Math 124 G, Fall 2023 Midterm I

October 24, 2023

Name

Student Number

Instructions

- These exams will be scanned. **Please write your name and student number clearly.**

- There are 4 questions. The exam is out of 50 points.

- You are allowed to use one page of notes written only on one side of the sheet in your own handwriting. No printed material allowed. **Hand in your notes with your exam.**

- You can only use a Ti-30x IIS calculator. Unless otherwise stated, you have to give exact answers to questions. ($\frac{2\ln 3}{\pi}$ and $1/3$ are exact, 0.699 and 0.333 are approximations for those numbers.)

- Show your work. If we cannot read or follow your work, we cannot grade it. **You may not get full credit for a right answer if your answer is not justified by your work.**

1. (14 points) Differentiate the following functions. You do not have to simplify your answers, but make sure your use of parentheses is correct.

   (a) $f(x) = 3x^4 - \dfrac{\pi}{x} + x^{4\pi} - 7\sqrt{x^5}$

   (b) $g(x) = \left(3x^3 - 4x^4\right)\left(5\tan x - 6e^x + 9\right)$

   (c) $h(x) = \dfrac{3x^3 - 7x + 1}{x\cos x}$

2. (10 points) Evaluate the following limits. Your answer must be an exact number, ∞, $-\infty$ or DNE. Show appropriate work. If the limit does not exist, explain why not.

(a) $\displaystyle\lim_{t\to 0} \frac{2\cos t - \sqrt{4\cos^2 t + 13\sin^2 t}}{23\sin^2 t}$.
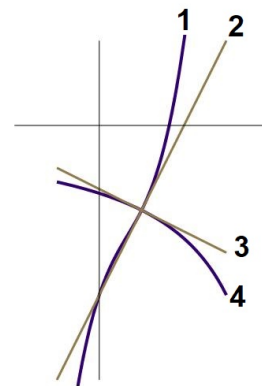
(b) $\displaystyle\lim_{t\to 7} \frac{t-7}{t^2 - 49}$

(c) $\displaystyle\lim_{x\to\infty} \frac{\sqrt{5x^2 + 1}}{7x + 8}$

3. (14 points) The tangent lines to the graphs of the functions

$$f(x) = x^3 - 2x^2 + 3x - 4 \quad \text{and} \quad g(x) = \frac{a}{b+x}$$

intersect at right angles at the point where $x = 1$ as shown on the right. Marking which graph belongs to which function is part of your question below.

(a) Find the equations of the two tangents.

(b) Find the values of a and b and write down the formula for the function $g(x)$.

(c) The graph of $y = f(x)$ is given by ☐ and the graph of $y = g(x)$ is given by ☐

The tangent line to $y = f(x)$ is ☐ and has equation ☐ .

The tangent line to $y = g(x)$ is ☐ and has equation ☐ .

4. (12 points) Below is the graph of the function $y = f(x)$. Answer the questions below and sketch a graph of the derivative $y = f'(x)$ on the axes provided. Note that the y-axes are not scaled, so we are not interested in the actual values of $f'(x)$, but its behavior. If you are not sure about the x-values, make your best guess.

y=f(x)

y=f'(x)
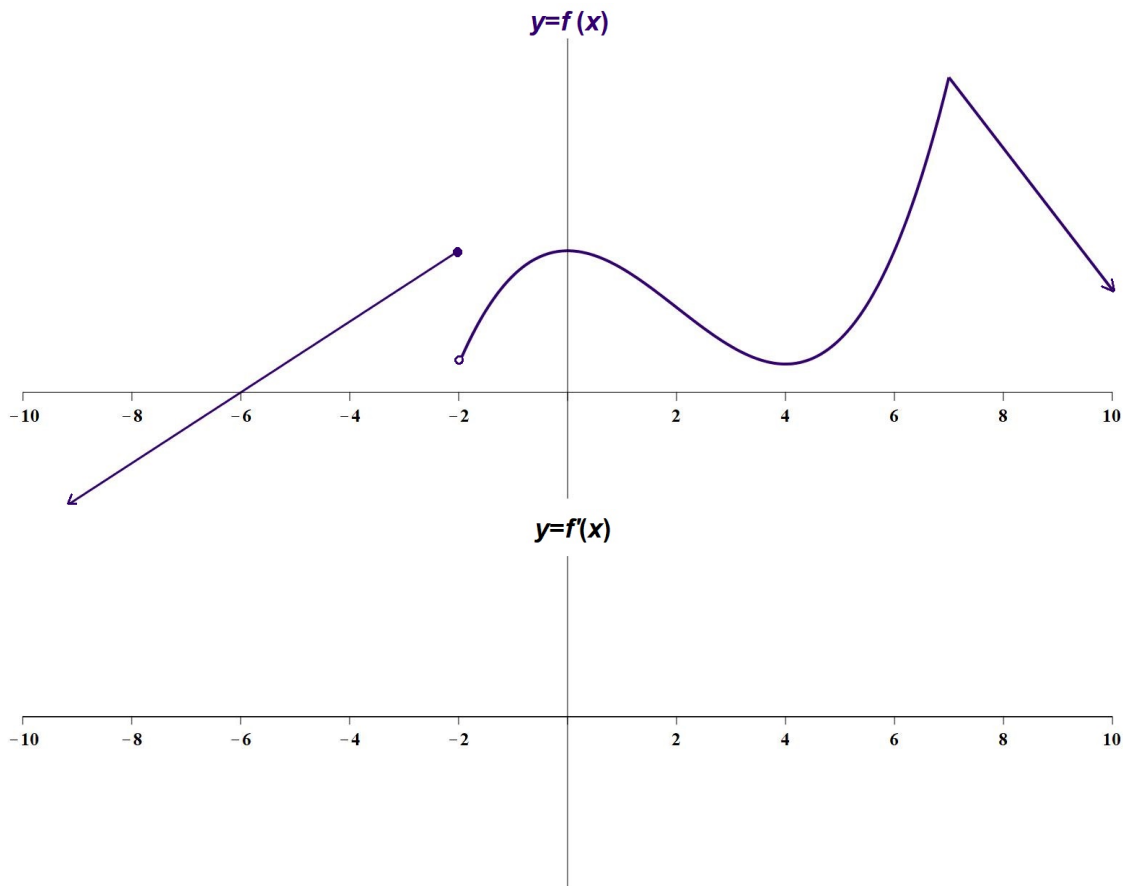
(a) List the x values where the function is not differentiable.

(b) List the x values where the derivative is zero.

(c) List the intervals where the derivative is positive.

(d) List the intervals where the derivative is increasing.

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