 40 \)

\[ 2 = 2t^2 \]
\[ 1 = t^2 \]
\[ t = 1 \text{ or } -1 \]

Slope = \( \frac{dy}{dx} \bigg|_{t=1} = \frac{3-12}{4} = -\frac{9}{4} \)

Tangent Line

\[ y + 11 = -\frac{9}{4}(x + 38) \]

Intersect with the curve:

\[ (t^3 - 12t) + 11 = -\frac{9}{4}\left(2t^2 - 40 + 38\right) \]

\[ 4(t^3 - 12t + 11) = -9(2t^2 - 2) \]
\[ 4t^3 - 48t + 44 + 18t^2 - 18 = 0 \]
\[ 4t^3 + 18t^2 - 48t + 26 = 0 \]

Since \( t = 1 \) is a double root of this equation

\[ (t-1)^2(4t+26) = 0 \]
\[ t = \frac{-26}{4} \]
\[ t = \left(\frac{-26}{4}\right)^3 = -\frac{1573}{8} \]
\[ t = \left(\frac{26}{4}\right)^3 = \frac{1728}{8} \]
3. (10 points) Use linear approximation near \( x = 1 \) to estimate a root \( b \) of the polynomial

\[ f(x) = 4x^3 - x^2 + 3x - 10. \]

Is the approximate value of \( b \) you found more than or less than the actual value of \( b \)? Explain.

\[
\begin{align*}
f'(x) &= 12x^2 - 2x + 3 \\
f'(1) &= 12 - 2 + 3 = 13 \\
\end{align*}
\]

Tangent line:

\[ y - (-4) = 13(x - 1) \]

\[ y = 13x - 17 \]

Root: \( f(b) = 0 \)

Linear approximation:

\[ 0 \approx 13b - 17 \]

\[ b \approx \frac{17}{13} \]

\[ f'(1) = 13 > 0 \quad f \text{ increasing} \]

\[ f''(x) = 24x - 2 \]

\[ f''(1) = 24 - 2 > 0 \quad f \text{ concave up} \]
4. (10 points) There is a lighthouse on a small rocky island 2 kilometers from the dock. It has a rotating light. On a clear and sleepless night you observe that the light from the lighthouse sweeps your house at a rate of 4 meters per second moving towards the dock. If your beach house is 3 kilometers from the docks, how fast is the light of the lighthouse turning? Give your answer in radians per second and in revolutions per minute. Be careful with the units. The picture is not to scale.

\[ \frac{dx}{dt} = -4 \text{ m/s} \quad \frac{d\theta}{dt} = ? \]

\[ \tan \theta = \frac{x}{2000} \]

Differentiate:
\[ \sec^2 \theta \frac{d\theta}{dt} = \frac{1}{2000} \frac{dx}{dt} \]

When \( x = 3000 \text{ m} \):

\[ \text{hypotenuse} = \sqrt{2000^2 + 3000^2} = (\sqrt{13})(1000) \]

\[ \sec \theta = \frac{\sqrt{13}(1000)}{2000} = \frac{\sqrt{13}}{2} \]

So:
\[ (\frac{\sqrt{13}}{2})^2 \frac{d\theta}{dt} = \frac{1}{2000} \cdot 4 \]

\[ \frac{d\theta}{dt} = \frac{4 \cdot 4}{2000 \cdot 13} = \frac{1}{13(125)} = \frac{1}{1625} \text{ rad/s} \]

\[ = \frac{60}{(1625)2\pi} \text{ rev/min}. \]