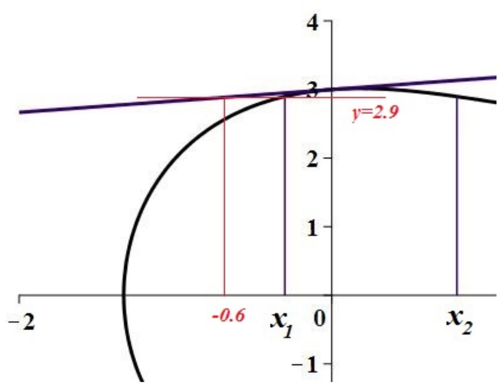
Math 124, Fall 2021 Solutions to Midterm II

1. (a) $f'(x) = \dfrac{10x}{5x^2+1} + 12e^{\cos x}\sin x + \dfrac{3x^2}{5(1+x^6)}$

   (b) $g'(t) = \dfrac{3 + \frac{5+\frac{\sqrt{2}}{\sqrt{t}}}{2\sqrt{5t+8}}}{2\sqrt{3t+\sqrt{5t+8}}} = \dfrac{1}{2}\left(3t+\sqrt{5t+8}\right)^{-1/2}\left(3+\dfrac{5}{2}(5t+8)^{-1/2}\right)$

   (c)
$$y = (1+\sin^2 x)^{\cos x}$$
$$\ln y = \ln\left((1+\sin^2 x)^{\cos x}\right) = (\cos x)\ln(1+\sin^2 x)$$
$$\frac{y'}{y} = -(\sin x)\ln(1+\sin^2 x) + \cos x \cdot \frac{2\sin x\cos x}{1+\sin^2 x}$$
$$h'(x) = \left(-(\sin x)\ln(1+\sin^2 x) + \frac{2\sin x\cos^2 x}{1+\sin^2 x}\right)(1+\sin^2 x)^{\cos x}$$

2. (a) Derivative
$$2yy' = 3x^2 - 6x + 1$$
when $x=0$, $y=3$
$$6y' = 1$$
so the tangent line is
$$y - 3 = \frac{1}{6}(x-0)$$
or $y = \frac{1}{6}x + 3$. Intersect with the curve to get
$$\left(\frac{1}{6}x+3\right)^2 = x^3 - 3x^2 + x + 9$$
simplifies to
$$0 = x^2\left(x - \frac{109}{36}\right)$$
since $x=0$ gives the point of tangency, the x-coordinate of point P is $\frac{109}{36}$.



   (b) Use the tangent line to get
$$2.9 \approx \frac{1}{6}a + 3$$
to get $a \approx -0.6$.

   (c) We approximated x_1. Our approximation is an underestimate, $-0.6 < x_1$, as shown on the picture.

   (d) Differentiating again
$$2y'y' + 2yy'' = 6x - 6$$
when $x=0$, $y=3$ and $y'=1/6$ we have
$$\frac{2}{36} + 6y'' = -6$$
so $y'' = -109/108$.

3. A particle is traveling on the xy-plane with parametric equations

$$x = t^2 + 5t + 2 \qquad \text{and} \qquad y = t^3 - 3t + 1.$$

(a)

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{3t^2 - 3}{2t + 5} = 0$$

when $3t^2 - 3 = 0$, so $t = \pm 1$.
When $t = 1$, the point is $(8, -1)$. When $t = -1$, the point is $(-2, 3)$.

(b)

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\frac{dy}{dx} = \frac{\frac{d}{dt}\frac{dy}{dx}}{dx/dt} = \frac{\frac{d}{dt}\frac{3t^2-3}{2t+5}}{2x+5} = \frac{\frac{6t(2t+5)-(3t^2-3)\cdot 2}{(2t+5)^2}}{2x+5} = \frac{6t^2 + 30t + 6}{(2t+5)^3}$$

(c) At $t = 1$, $x(1) = 8$, and $x'(1) = 2 + 5 = 7$ so

$$x = 7t + 1.$$

At $t = 1$, $y(1) = -1$ and $y'(1) = 0$ so

$$y = -1.$$

4. (a) Using similar triangles

$$\frac{h}{20} = \frac{y}{8}$$

so

$$\frac{1}{20}\frac{dh}{dt} = \frac{1}{8}\frac{dy}{dt}$$

So, $\frac{dh}{dt} = 0.75$ meters per second.

(b) Using similar triangles again

$$\frac{x+h}{20} = \frac{y+4}{8}$$

so

$$\frac{1}{20}\left(\frac{dx}{dt} + \frac{dh}{dt}\right) = \frac{1}{8}\frac{dy}{dt}$$

so $dx/dt = 0$.