Solutions to Math 124 G Fall 2023 Midterm II

1. (a) $\dfrac{dy}{dx} = \dfrac{15x^2}{2\sqrt{2+5x^3}} - 17(4e^x+9x)^{18} \cdot (4e^x+9)$

   (b) $y' = -\cos(\cos(\ln x)) \cdot \sin(\ln x) \cdot \dfrac{1}{x}$

   (c) Logaritmic differentiation:

$$\ln y = \ln\left((1+x^2)^{(1+\sin x)}\right) = (1+\sin x)\ln\left(1+x^2\right)$$

$$\frac{y'}{y} = (\cos x)\ln(1+x^2) + (1+\sin x)\frac{2x}{1+x^2}$$

$$y' = \left[(\cos x)\ln(1+x^2) + (1+\sin x)\frac{2x}{1+x^2}\right] \cdot (1+x^2)^{(1+\sin x)}$$

2. A curve is given by the implicit equation

$$xy^2 + \ln(1+x^2+y) = 2^x - 1.$$

   (a) Implicit Differentiation:

$$y^2 + x \cdot 2yy' + \frac{2x+y'}{1+x^2+y} = \ln 2 \cdot 2^x$$

   when $x=0$ and $y=0$

$$0 + 0 + \frac{0+y'}{1+0+0} = \ln 2 \cdot 2^0$$

   so $y' = \ln 2$ and the equation of the tangent line is $y = \ln 2 \cdot x$. Therefore, $0.3 \approx \ln 2 \cdot b$ and so

$$b \approx \frac{0.3}{\ln 2}.$$

   (b) Differentiate

$$y^2 + x \cdot 2yy' + \frac{2x+y'}{1+x^2+y} = \ln 2 \cdot 2^x$$

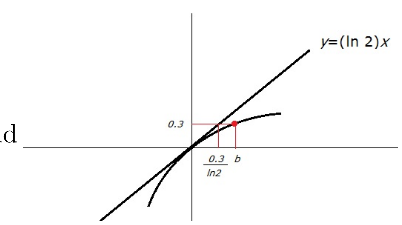
   to get

$$2yy' + 2yy' + 2xy'y' + 2xy'y'' + \frac{(2+y'')(1+x^2+y) - (2x+y')^2}{(1+x^2+y)^2} = (\ln 2)^2 \cdot 2^x.$$

   Now plug in $x=0$, $y=0$ and $y' = \ln 2$, so

$$(2+y'') - (\ln 2)^2 = (\ln 2)^2$$

   and $y'' = 2(\ln 2)^2 - 2 < 0$. So the graph at $(0,0)$ is concave down and increasing and therefore $\dfrac{0.3}{\ln 2} < b$.



3. (a) Point C has coordinates $(x(3), y(3)) \approx (165.07, 112.93)$. The slope is given by

$$\left.\frac{dy}{dx}\right|_{t=3} = \left.\frac{200(0.2)\cos(0.2t)}{-200(0.2)\sin(0.2t)}\right|_{t=3} \approx -1.46$$

so the tangent line has equation $y - 112.93 = -1.46(x - 165.07)$ or $y = -1.46x + 353.93$.

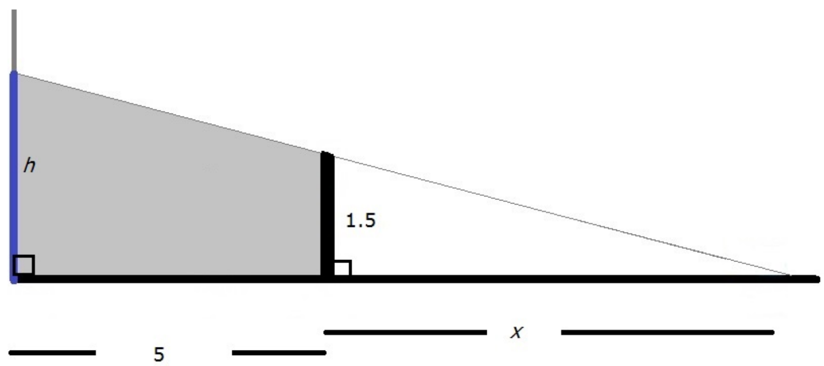
(b) When $x = 0$, $y = 353.93$ so $B(0, 353.93)$.

(c) Copper has angular speed 0.2 radians per second which gives a linear speed of $200(0.2) = 40$ meters per second. The distance from C to B is

$$\sqrt{(353.93 - 112.93)^2 + (0 - 165.07)^2} \approx 292.11$$

so it takes Copper $292.11/40 \approx 7.30$ seconds to get there.

4. Simpler picture:



$$\frac{dh}{dt} = 1.3 \qquad\qquad \frac{dx}{dt} = ?$$

Similar triangles:

$$\frac{h}{1.5} = \frac{5+x}{x}$$

so $xh = 1.5(5 + x)$. Implicit differentiation:

$$\frac{dx}{dt}h + x\frac{dh}{dt} = 1.5\frac{dx}{dt}.$$

From the equation $xh = 1.5(5 + x)$, when $h = 3$ we get $x = 5$. When $\frac{dh}{dt} = 1.3$,

$$3\frac{dx}{dt} + 5 \cdot 1.3 = 1.5\frac{dx}{dt},$$

so $\frac{dx}{dt} = -\frac{13}{3}$ m/s.